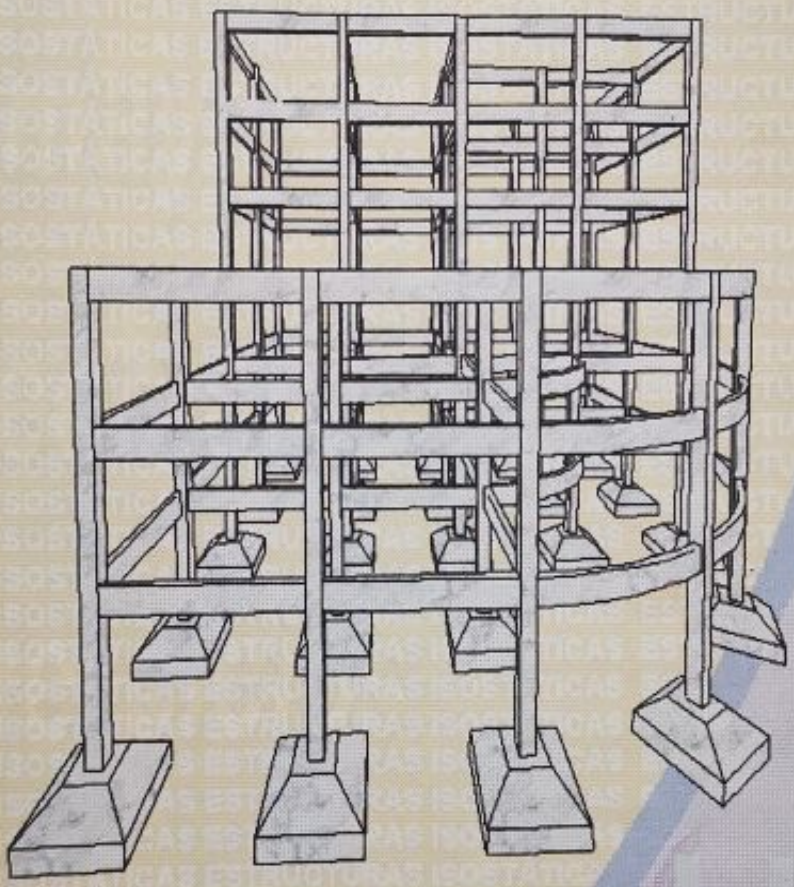
Estructuras Isostáticas

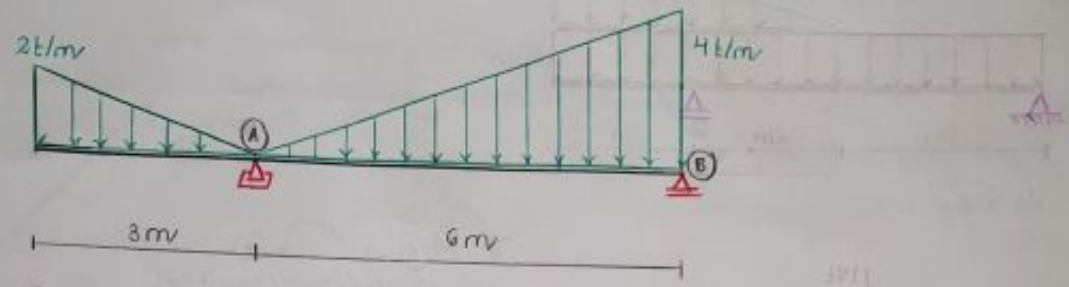
CIENTO POR CIENTO PRÁCTICO



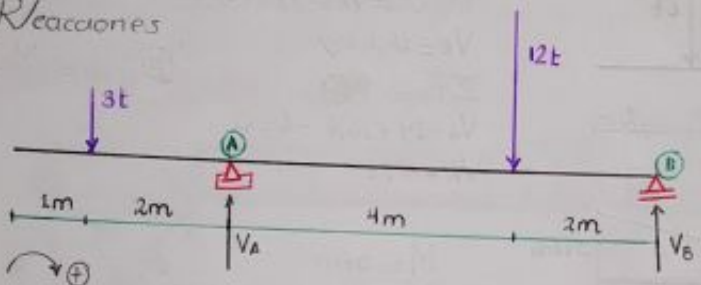
La mayor colección de ejercicios prácticos

Tomás Wilson Alemán Ramírez

13 Graficar los diagramas de esfuerzos caracteristicas utilizando el método gráfico - numérico.



1.- Calculo de Reacciones



$\Sigma M_{(A)=0}$ ⊕

$-3(2)+12(4)-V_B(6)=0$

$V_B = 7t$ //

$\Sigma F_V = 0$ ↑ ⊕

$-3+V_A-12+7=0$

$V_A = 8t$ //

2.- Momentos

$M_A = -3(2)$

$M_A = -6tm$

$M_B = 0tm$

3.- Cortantes

$Q_A = -3t$ (Izq)

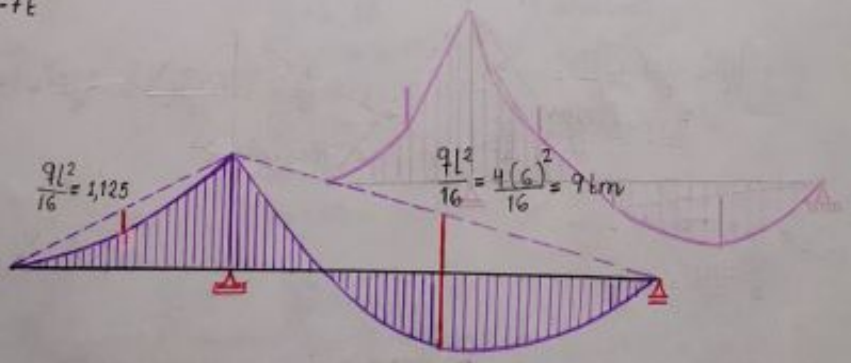
$Q_A = -3+8 = 5t$ (Der)

$Q_B = -7t$

4.- Diagramas

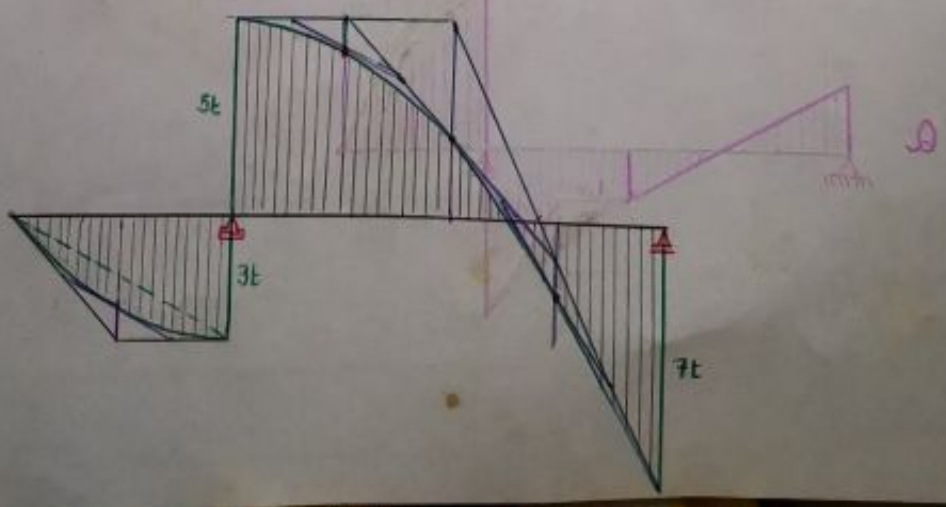
-Momento

Escala = 4tm/cm

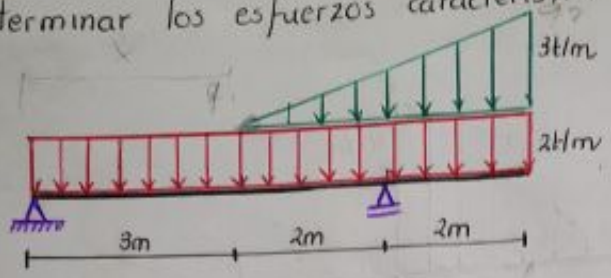


Cortante

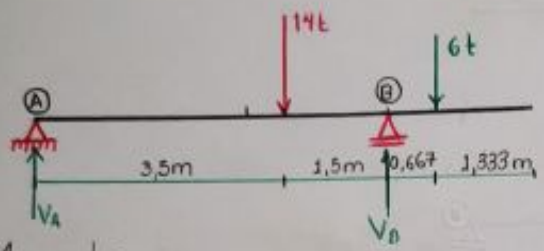
Escala = 2tm/cm



14.- Determinar los esfuerzos característicos



1.- Cálculo de Reacciones



$\sum M_{(A)} = 0$ ↺ (+)

$14(3,5) - V_B(5) + 6(5,667) = 0$

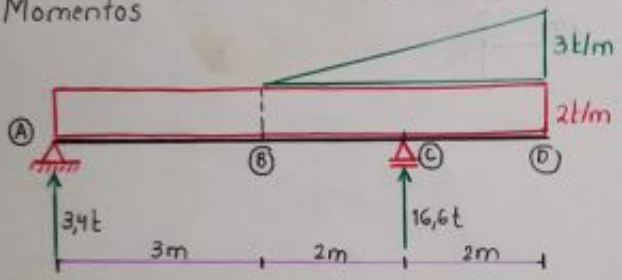
$V_B = 16,6\,t$ //

$\sum F_V = 0$ ↑ (+)

$V_A - 14 + 16,6 - 6 = 0$

$V_A = 3,4\,t$

2.- Momentos



$M_A = 0\,tm$

$M_B = 3,4(3) - 2(3)(1,5) = 1,2\,tm$

$M_C = 3,4(5) - 2(5)(2,5) - \frac{1,5(2)}{2}\left(\cdot\frac{1}{3}\cdot 2\right) = -9\,tm$

$M_D = 0\,tm$

3.- Cortantes

$Q_A = 3,4\,t$

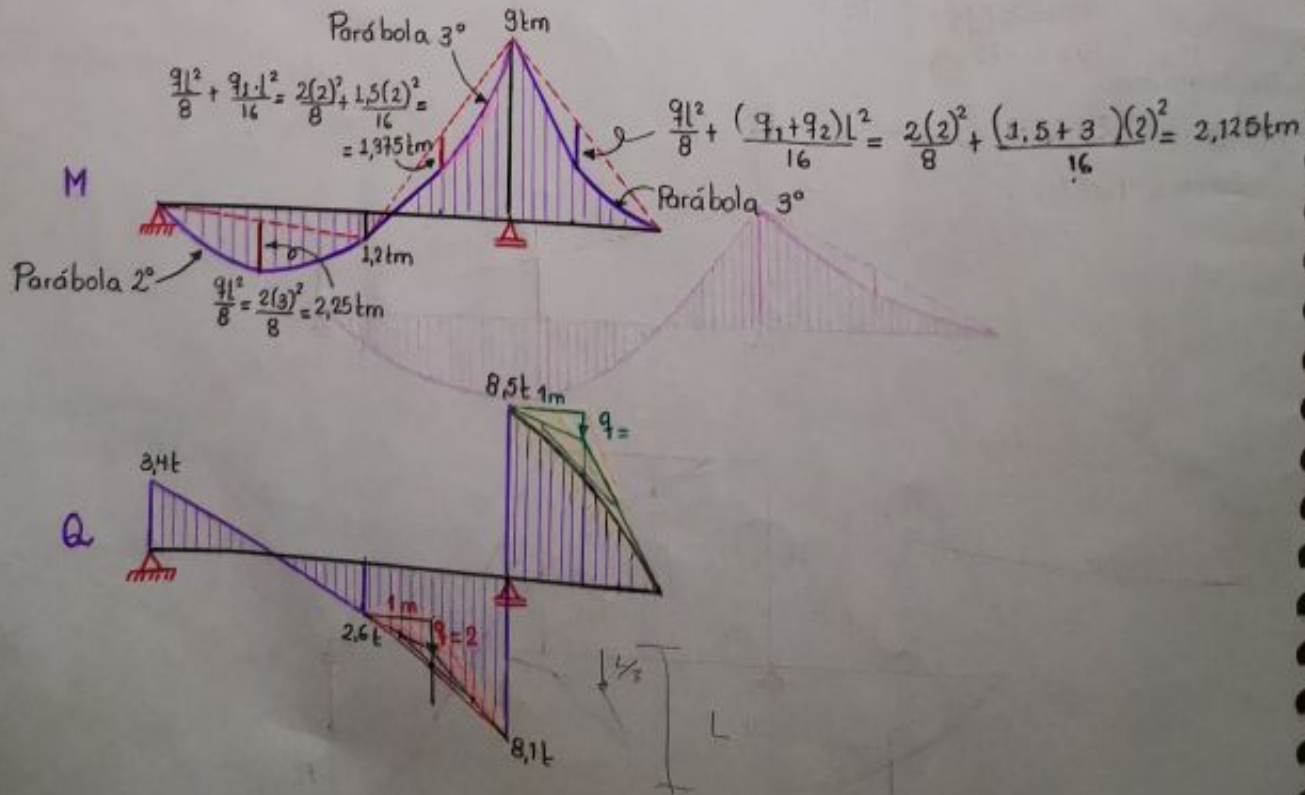
$Q_B = 3,4 - 2(3) = -2,6\,t$

$Q_{C(Izq)} = 3,4 - 2(5) - \frac{1,5(2)}{2} = -8,1\,t$

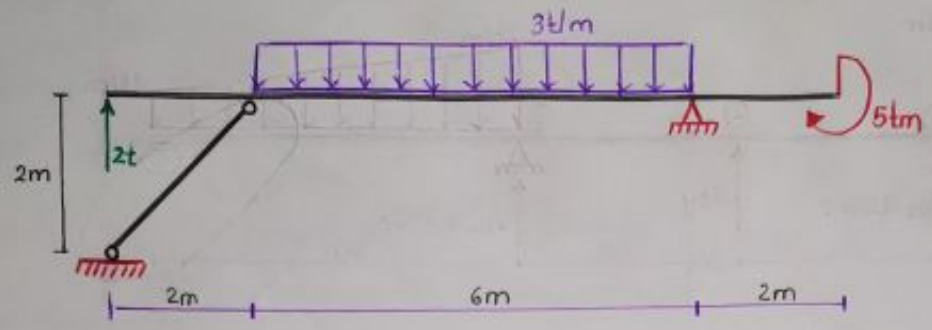
$Q_{C(Der)} = Q_{C(Izq)} + 16,6 = -8,1 + 16,6 = 8,5\,t$

$Q_D = 0\,t$

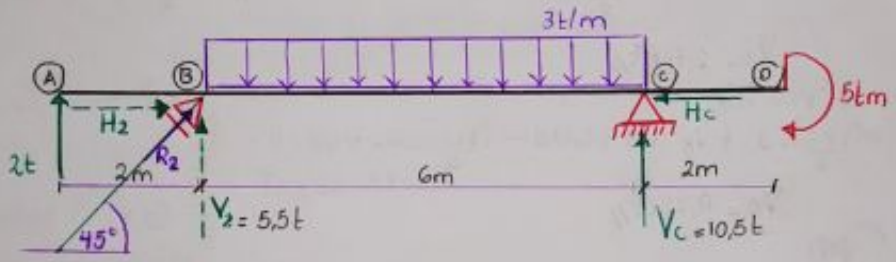
4.- Diagramas



15.- Graficar los diagrama de Momento, cortante y normal



1.- Idealizacion de apoyo



2- Reacciones

$\Sigma M_B = 0$ ↻ ⊕ $\quad 2(2) + 3(6)(3) + 5 - V_c(6) = 0$

$V_c = 10,5t$

$\Sigma F_V = 0$ ↑ ⊕ $\quad 2 + V_2 - 3(6) + 10,5$

$V_2 = 5,5t$ //

$\text{Sen}45° = \frac{V_2}{R_2} \Rightarrow R_2 = V_2/\text{Sen}45$

$R_2 = 5,5/\text{Sen}45 = 7,778t$ //

$H_2 = R_2 \cos 45 = 5,5t$

$\Sigma F_H = 0$ → ⊕ $\quad +5,5 - H_c = 0$

$H_c = 5,5t$ //

3.-Momentos

$M_A = 0$

$M_B = 2(2) = 4tm$

$M_C = -5\,tm$

$M_D = -5tm$

4.- Cortantes

$Q_A = 2t$

$Q_{B(IZQ)} = 2t$

$Q_{B(DER)} = 2 + 5,5 = 7,5t$

$Q_{C(IZQ)} = 2 + 5,5 - 3(6) = -10,5t$

$Q_{C(DER)} = -10,5 + 10,5 = 0t$

$Q_D = 0t$

5 - Normales

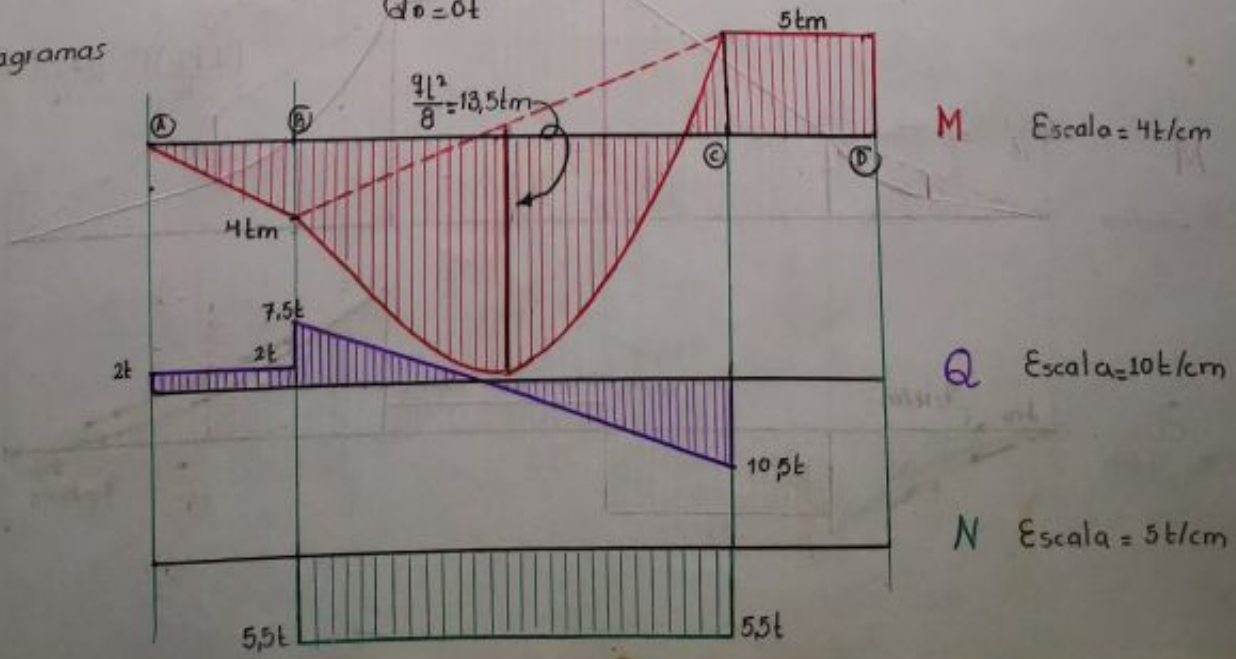
$N_A = 0$

$N_B = -5,5t$

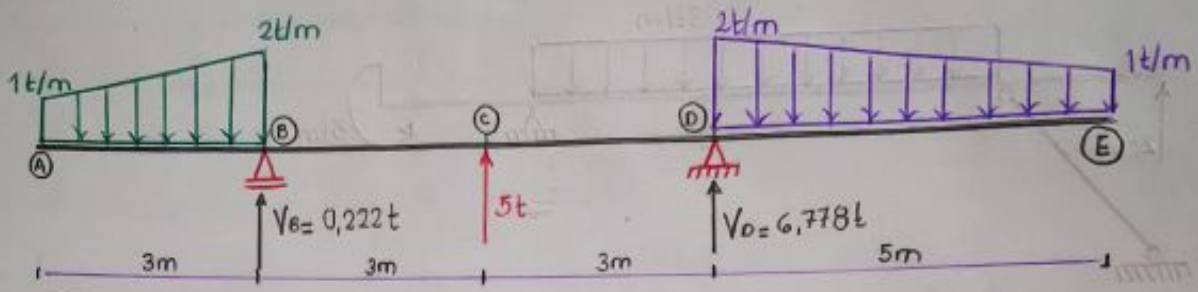
$N_C = -5,5t$

$N_D = 0t$

6 -Diagramas



16.- Para la siguiente viga obtener las diagrama de Momento y cortante



1.- Reacciones

$$\Sigma M_B = 0 \curvearrowright +$$

$$-1(3)(1,5) - \frac{1(3)}{2}(1) - 5(3) - V_D(6) + 1(5)(8,5) + \frac{1(5)}{2}\left(6 + \frac{1}{3}\cdot 5\right) = 0$$

$$V_D = 6,778t //$$

$$\Sigma F_V = 0 \uparrow +$$

$$-\frac{(1+2)}{2}.3 + V_B + 5 + 6,778 - \frac{(1+2)}{2}.5 = 0$$

$$V_B = 0,222t //$$

2- Momento $\curvearrowright +$

$M_A = 0tm$

$M_B = -1(3)(1,5) - \frac{1(3)}{2}(1) = -6tm$

$M_C = -1(3)(4,5) - \frac{1(3)}{2}(4) + 0,222(3) = -18,834tm$

$M_D = -1(5)(2,5) - \frac{1(5)}{2}.\left(\frac{5}{3}\right) = -16,667\ tm$

3.- Cortantes $\uparrow +$

$Q_A = 0t$

$Q_{B(IZQ)} = -\frac{(1+2)}{2}.3 = -4,5t$

$Q_{B(DER)} = -4,5 + 0,222 = -4,278t$

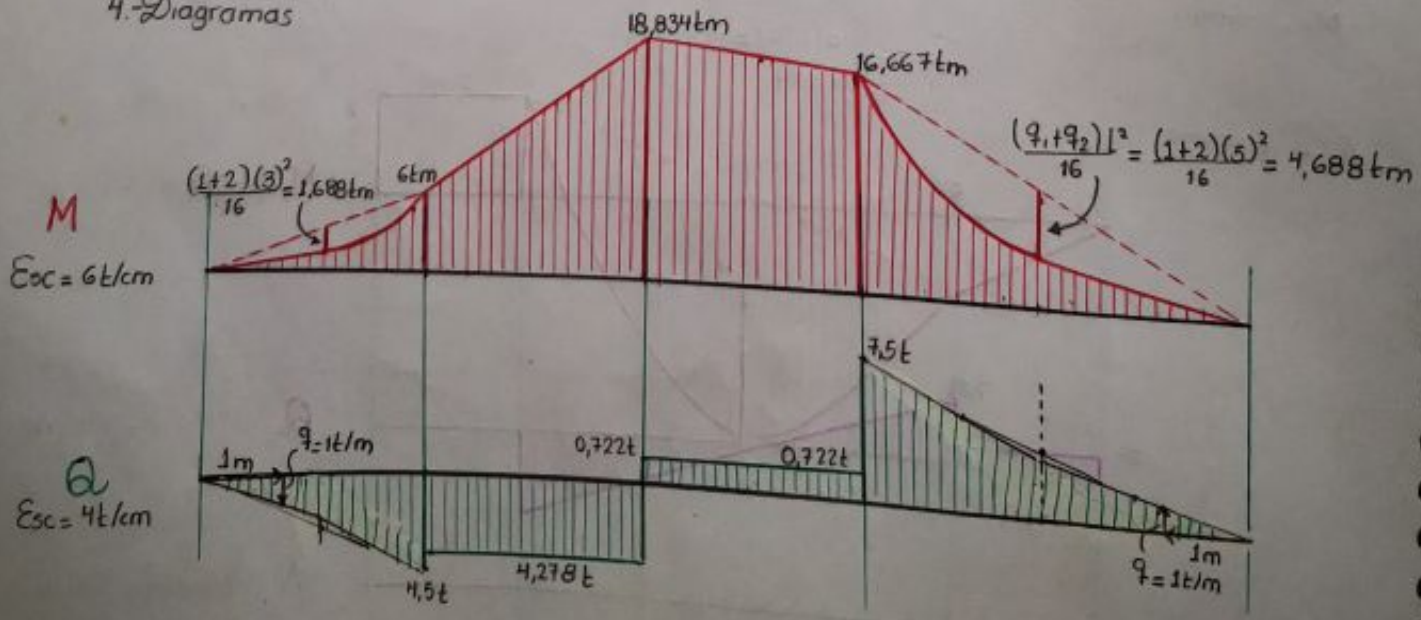
$Q_{C(IZQ)} = -4,278t$

$Q_{C(DER)} = -4,278 + 5 = 0,722t$

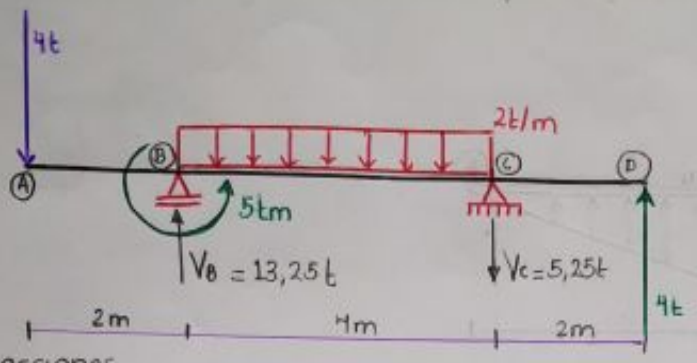
$Q_{D(IZQ)} = 0,722t$

$Q_{D(DER)} = \frac{(1+2)}{2}.5 = 7,5t$

4.- Diagramas



17.- Diagramar los esfuerzos caracteristicos



1.- Reacciones

$\Sigma M_{B=0}$ ↻ ⊕

$-4(2) - 5 + 2(4)(2) + V_C(4) - 4(6) = 0$

$V_C = 5{,}25t$

$\Sigma F_{V=0}$ ↑ ⊕

$-4 + V_B - 2(4) - 5{,}25 + 4 = 0$

$V_B = 13{,}25t$

2.- Momentos ↻ ⊕

$M_{A=0tm}$

(IZQ) $M_B = -4(2) = -8tm$

(DER) $M_B - 8 - 5 = -13tm$

$M_C = 4(2) = 8tm$

3.- Cortantes ↑ ⊕

$Q_{A=-4t}$

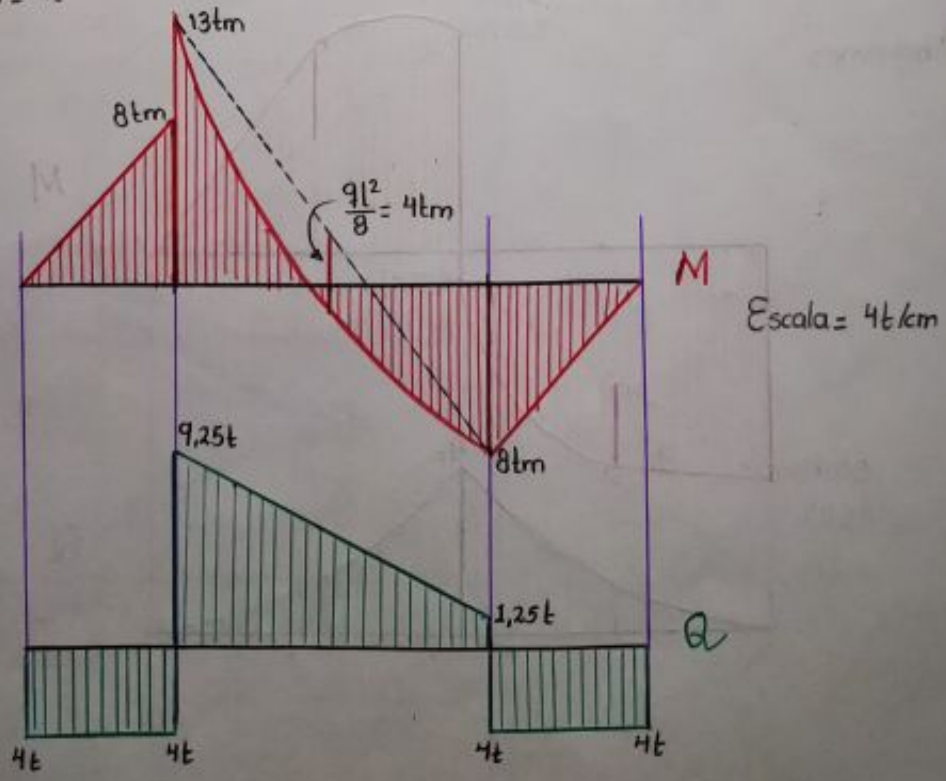
$Q_{B(IZQ)} = -4t$

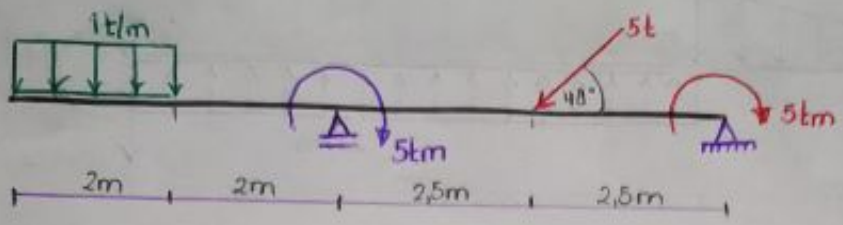
$Q_{B(DER)} = -4 + 13{,}25 = 9{,}25t$

$Q_{C(IZQ)} = -4 + 5{,}25 = 1{,}25t$

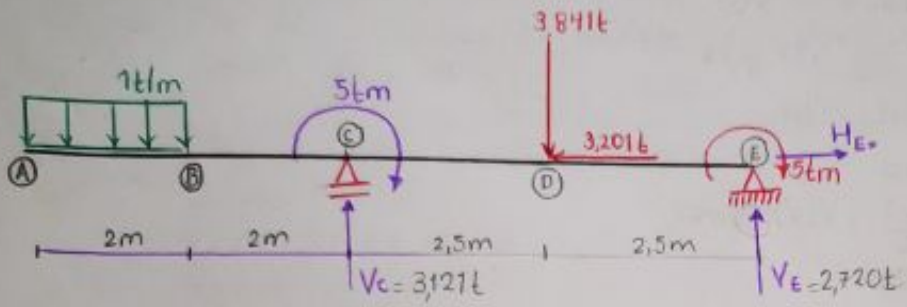
$Q_{C(DER)} = -4t$

$Q_D = -4t$





1- Cálculo de Reacciones



$\Sigma M_{(C)} = 0$ ↷ (+)

$-1(2)(3) + 5 + 3,841(2,5) + 5 - V_E(5) = 0$

$V_E = 2,720t$

$\Sigma F_V = 0$ ↑(+)

$-1(2) + V_C - 3,841 + 2,7205$

$V_C = 3,121t$

$\Sigma F_H = 0$ → (+)

$-3,201 + H_E = 0$

$H_E = 3,201t$

2- Momentos ↷ (+)

$M_A = 0$

$M_B = -1(2)(1) = -2tm$

$M_{C(IZQ)} = -1(2)(3) = -6tm$

$M_{C(DER)} = -6 + 5 = -1tm$

$M_D = -1(2)(5,5) + 5 + 3,121(2,5) = 1,803\,tm$

$M_E = -5tm$ (de derecha a izquierda)

3.-Cortante ↑(+)

$Q_A = 0$

$Q_B = -2t$

$Q_{C(IZQ)} = -2t$

$Q_{C(DER)} = -2 + 3,121 = 1,121t$

$Q_{D(IZQ)} = 1,121t$

$Q_{D(DER)} = 1,121 - 3,841 - 2,720t$

$Q_E = -2,720t$

4.- Normales

$N_A = 0$

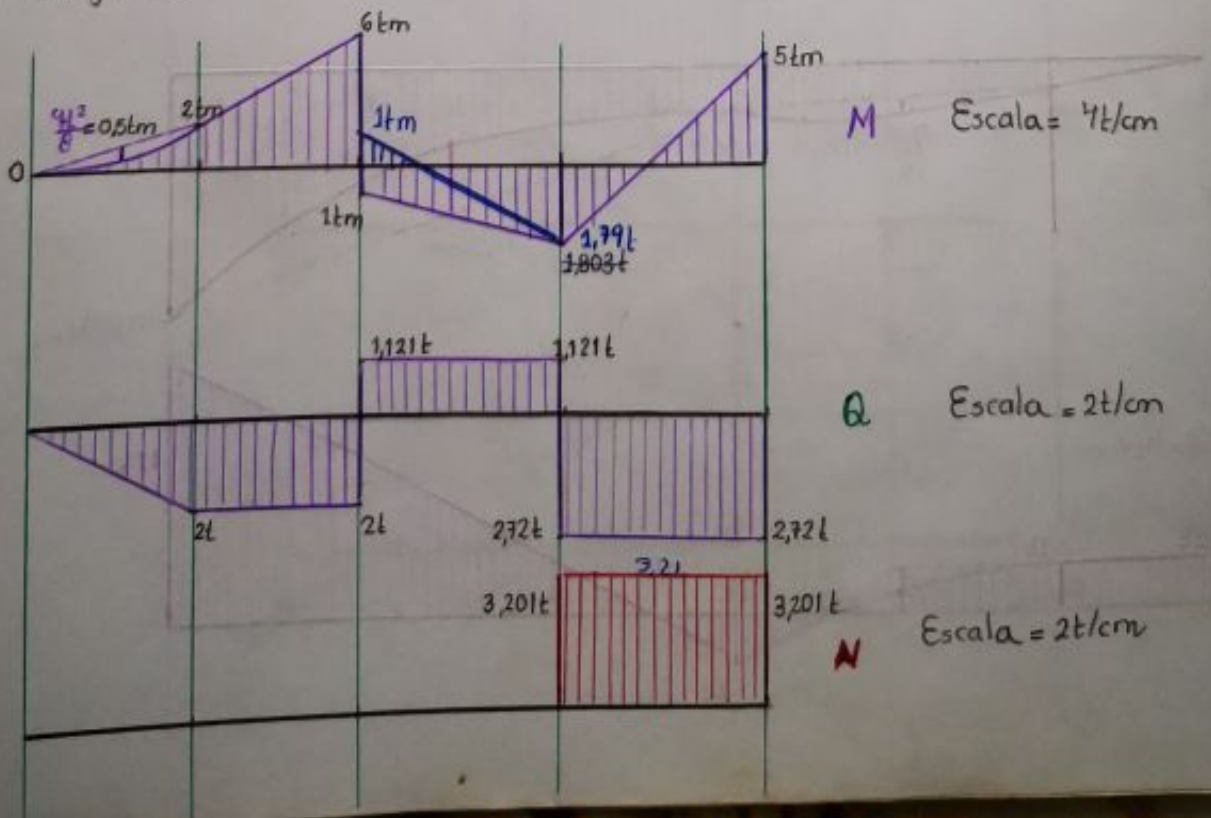
$N_B = 0$

$N_C = 0$

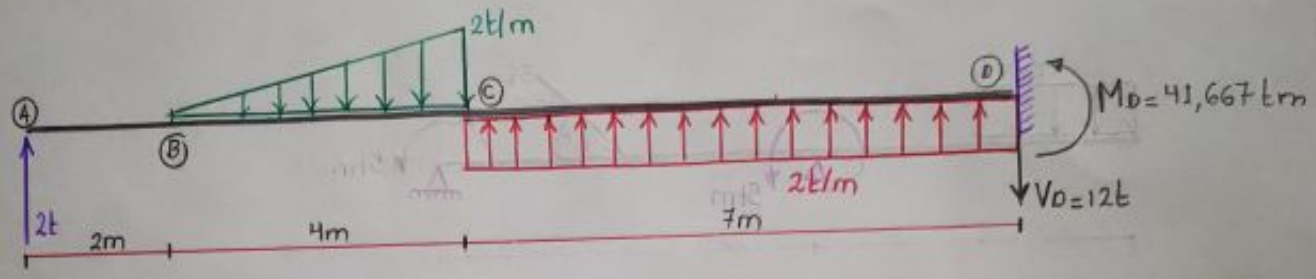
$N_D = 3,201t$

$N_E = 3,201t$

5.- Diagramas



20 Determinar los esfuerzos característicos



1- Cálculo de Reacciones

$\Sigma M_{D=0}$ ↺ (+)

$2(13) - \frac{2(4)}{2}\left(7+\frac{4}{3}\right) + 2(7)(3,5) - M_D = 0$

$M_D = 41,667\ tm$

$\Sigma F_{V=0}$ ↑ (+)

$2 - \frac{2(4)}{2} + 2(7) - V_D = 0$

$V_D = 12t$

2.- Momentos ↺ (+)

$M_{A=0}$

$M_B = 2(2) = 4\ tm$

$M_C = 2(6) - \frac{2(4)}{2}\left(\frac{4}{3}\right) = 6,667\ tm$

$M_D = 41,667\ tm$

3.- Cortantes ↑ (+)

$Q_A = 2t$

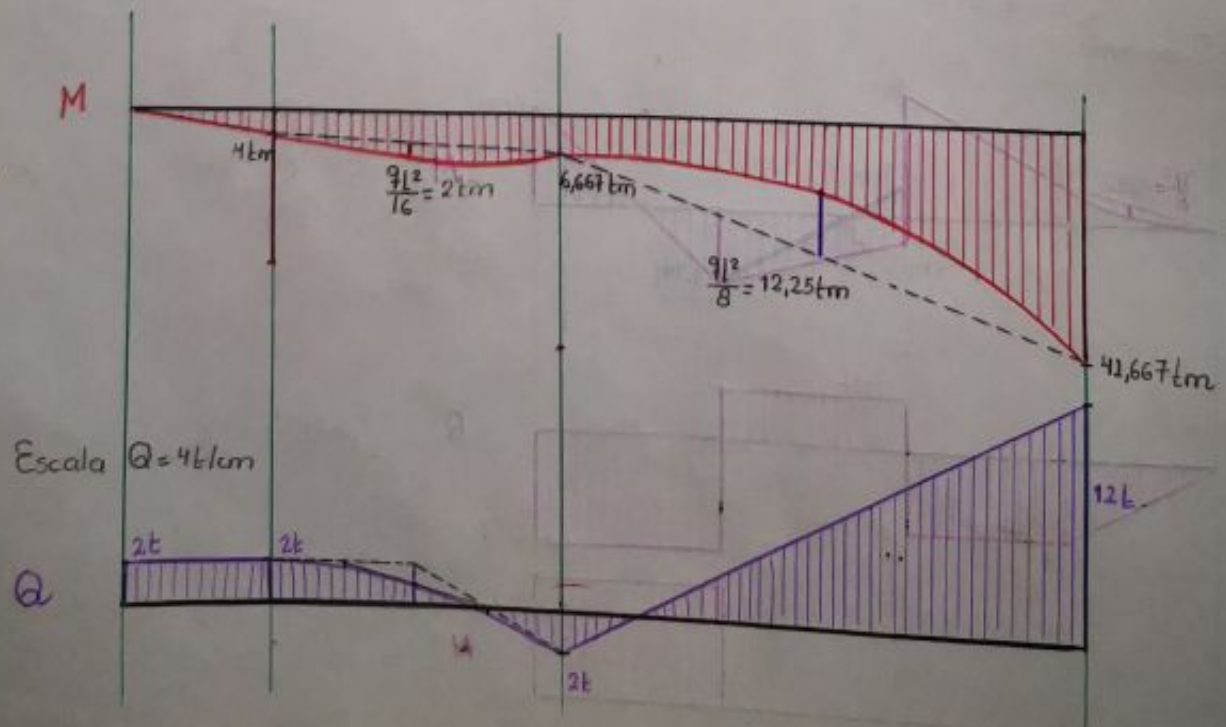
$Q_B = 2t$

$Q_C = 2 - \frac{2(4)}{2} = -2t$

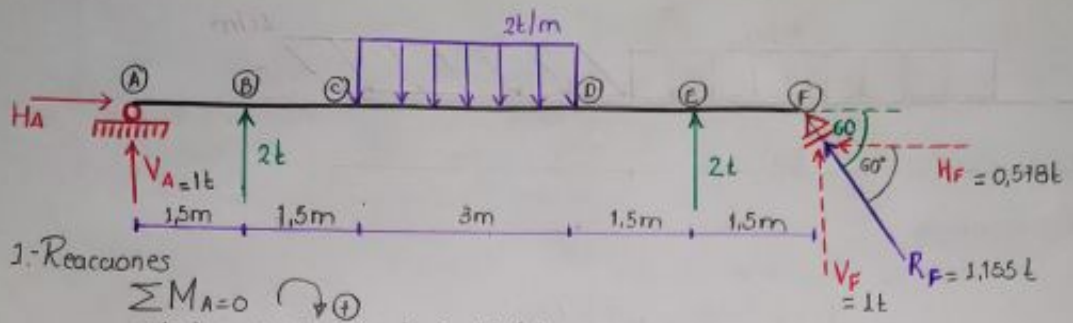
$Q_D = 2 - \frac{2(4)}{2} + 2(7) = 12t$ //

4 - Diagramas

Escala M = 15t/cm



21.- Diagramar los esfuerzos característicos



1.-Reacciones

$\Sigma M_A = 0 \quad \circlearrowleft +$

$-2(1,5) + 2(3)(4,5) - 2(7,5) - V_F(9) = 0$

$V_F = 1t$

$\text{Sen}60° = \frac{V_F}{R_F} \Rightarrow R_F = \frac{1}{\text{Sen}60} = 1,155t$

$H_F = R_F \text{Cos}60 = 1,155 \text{Cos}60 = 0,578t$

$\Sigma F_V = 0 \quad \uparrow +$

$V_A + 2 - 2(3) + 2 + 1 = 0$

$V_A = 1t$

$\Sigma F_H = 0 \rightarrow +$

$H_A = 0,578t$

2.-Momentos

$M_A = 0tm$

$M_B = 1(1,5) = 1,5tm$

$M_C = 1(3) + 2(1,5) = 6tm$

$M_D = 1(3) + 2(1,5) = 6tm$

$M_E = 1(1,5) = 1,5tm$

$M_F = 0tm$

3.-Cortantes

$Q_A = 1t$

$Q_{B(IZQ)} = 1t$

$Q_{B(DER)} = 3t$

$Q_C = 3t$

$Q_D = 3 - 2(3) = -3t$

$Q_{E(IZQ)} = -3t$

$Q_{E(DER)} = -3 + 2 = -1t$

$Q_F = -1t$

4.-Normales

$N_A = -0,578t$

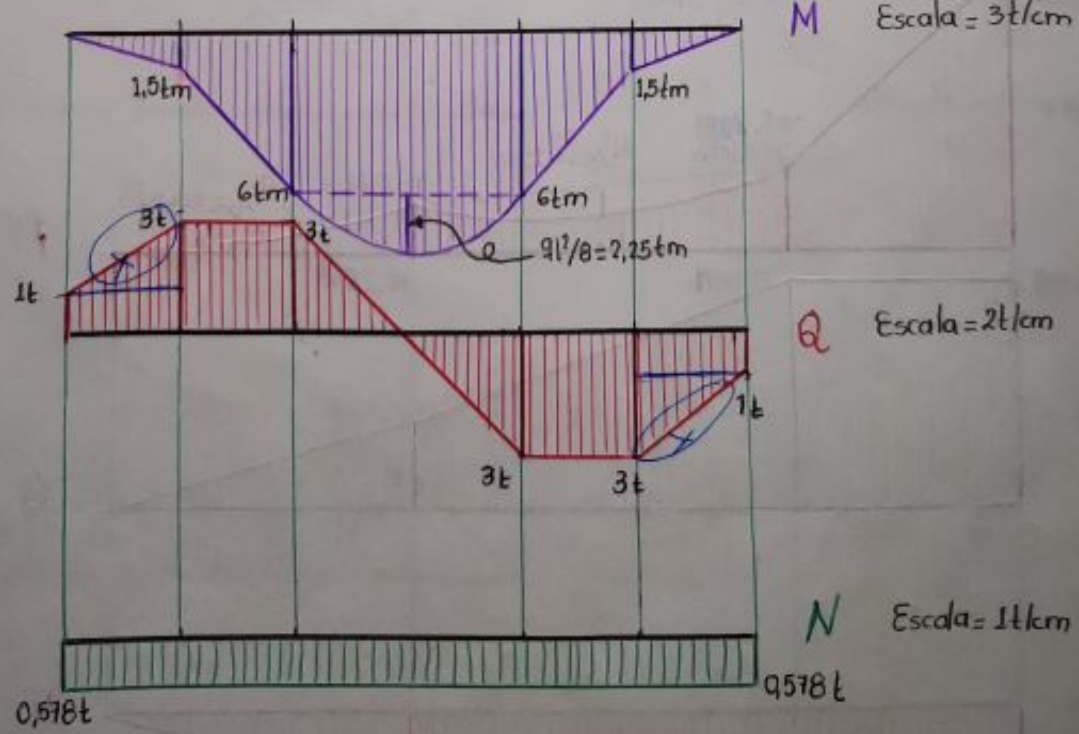
$N_B = -0,578t$

$N_C = -0,578t$

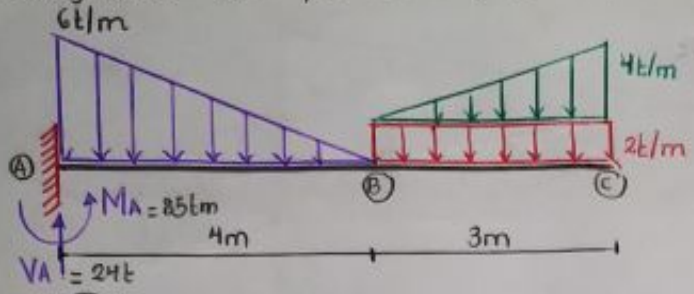
$N_D = -0,578t$

$N_E = -0,578t$

$N_F = -0,578t$



23.- Diagramar los esfuerzos caracteristicos



1.- Reacciones

$\sum F_V = 0 \uparrow (+)$

$V_A - \frac{6(4)}{2} - \frac{4(3)}{2} - 2(3) = 0$

$V_A = 24t$ //

$\sum M_A = 0 \curvearrowright (+)$

$-M_A + \frac{6(4)}{2}\left[\frac{1}{3}.4\right] + 2(3)(5.5) + \frac{4(3)}{2}.(6) = 0$

$M_A = 85tm$

2.- Momentos

$M_A = -85tm$

$M_B = -85 + 24(4) - \frac{6(4)}{2}\left(\frac{2}{3}.4\right) = -21tm$

$M_C = 0tm$

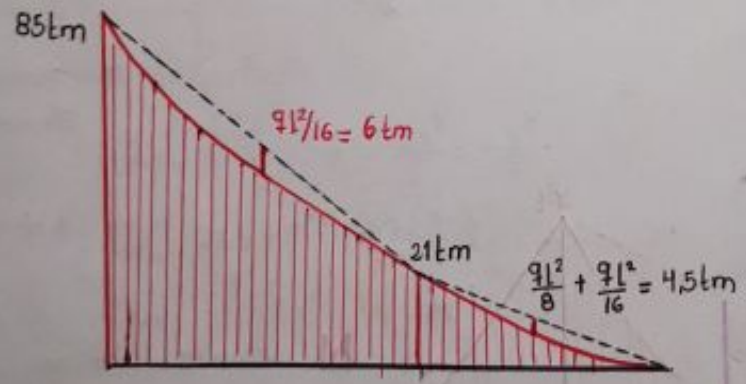
3.- Cortantes

$Q_A = 24t$

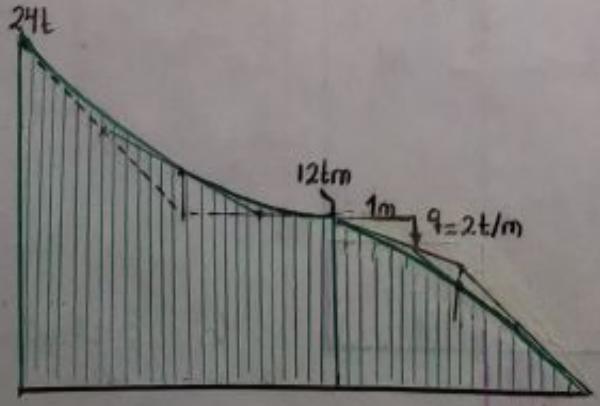
$Q_B = 24 - \frac{6.4}{2} = 12t$

$Q_C = 0tm$

4.- Diagramas



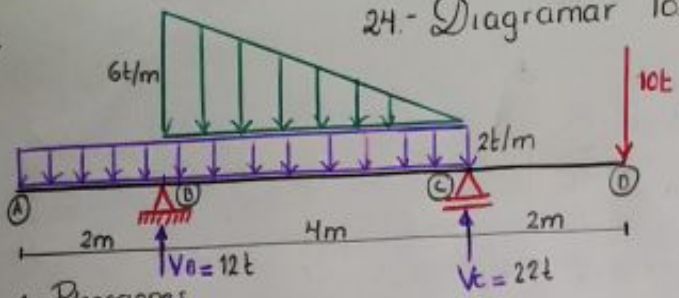
M Escala = 20tm/cm



Q Escala = 6t/cm

24.- Diagramar los esfuerzos caracteristicos

24:



1.- Reacciones

$\sum M_{(B)} = 0$ ↻ (+)

$-2(2)(1) + 2(4)(2) + \frac{6(4)}{2}\left(\frac{1}{3}.4\right) - V_C(4) + 10(6) = 0$

$V_C = 22t$ //

$\sum F_V = 0$ ↑ (+)

$-2(2) + V_B - 2(4) - \frac{6(4)}{2} + 22 - 10 = 0$

$V_B = 12t$

2.- Momentos ↻ (+)

$M_A = 0tm$

$M_B = -2(2)(1) = -4tm$

$M_C = -10(2) = -20tm$

$M_D = 0tm$

3.- Cortantes ↑ (+)

$Q_A = 0t$

$Q_{B(IZQ)} = -2(2) = -4t$

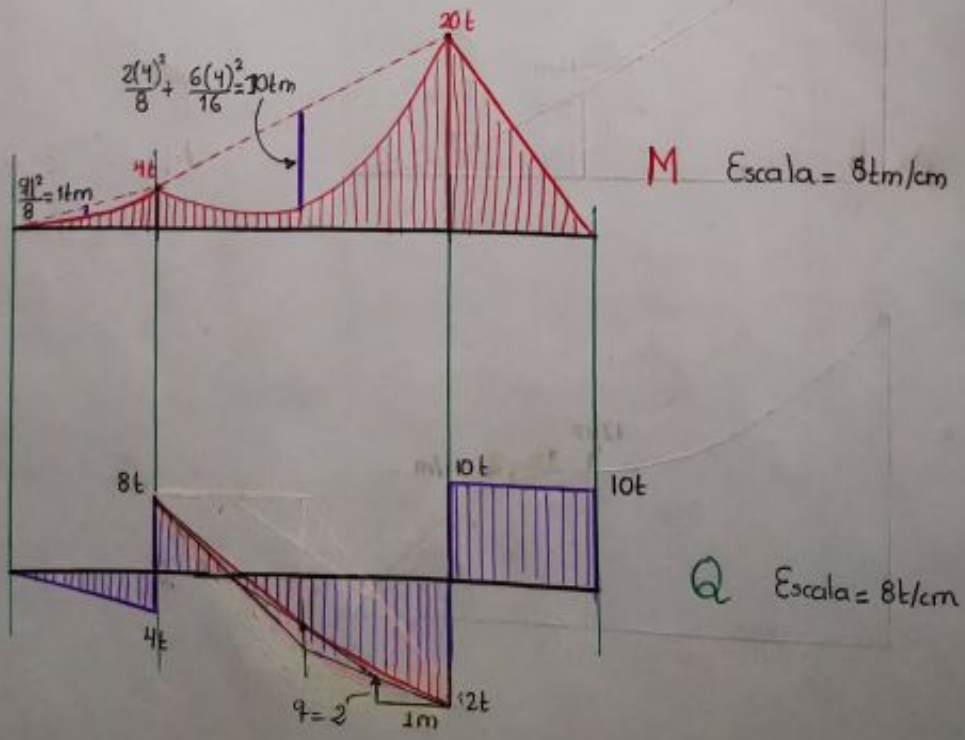
$Q_{B(DER)} = -4 + 12 = 8t$

$Q_{C(IZQ)} = 10 - 22 = -12t$

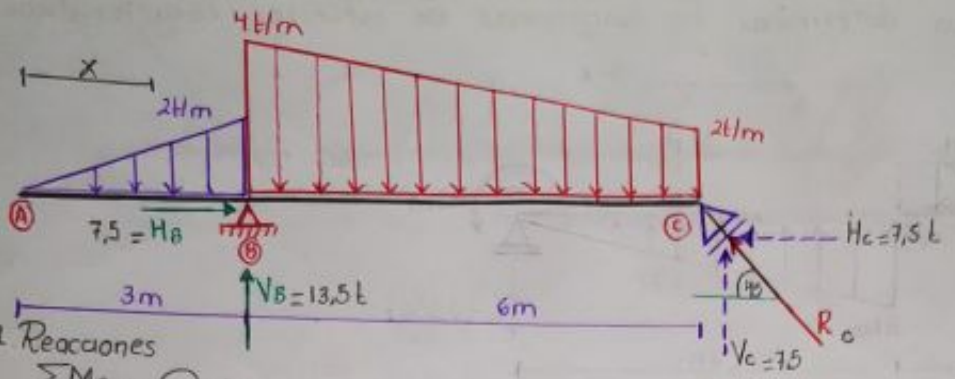
$Q_{C(DER)} = 10t$

$Q_D = 10t$

4.- Diagramas



25.- Diagramar Momento y cortante



1 Reacciones

$\Sigma M_B=0 \circlearrowright +$

$-\frac{2(3)}{2}.(1) + 2(6)(3) + \frac{2(6)}{2}.2 - V_c.6 = 0 \quad \therefore V_c = 7,5t$

$H_c = V_c = 7,5t$

$R_c = 10,607t$

$\Sigma F_V=0 \uparrow +$

$-\frac{2(3)}{2} + V_B - \frac{(2+4)}{2}.6 + 7,5 = 0 \quad \therefore V_B = 13,5t$

$\Sigma F_H=0 \rightarrow +$ $\quad \therefore H_B = 7,5t$

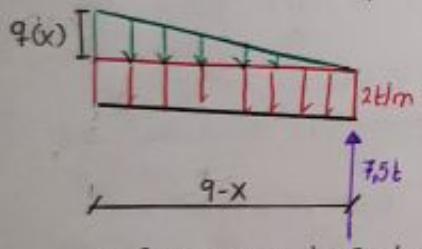
2 - Ecuaciones de Momento

* Tramo A-B

$\frac{q(x)}{x} = \frac{2}{3} \Rightarrow q(x) = \frac{2x}{3} \Rightarrow M_{AB} = \frac{-\frac{2x}{3}.x}{2}.\frac{x}{3} = \frac{-2x^3}{18}$

$M_{AB} = -\frac{x^3}{9}$ //

* Tramo BC



$\frac{q(x)}{9-x} = \frac{2}{6} \Rightarrow q(x) = 3 - \frac{x}{3}$ //

$M_{BC} = 7,5(9-x) - \frac{2(9-x)^2}{2} - \frac{(3-\frac{x}{3})(9-x).(9-x)}{2}.\frac{1}{3}$

$M_{BC} = 0,0556x^3 - 2,5x^2 + 24x - 54$

3.- Ecuaciones de Cortante

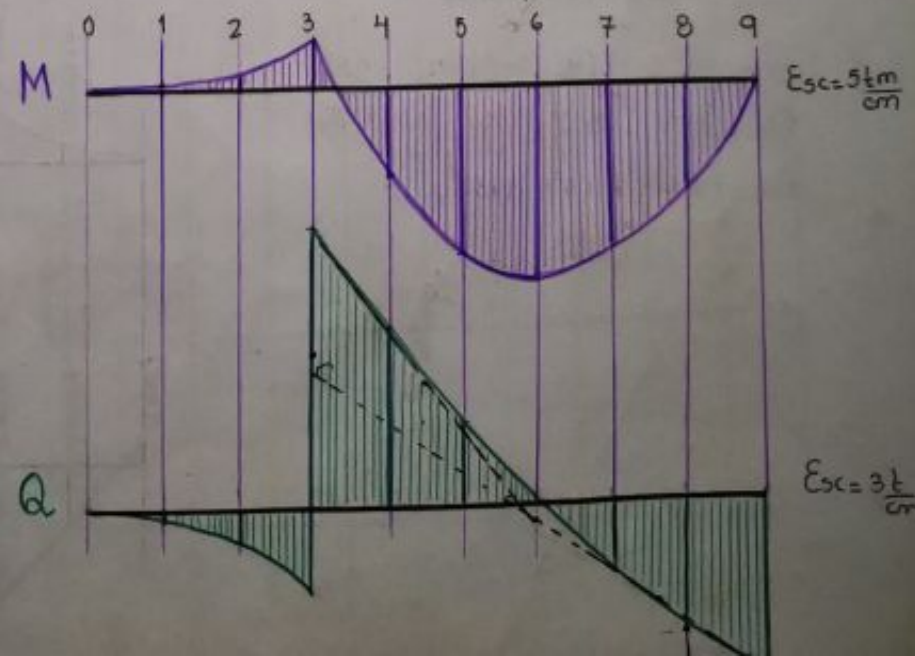
* Tramo AB $\quad Q_{AB} = \frac{(-\frac{2x}{3}).x}{2} = -\frac{2x^2}{6} = -\frac{x^2}{3}$

* Tramo BC $\quad Q_{BC} = -7,5 + 2(9-x) + \frac{(3-\frac{x}{3})(9-x)}{2}$

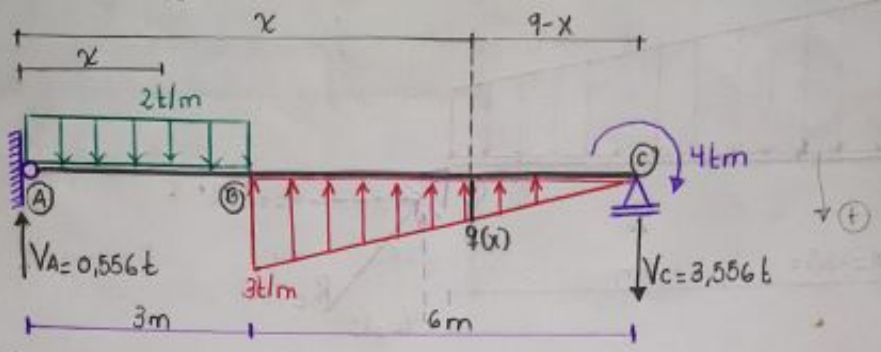
$Q_{BC} = 0,1667x^2 - 5x + 24$

4.- Diagramas

x	Momento	Cortante
0	0,000	0,000
1	-0,111	-0,333
2	-0,889	-1,333
3	-3	-3 / 10,50
4	5,558	6,667
5	10,45	3,168
6	12,01	0,00
7	10,571	-2,832
8	6,467	-5,3312
9	0,000	-7,5



26: Para la siguiente viga determinar los diagramas de esfuerzos caracteristicos



1.- Reacciones

$\Sigma M_{A=0}$ ↻ ⊕

$2(3)(1,5) - \frac{3(6)}{2}(5) + 4 + V_c(9) = 0$

$V_c = 3,556t$ //

$\Sigma F_v = 0$ ↑ ⊕

$V_A - 2(3) + \frac{3(6)}{2} - 3,556$

$V_A = 0,556t$ //

2.- Ecuaciones de Momento

Tramo A-B

$M_{AB} = 0,556x - 2(x)\frac{(x)}{2}$

$M_{AB} = 0,556x - x^2$ //

Tramo BC

$\frac{q(x)}{9-x} = \frac{3}{6} \Rightarrow q(x) = 4,5 - 0,5x$

$M_{BC} = -3,556(9-x) - 4 + \frac{(4,5-0,5x)(9-x)}{2} \cdot \frac{1}{3}(9-x)$

$M_{BC} = -32,004 + 3,556x - 4 + \frac{1}{6}(4,5-0,5x)(81-18x+x^2)$

$M_{BC} = -36,004 + 3,556x + \frac{1}{6}\left[364,5 - 81x + 4,5x^2 - 40,5x + 9x^2 - 0,5x^3\right]$

$M_{BC} = -36,004 + 3,556x + \frac{1}{6}\left[364,5 - 121,5x + 13,5x^2 - 0,5x^3\right]$

$M_{BC} = -36,004 + 3,556x + 60,75 - 20,25x + 2,25x^2 - 0,083x^3$

$M_{BC} = 24,746 - 16,694x + 2,25x^2 - 0,083x^3$ //

3.- Ecuaciones de Cortante

$Q_{AB} = 0,556 - 2(x)$ //

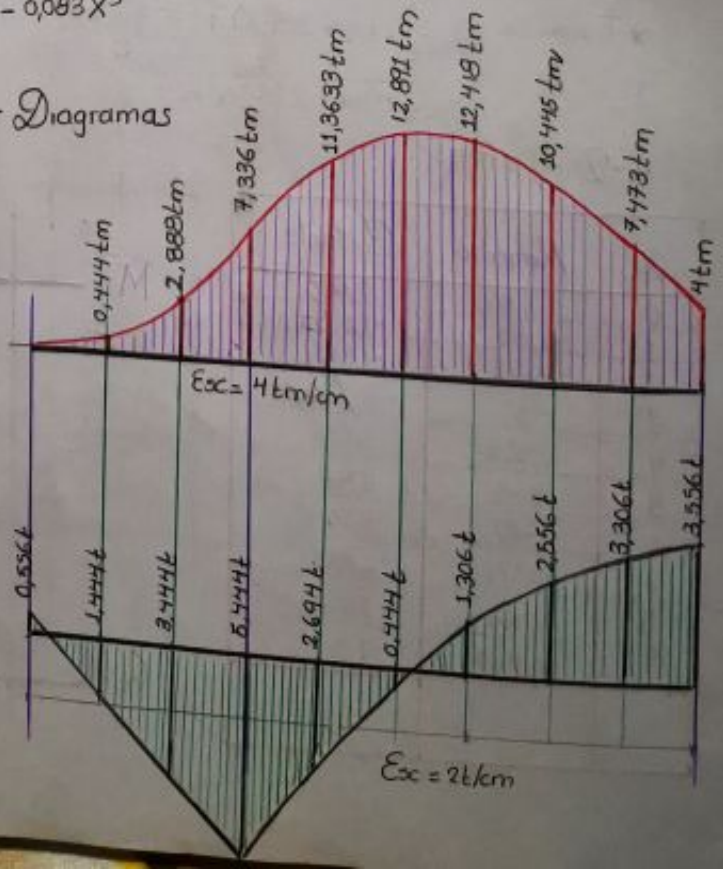
$Q_{BC} = 3,556 - \frac{(4,5-0,5x)(9-x)}{2}$

$Q_{BC} = 3,556 - \frac{1}{2}(40,5 - 4,5x - 4,5x + 0,5x^2)$

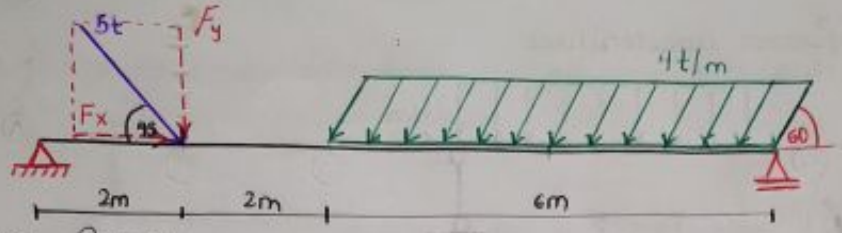
$Q_{BC} = 3,556 - 20,25 + 2,25x + 2,25x - 0,25x^2$

$Q_{BC} = -16,694 + 4,5x - 0,25x^2$ //

4.- Diagramas



27.- Diagramar los esfuerzas característicos

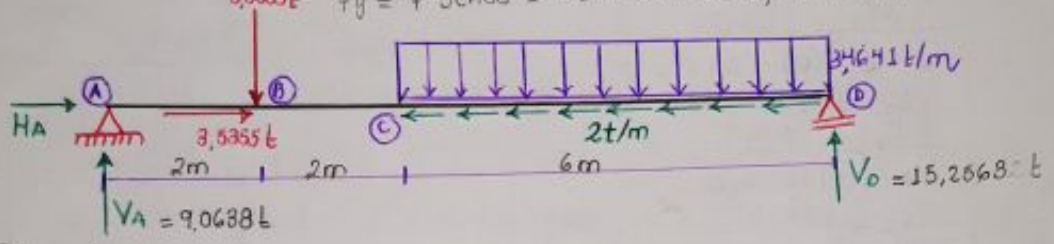


1.- Idealizar Cargas

$F_x = F_y = 5t \cdot Cos45 = 3,5355\ t$

$q_x = q\ Cos60 = 4t/m\ Cos60 = 2t/m$

$q_y = q\ Sen60 = 4t/m\ Sen60 = 3,4641\ t/m$



2.- Reacciones

$\sum M_{A=0}$ ↻ ⊕

$3,5355\,(2) + 3,4641(6)(7) - V_D(10) = 0 \quad \therefore V_D = 15,2563\ t$

$\sum F_V = 0$ ↑ ⊕

$V_A - 3,5355 - 3,4641\,(6) + 15,2563 = 0 \quad \therefore V_A = 9,0638\ t$

$\sum F_H = 0$ → ⊕

$H_A + 3,5355 - 2(6) = 0 \quad \therefore H_A = 8,4645\ t$

3.- Ecuaciones de Momento

* Tramo A-B

$M_x = V_A \cdot x = 9,0638 \cdot x$

* Tramo BC

$M_x = 9,0638 \cdot x - 3,5355\,(x-2)$

$M_x = 5,5283x + 7,0710$ //

* Tramo CD

$M_x = 15,2563 \cdot (10-x) - 3,4641 \frac{(10-x)^2}{2}$

$M_x = -1,7321x^2 + 19,3847x - 20,642$ //

4 Ecuaciones de Corte

* Tramo AB

$Q_x = 9,0638\ t$

* Tramo BC

$Q_x = 9,0638 - 3,5355$

$Q_x = 5,5283$

* Tramo CD

$Q_x = -15,2568 + 3,4641(10-x)$

$Q_x = -3,4641x + 19,3842$

5.- Ecuaciones de Normal

* Tramo AB

$N_x = -8,4645\ t$

* Tramo BC

$N_x = -8,4645 - 3,5355$

$N_x = -12\ t$

* Tramo CD

$N_x = -2(10-x)$

$N_x = 2x - 20$

6- Diagramas

x	Mto	Corte	Normal
0	0	9,0638	-8,4645
1	9,0638	9,0638	-8,4645
2	18,1276	9,06 | 5,528	-8,46 | -12
3	23,6559	5,5283	-12
4	29,1842	5,5283	-12
5	32,979	2,0637	-10
6	33,3106	-1,4004	-8
7	30,178	-4,8645	-6
8	23,5812	-8,3286	-4
9	13,5202	-11,7927	-2
10	0	-15,2563	0

28.- Diagramar los esfuerzos característicos

1.- Reacciones

$\Sigma M_A = 0 \circlearrowright \oplus$

$6 + 2(4)(6) - V_c(8) = 0$

$V_c = 6,75t$ //

$\Sigma F_V = 0 \uparrow \oplus$

$V_A - 2(4) + 6,75 = 0$

$V_A = 1,25t$

2.- Ecuaciones de Momento

* Tramo AB $\quad M_x = 1,25 \cdot x + 6$ //
* Tramo BC $\quad M_x = 6,75 \cdot (8-x) - \frac{2(8-x)^2}{2}$

$M_x = -x^2 + 9,25x - 10$ //

3.- Ecuaciones de Corte

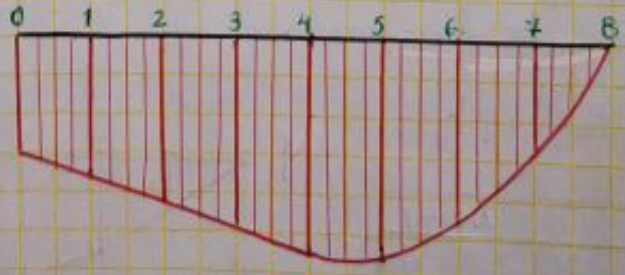
* Tramo AB $\quad Q_x = 1,25t$ //
* Tramo BC $\quad Q_x = 1,25 - 2(x-4)$

$Q_x = 9,25 - 2x$ //

4.- Diagramas

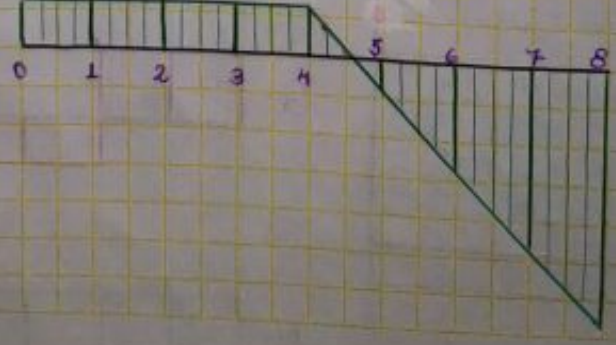
x	Momento	Cortante
0	6	1,25
1	7,25	1,25
2	8,5	1,25
3	9,75	1,25
4	11	1,25
5	11,25	-0,75
6	9,5	-2,75
7	5,75	-4,75
8	0	-6,75

M



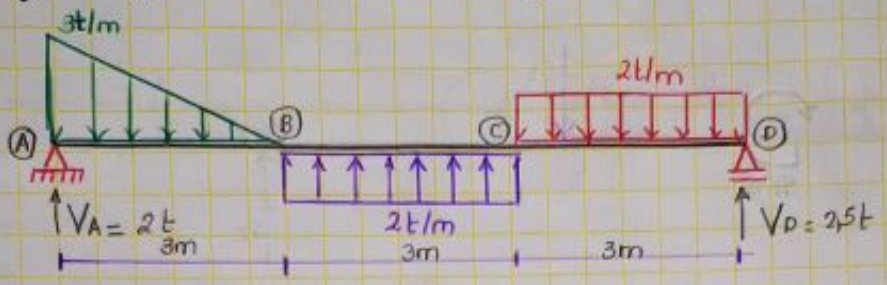
E= 4tm/cm

Q



E= 2t/cm

29.- Para la viga mostrada determinar los esfuerzos característicos



1.- Reacciones

$\sum M_{A=0}$ ↻ ⊕

$\frac{3(3)}{2}.(1) - 2(3)(4,5) + 2(3)(7,5) - V_D(9) = 0 \quad \therefore V_D = 2,5t$

$\sum F_V = 0$ ↑ ⊕

$V_A - \frac{3(3)}{2} + 2(3) - 2(3) + 2,5 = 0 \quad \therefore V_A = 2t$

2 - Ecuaciones de Momento

* Tramo AB

$Mx = 2\cdot x - \left(\frac{3x^2}{2} - \frac{3x^3}{6(3)}\right)$

$Mx = [x^3 - 9x^2 + 12x]/6$ //

* Tramo BC

$Mx = 2x - \frac{3(3)}{2}.(x-1) + \frac{2(x-3)^2}{2}$

$Mx = \frac{2x^2 - 17x + 27}{2}$ //

* Tramo CD

$Mx = 2,5(9-x) - \frac{2(9-x)^2}{2}$

$Mx = -x^2 + 15,5x - 58,5$ //

3.- Ecuaciones de Corte

* Tramo AB

$Qx = 2 - \left[3\cdot x - \frac{3x^2}{2(3)}\right]$

$Qx = 0,5x^2 - 3x + 2$ //

* Tramo BC

$Qx = 2 - \frac{3(3)}{2} + 2(x-3)$

$Qx = 2x - 8,5$ //

* Tramo CD

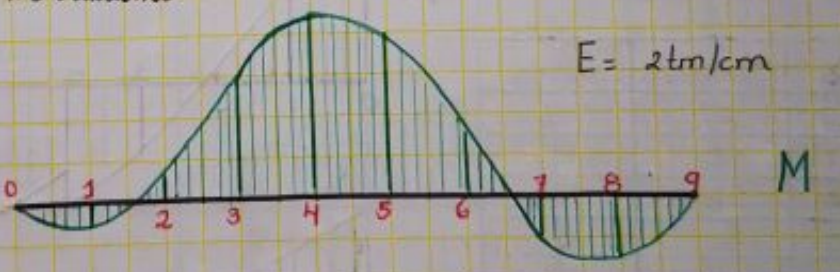
$Qx = -2,5 + 2(9-x)$

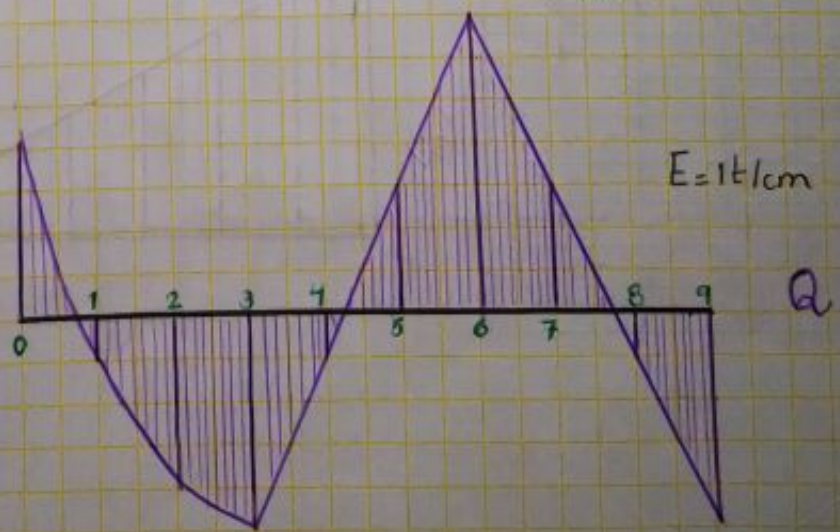
$Qx = -2x + 15,5$ //

3 - Diagramas

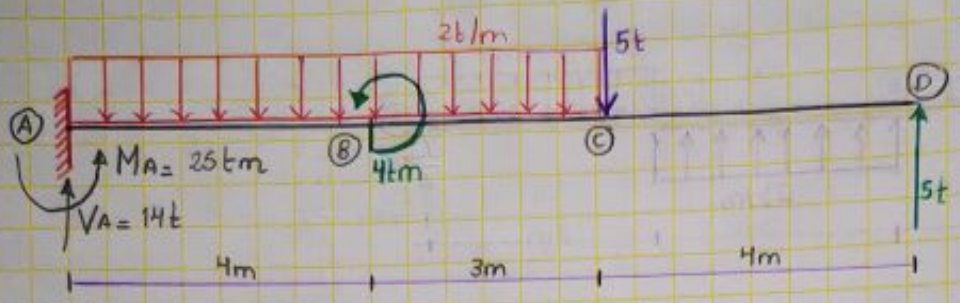
x	Momentos	Cortantes
0	0	2
1	0,667	-0,5
2	-0,667	-2
3	-3	-2,5
4	-4,5	-0,5
5	-4	1,5
6	-3,5	3,5
7	1	1,5
8	1,5	-0,5
9	0	-2,5

1. Reacciones





30.- Diagramar los esfuerzos caracteristicos



1- Reacciones

$\sum F_v = 0 \quad \uparrow (+)$

$V_A - 2(7) - 5 + 5 = 0 \quad \therefore V_A = 14t$

$\sum M_A = 0 \quad \curvearrowright (+)$

$-M_A + 2(7)(3,5) + 5(7) - 4 - 5(11) = 0$

$M_A = 25tm$

2- Ecuaciones de Momento

* Tramo AB

$M_x = 14(x) - 25 - \frac{2x^2}{2}$

$M_x = -x^2 + 14x - 25$ //

* Tramo BC

$M_x = 14 \cdot x - 25 - \frac{2x^2}{2} - 4$

$M_x = -x^2 + 14x - 29$ //

* Tramo CD

$M_x = 5(11 - x)$

$M_x = 55 - 5x$ //

3.- Ecuaciones de Corte

* Tramo AC

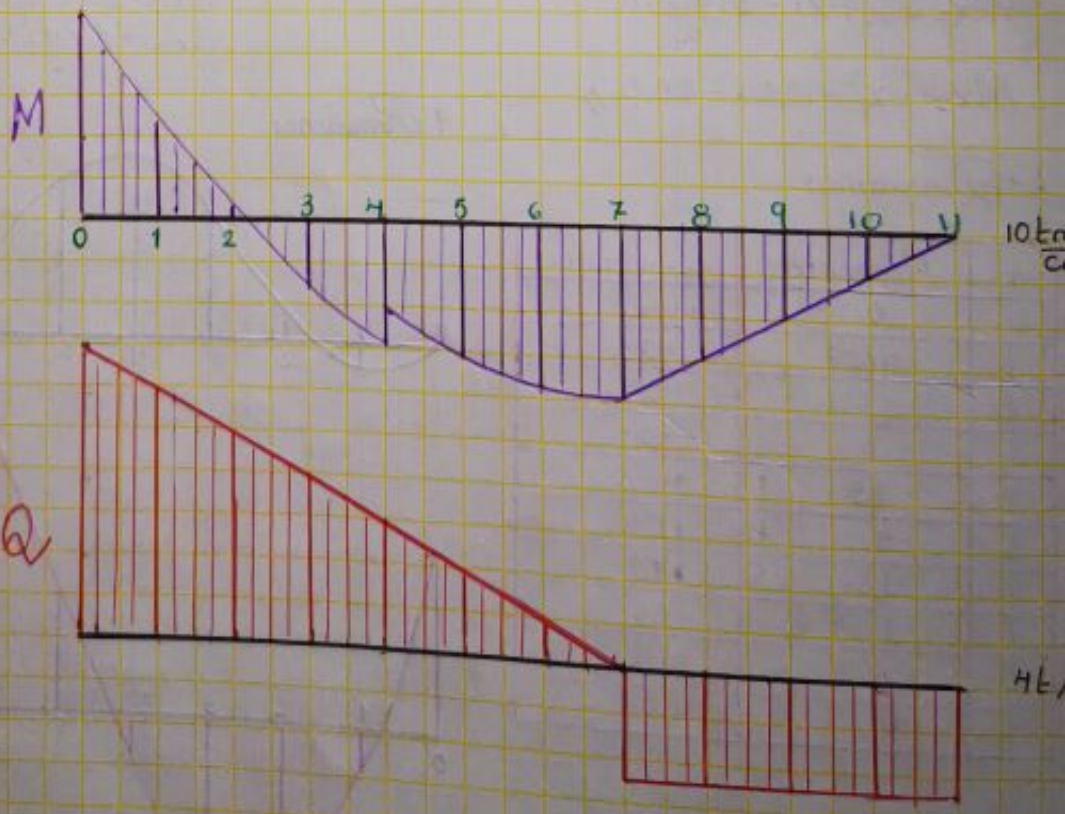
$Q_x = 14 - 2x$ //

* Tramo CD

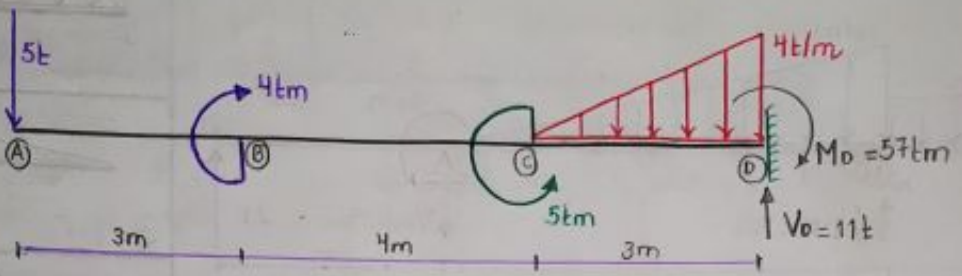
$Q_x = -5t$ //

4- Diagramas

x	Momento	Corte
0	-25	14
1	-12	12
2	-1	10
3	8	8
4	15 / 11	6
5	16	4
6	19	2
7	20	0 / -5
8	15	-5
9	10	-5
10	5	-5
11	0	-5



31.- Diagramar los esfuerzos internos



1.- Reacciones

$\Sigma F_{V=0} \uparrow \oplus$

$-5 - \frac{4(3)}{2} + V_D = 0 \quad \therefore V_D = 11t$

$\Sigma M_{D=0} \curvearrowright \oplus$

$-5(10) + 4 - 5 - \frac{4(3)}{2}.(1) + M_D = 0 \quad \therefore M_D = 57tm$

2.- Ecuaciones de Momento

* Tramo AB $\quad M_{AB} = -5X$ //
* Tramo BC $\quad M_{BC} = -5X + 4$ //
* Tramo CD $\quad M_{CD} = -5X + 4 - 5 - \frac{4(X-7)^3}{6(3)}$

$M_{CD} = -\frac{2X^3 - 42X^2 + 339X - 677}{9}$

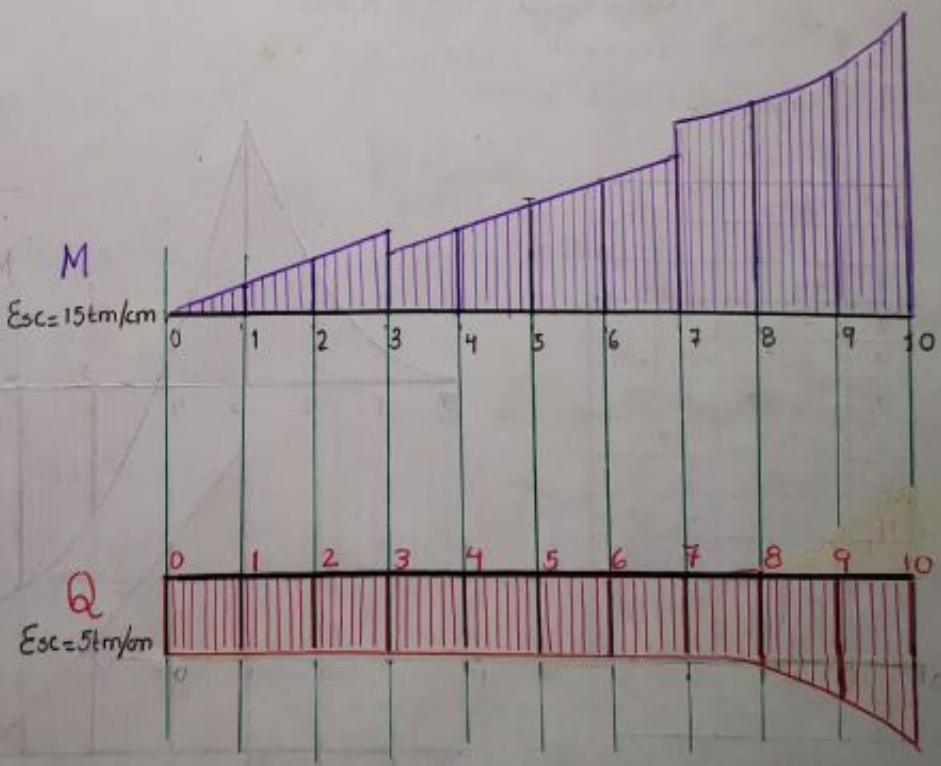
3.- Ecuaciones de Corte

* Tramo AB $\quad \theta_{AB} = -5t$
* Tramo BC $\quad \theta_{BC} = -5t$
* Tramo CD $\quad Q_{CD} = -5 - \frac{4(X-7)^2}{2 \cdot 3}$

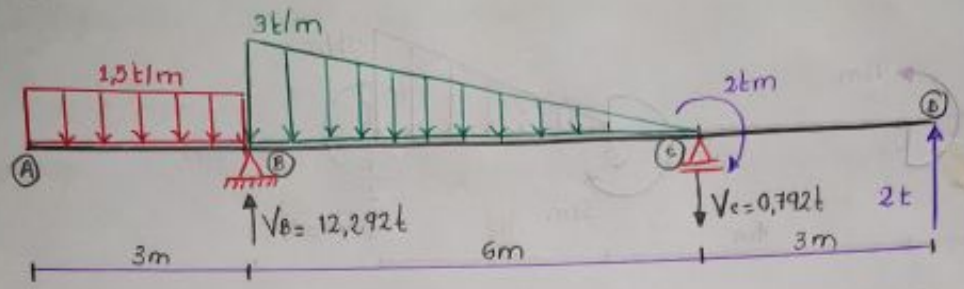
$\theta_{CD} = \frac{-2X^2 + 28X - 113}{3}$

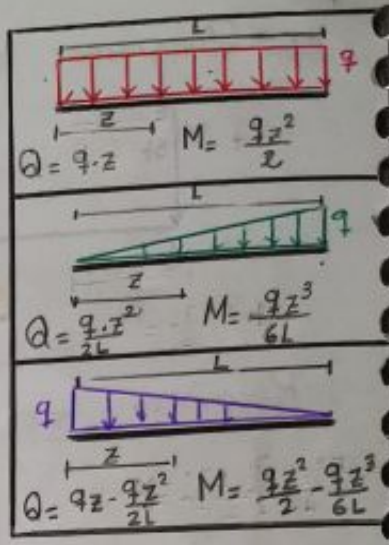
4.- Diagramas

x	Mto	Corte
0	0	-5
1	-5	-5
2	-10	-5
3	-15 | -11	-5
4	-16	-5
5	-21	-5
6	-26	-5
7	-31 | -36	-5
8	-41,222	-5,667
9	-47,778	-7,667
10	-57	-11



32.- Diagramar los esfuerzos internos





1.- Reacciones

$\Sigma M_{(B)} = 0$ ↷ ⊕

$-1,5(3)(1,5) + \frac{3(6)}{2}(2) + 2 - 2(9) + V_c(6) = 0 \quad \therefore V_c = 0,792t$

$\Sigma F_v = 0$ ↑ ⊕

$-1,5(3) + V_B - \frac{3(6)}{2} - 0,792 + 2 = 0 \quad \therefore V_B = 12,292t$

2.- Ecuaciones de Momento

* Tramo AB $M_{AB} = -\frac{1,5x^2}{2} = -0,75x^2$ //

* Tramo BC $M_{BC} = -1,5(3)(x-1,5) + 12,292(x-3) - \left[\frac{3(x-3)^2}{2} - \frac{3(x-3)^3}{6(6)}\right]$

$M_{BC} = 0,0833x^3 - 2,25x^2 + 19,042x - 45,876$ //

* Tramo CD $M_{CD} = 2(12-x) = 24 - 2x$

3.- Ecuaciones de Corte

* Tramo AB $Q_{AB} = -1,5x$

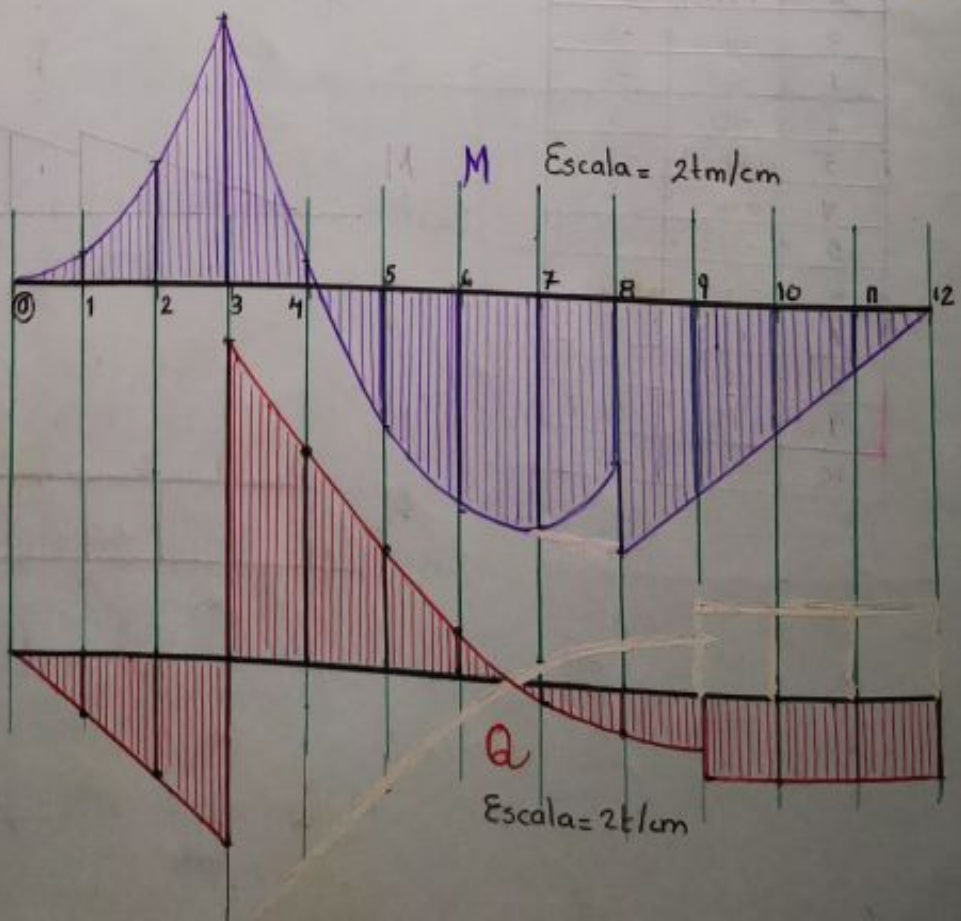
* Tramo BC $Q_{BC} = -1,5(3) + 12,292 - \left[3(x-3) - \frac{3(x-3)^2}{2(6)}\right]$

$Q_{BC} = 0,25x^2 - 4,5x + 19,042$

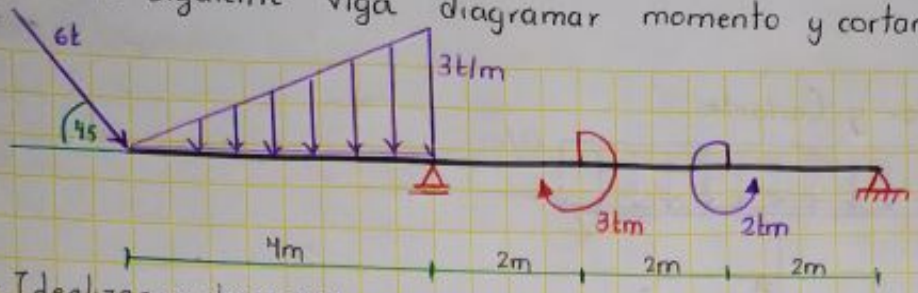
* Tramo CD $Q_{CD} = -2t$

4.- Diagramas

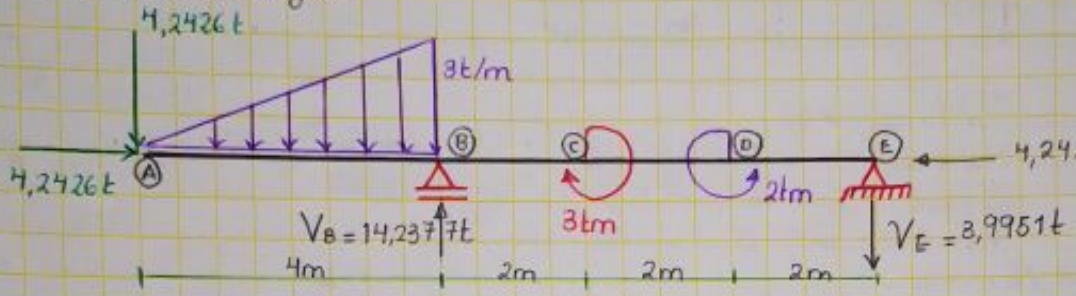
x	Mto	Corte
0	0	0
1	-0,75	-1,5
2	-3	-3
3	-6,75	-4,5 | 7,792
4	-0,377	5,092
5	3,497	2,792
6	5,369	1,042
7	5,740	-0,208
8	5,1096	-0,958
9	4 | 6	-1,208 | -2
10	4	-2
11	2	-2
12	0	-2



33. Para la siguiente viga diagramar momento y cortante



1.- Idealizacion de cargas



2.- Reacciones

$\sum M_B = 0 \quad \circlearrowright \oplus$

$-4,2426(4) - \frac{3(4)}{2}\cdot\left[\frac{1}{3}\cdot 4\right] + 3 - 2 + V_E(6) = 0 \quad \therefore V_E = 3,9951t$ //

$\sum F_V = 0 \quad \uparrow \oplus$

$-4,2426 - \frac{3(4)}{2} + V_B - 3,9951 = 0 \quad \therefore V_B = 14,2377t$ //

$\sum F_H = 0 \quad \rightarrow \oplus \quad \therefore H_E = 4,2426t$ //

3.- Ecuaciones de Momento

* Tramo AB

$M_x = \frac{-3x^3}{6(4)} - 4,2426\cdot x$

$M_x = -0,125x^3 - 4,2426x$ //

* Tramo BC

$M_x = -4,2426\cdot x - \frac{3(4)}{2}\cdot\left[x - \frac{2\cdot 4}{3}\right] + 14,2377(x-4)$

$M_x = 3,9951x - 40,9508$ //

* Tramo CD

$M_x = -3,9951\cdot(10-x) + 2 = 3,9951x - 37,951$ //

* Tramo DE

$M_x = -3,9951(10-x)$

$M_x = 3,9951x - 39,951$ //

4.- Ecuaciones de Corte

* Tramo AB

$Q_x = -4,2426 - \frac{3x^2}{2(4)}$

$Q_x = -4,2426 - 0,375x^2$ //

* Tramo BE

$Q_x = -4,2426 - \frac{3(4)}{2} + 14,2377 = 3,9951t$ //

5.- Normal

* Tramo AE

$N = -4,2426t$ //

4.- Diagramas

x	Momento	Corte
0	0	-4,2426
1	-4,3676	-9,6176
2	-9,4852	-5,7426
3	-16,1028	-7,6176
4	-24,9704	-10,24 / 3,9951
5	-20,9753	3,9951
6	-16,98 / -13,98	3,9951
7	-9,9858	3,9951
8	-5,99 / -7,99	3,9951
9	-3,9951	3,9951
10	0	3,9951

Diagrama de Momento

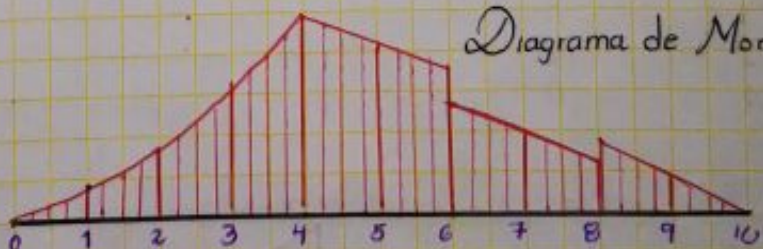
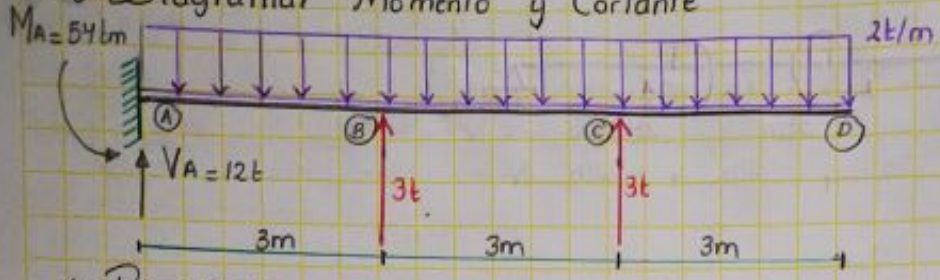


Diagrama de Cortante

34.- Diagramar Momento y Cortante



1- Reacciones

$\Sigma M_A = 0 \curvearrowright +$

$-M_A + 2(9)(4,5) - 3(3) - 3(6) = 0 \quad \therefore M_A = 54\,tm$

$\Sigma F_V = 0 \uparrow +$

$V_A + 3 + 3 - 2(9) = 0 \quad \therefore V_A = 12\,t$ //

2- Ecuaciones de Momento

* Tramo AB

$M_x = 12x - 54 - \frac{2x^2}{2}$

$M_x = -x^2 + 12x - 54$

* Tramo BC

$M_x = 12x + 3(x-3) - 54 - \frac{2x^2}{2}$

$M_x = -x^2 + 15x - 63$

* Tramo CD

$M_x = -\frac{2(9-x)^2}{2}$

$M_x = -x^2 + 18x - 81$

3.- Ecuaciones de Corte

* Tramo AB

$Q_x = 12 - 2x$ //

* Tramo BC

$Q_x = 12 - 2x + 3$

$Q_x = 15 - 2x$ //

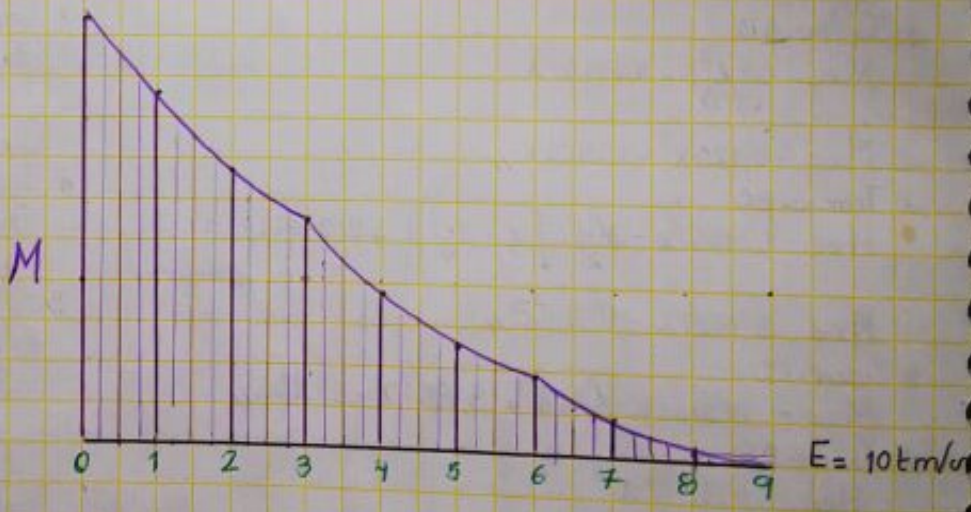
* Tramo CD

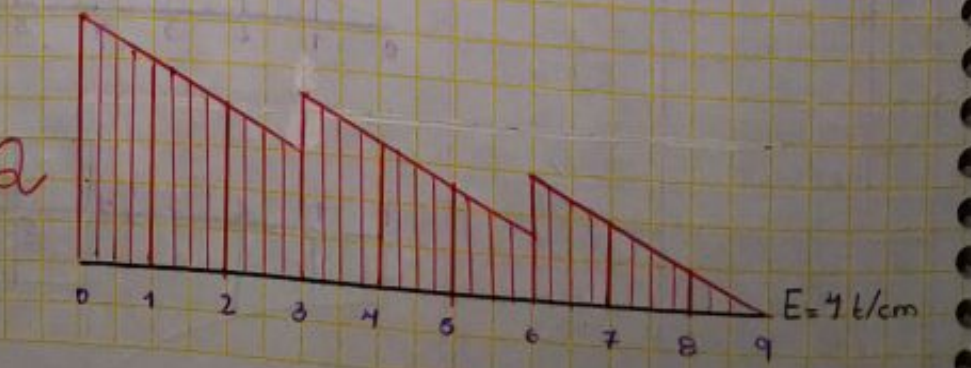
$Q_x = 2(9-x)$

$Q_x = 18 - 2x$ //

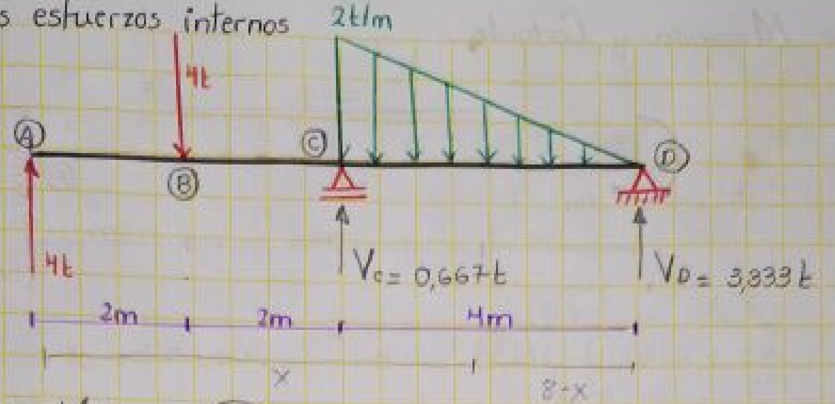
4.- Diagramas

x	Momentos	Cortantes
0	-54	12
1	-43	10
2	-34	8
3	-27	6 / 9
4	-19	7
5	-13	5
6	-9	3 / 6
7	-4	4
8	-1	2
9	0	0





35.- Diagramar los esfuerzos internos



1- Reacciones

$\sum M_C = 0$ (+)

$-V_D(4) + \frac{2(4)}{2} \cdot \left[\frac{1}{3}(4)\right] - 4(2) + 4(4) = 0$

$V_D = 3{,}333\ t$

$\sum V = 0$ ↑(+)

$4 - 4 + V_C - \frac{2(4)}{2} + 3{,}333 = 0 \quad \therefore V_C = 0{,}667\ t$

2- Ecuaciones de Momento

* Tramo AB

$M_x = 4x$

* Tramo BC

$M_x = 4x - 4(x-2)$

$M_x = 8$ //

* Tramo CD

$M_x = 3{,}333(8-x) - \frac{2(8-x)^3}{6(4)}$

$M_x = 0{,}083x^3 - 2x^2 + 12{,}667x - 16{,}003$ //

3.- Ecuaciones de Corte

* Tramo AB

$Q_x = 4t$ //

* Tramo BC

$Q_x = 4 - 4 = 0t$ //

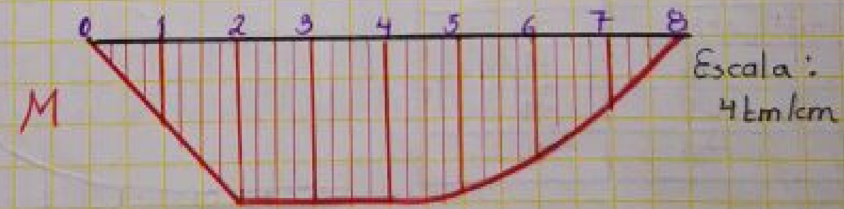
* Tramo CD

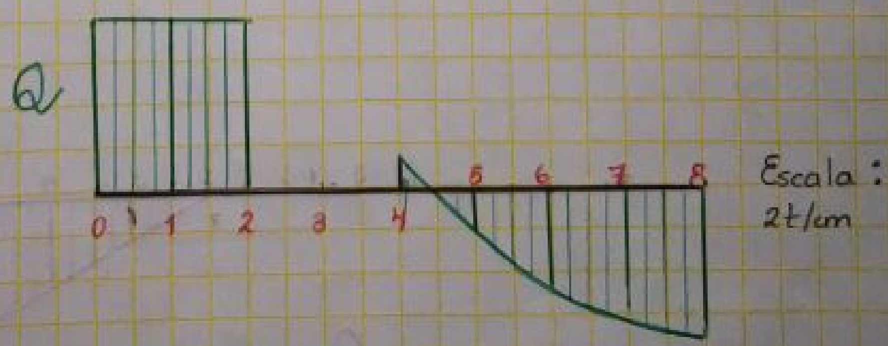
$Q_x = -3{,}333 + \frac{2(8-x)^2}{2(4)}$

$Q_x = 0{,}25x^2 - 4x + 12{,}667$ //

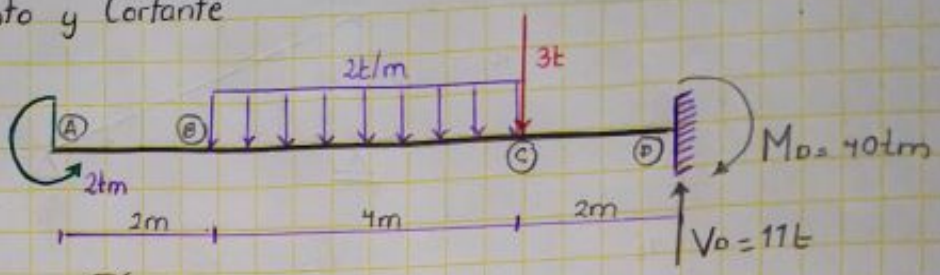
4.- Diagramas

x	Momento	Cortante
0	0	4
1	4	4
2	8	4/0
3	8	0
4	8	0/0,667
5	7,748	-1,083
6	5,998	-2,333
7	3,248	-3,083
8	0	-3,333





36- Diagramar Momento y Cortante



1- Reacciones

$\sum M_D = 0 \quad \curvearrowleft \oplus$

$-2 - 2(4)(4) - 3(2) + M_D = 0 \quad \therefore M_D = 40\,tm$ //

$\sum F_V = 0 \quad \uparrow \oplus$

$-2(4) - 3 + V_D = 0 \quad \therefore V_D = 11\,t$ //

2- Ecuaciones de Momento

* Tramo AB

$M_x = -2\,tm$ //

* Tramo BC

$M_x = -2 - \frac{2(x-2)^2}{2}$

$M_x = -x^2 + 4x - 6$ //

* Tramo CD

$M_x = 11(8-x) - 40$

$M_x = -11x + 48$ //

3.- Ecuaciones de Corte

* Tramo AB

$Q_x = 0$

* Tramo BC

$Q_x = -2(x-2)$

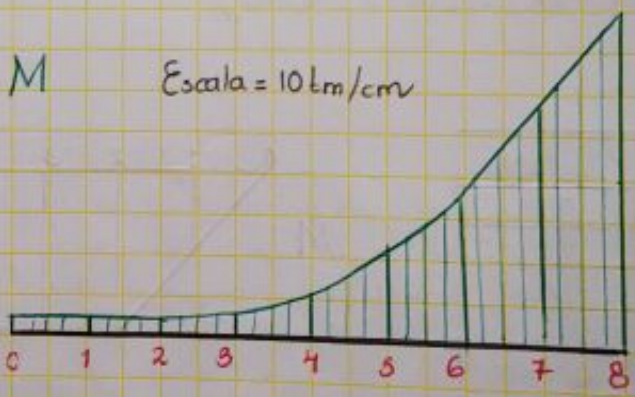
$Q_x = 4 - 2x$ //

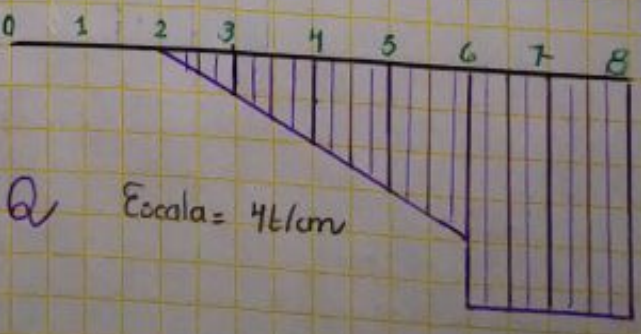
* Tramo CD

$Q_x = -11\,t$ //

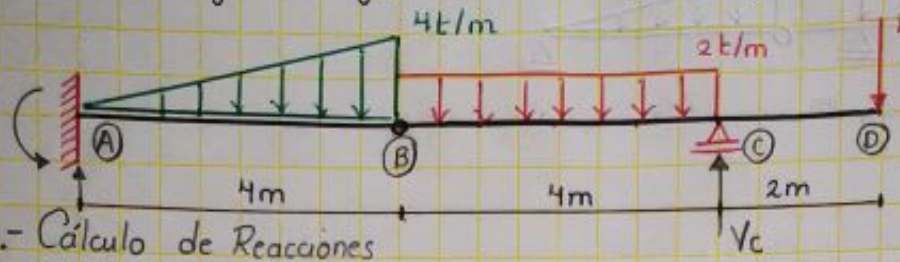
4- Diagramas

x	Momento	Cortante
0	-2	0
1	-2	0
2	-2	0
3	-3	-2
4	-6	-4
5	-11	-6
6	-18	-8 / -11
7	-29	-11
8	-40	-11





39.- Para la siguiente viga determinar los diagramas de los esfuerzos característicos



1.- Cálculo de Reacciones

$\sum M_B = 0$ ↻⊕ a la derecha de la articulación

$2(4)(2) + 4(6) - V_C(4) = 0$

$V_C = 10t$

$\sum F_V = 0$ ↑⊕

$V_A - \frac{4(4)}{2} - 2(4) + 10 - 4 = 0$

$V_A = 10t$ //

$\sum M_B = 0$ ↻⊕ a la izquierda de la articulación

$-M_A + 10(4) - \frac{4(4)}{2} \cdot \frac{1}{3}(4) = 0$

$M_A = 29{,}333\,tm$ //

2.- Momentos

Barra AB $M_A = -29{,}333\,tm$

$M_B = 0\,tm$

Barra BC $M_B = 0\,tm$

$M_C = -4(2) = -8\,tm$

Barra CD $M_C = -4(2) = -8\,tm$

$M_D = 0\,tm$

3.- Cortantes

Barra AB $Q_A = 10t$

$Q_B = 10t - \frac{4(4)}{2} = 2t$ //

Barra BC $Q_B = 10 - \frac{4(4)}{2} = 2t$

$Q_C = 10 - \frac{4(4)}{2} - 2(4) = -6t$

Barra CD $Q_C = 10 - \frac{4(4)}{2} - 2(4) + 10 = 4t$

$Q_D = 4t$

3.- Diagramas

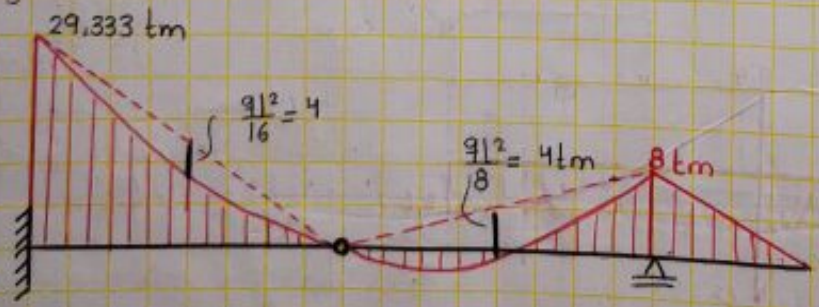


Diagrama de Momento

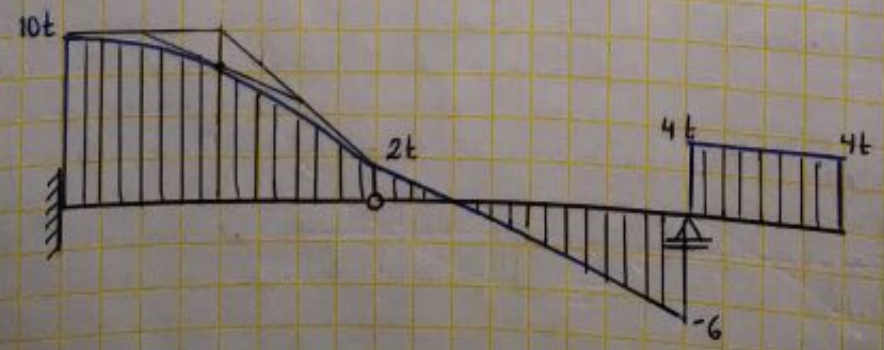
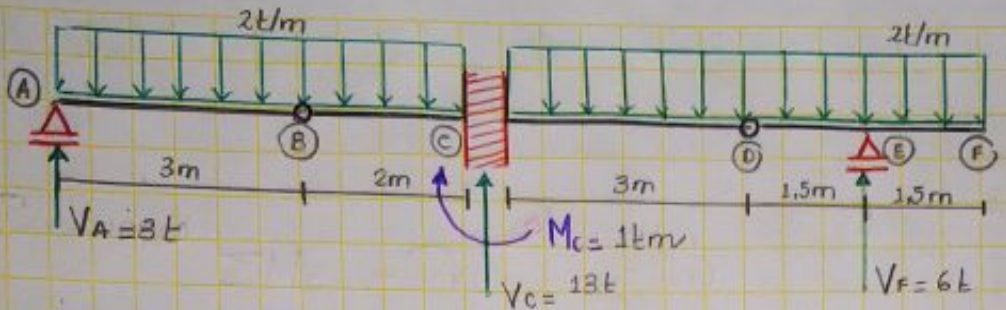


Diagrama de Cortante

40 Diagramar los esfuerzos característicos



1- Cálculo de Reacciones

$\sum M_{B=0}$ ↻ ⊕ (Izquierda)

$V_A(3) - 2(3)(1,5) = 0$

$V_A = 3t$ //

$\sum M_{D=0}$ ↻ ⊕ (Derecho)

$2(3)(1,5) - V_F(1,5) = 0$

$V_f = 6t$

$\sum F_{V=0}$ ↑ ⊕

$3 - 2(5) + V_C - 2(6) + 6 = 0$

$V_C = 13t$

$\sum M_{A=0}$ ↻ ⊕

$2(5)(2,5) - 13(5) + M_C + 2(6)(8) - 6(9,5) = 0$

$M_C = 1tm$

2- Ecuaciones de Momento

* Tramo AC

$$M_x = 3 \cdot x - \frac{2x^2}{2} = 3x - x^2 \text{ //}$$

* Tramo CE

$$M_x = 3x - 2(5)(x-2,5) + 1 + 13(x-5) - \frac{2(x-5)^2}{2}$$

$$M_x = -x^2 + 16x - 64 \text{ //}$$

* Tramo EF

$$M_x = -\frac{2(11-x)^2}{2}$$

$$M_x = -x^2 + 22x - 121 \text{ //}$$

3- Ecuaciones de Corte

* Tramo AC

$Q_x = 3 - 2x$ //

* Tramo CE

$Q_x = 3 - 2x + 13$

$Q_x = 16 - 2x$ //

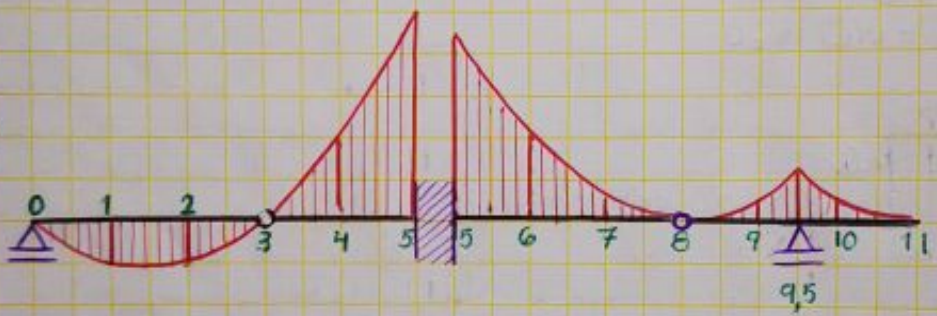
* Tramo EF

$Q_x = 2(11-2x) = 22 - 2x$ //

4- Diagramas

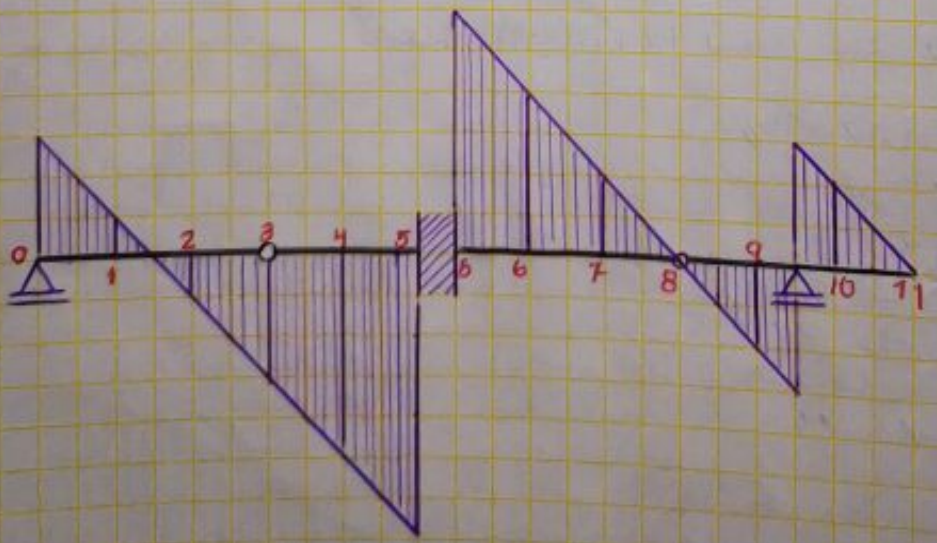
x	Momento	Cortante
0	0	3
1	2	1
2	2	-1
3	0	-3
4	-4	-5
5	-10 / -9	-7 / 6
6	-4	4
7	-1	2
8	0	0
9	-1	-2
9,5	-2,25	-3 / 3
10	-1	2
11	0	0

M



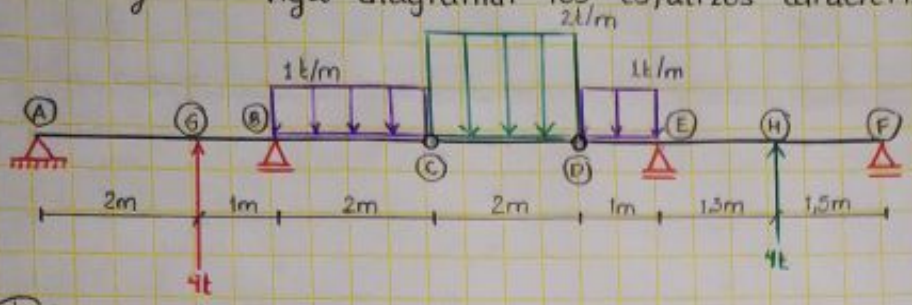
Escala = 4 tm/cm

Q

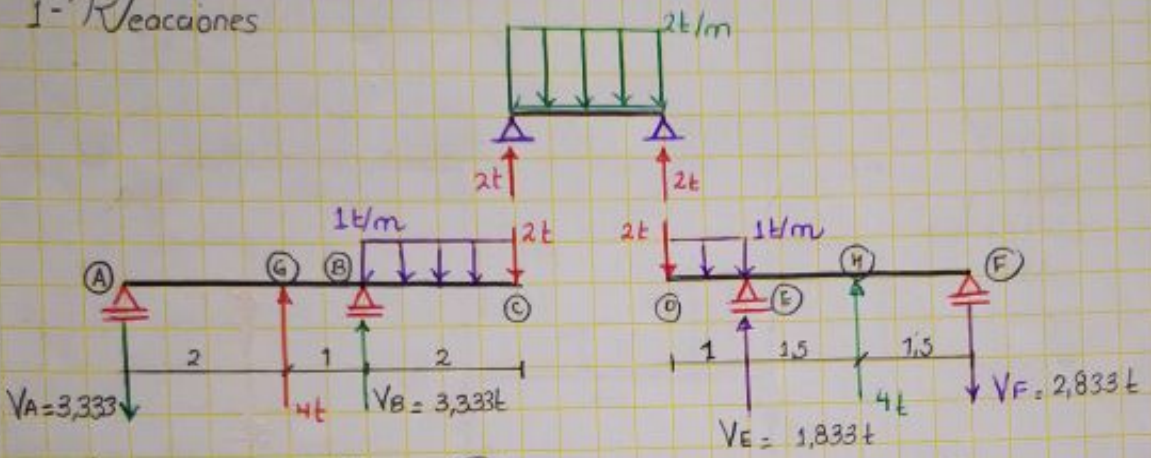


Escala = 2 t/cm

41 - Para la siguiente viga diagramar los esfuerzos característicos



1- Reacciones



Viga ABC $\sum M_{A=0}$ ↻ ⊕

$-4(2) + 1(2)(4) + 2(5) - V_B(3) = 0$ ∴ $V_B = 3,333\ t$

$\sum F_{V=0}$ ↑ ⊕

$V_A + 4 + 3,333 - 2 - 1(2) = 0$ ∴ $V_A = 3,333\ t$

Viga DE F

$\sum M_{E=0}$ ↻ ⊕

$-2(1) - 1(1)(0,5) - 4(1,5) + V_F(3) = 0$ ∴ $V_F = 2,833\ t$ //

$\sum F_{V=0}$ ↑ ⊕

$-2 - 1(1) + 4 - 2.833 + V_E = 0$ ∴ $V_E = 1,833\ t$ //

2- Ecuaciones de Momento

* Tramo AG

$M_x = -3,333X$ //

* Tramo GB

$M_x = -3,333X + 4(X-2)$

$M_x = 0,667X - 8$ //

* Tramo BC

$M_x = -3,333X + 4(X-2) + 3,333(X-3) - \frac{1(X-3)^2}{2}$

$M_x = -0,5X^2 + 7X - 22,499$ //

* Tramo CD

$M_x = 2(X-5) - \frac{2(X-5)^2}{2} = -X^2 + 12X - 35$ //

* Tramo DE

$M_x = -2(X-7) - \frac{1(X-7)^2}{2} = -0,5X^2 + 5X - 10,5$ //

* Tramo EH

$M_x = -2,833(11-X) + 4[11-X-1,5]$

$M_x = -1,167X + 6,837$ //

* Tramo HF

$M_x = -2,833(11-X)$

3- Ecuaciones de Corte

* Tramo AG

$Q_x = -3,333\ t$

* Tramo GB

$Q_x = -3,333 + 4 = 0,667\ t$

* Tramo BC

$Q_x = -3,333 + 4 + 3,333 - 1(X-3)$

$Q_x = 7 - X$

* Tramo CD

$Q_x = 2 - 2(X-5)$

$Q_x = 12 - 2X$ //

* Tramo DE

$Q_x = -2 - 1(X-7)$

$Q_x = 5 - X$ //

* Tramo EH

$Q_x = 2,833 - 4 = 1,167\ t$ //

* Tramo HF

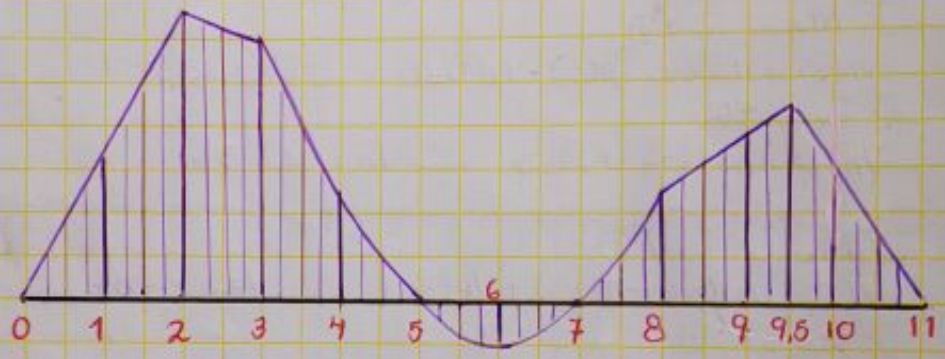
$Q_x = 2,833\ t$ //

x	Momento	Cortante
0	0	-3,333
1	-3,333	-3,333
2	-6,666	-3,333/0,667
3	-5,999	0,667 / 4
4	-2,499	3
5	0	2
6	1	0
7	0	-2
8	-2,5	-3/1,167
9	-3,666	1,167
9,5	-4,2495	1,167/2.833
10	-2,833	2,8333
11	0	2,8333

Diagramas

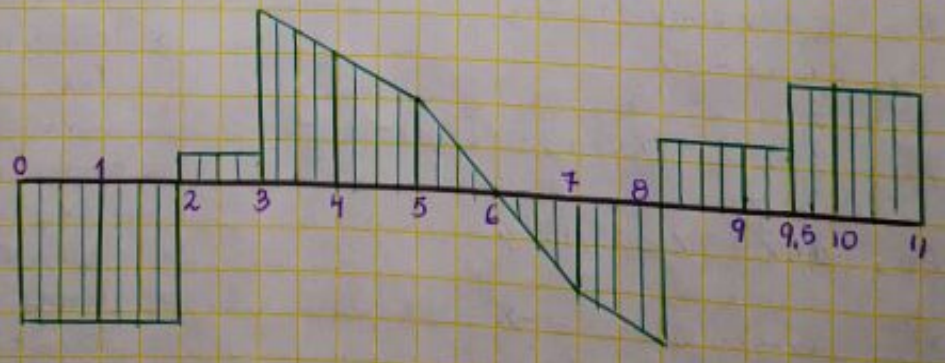
Momentos

Escala = 2tm/cm

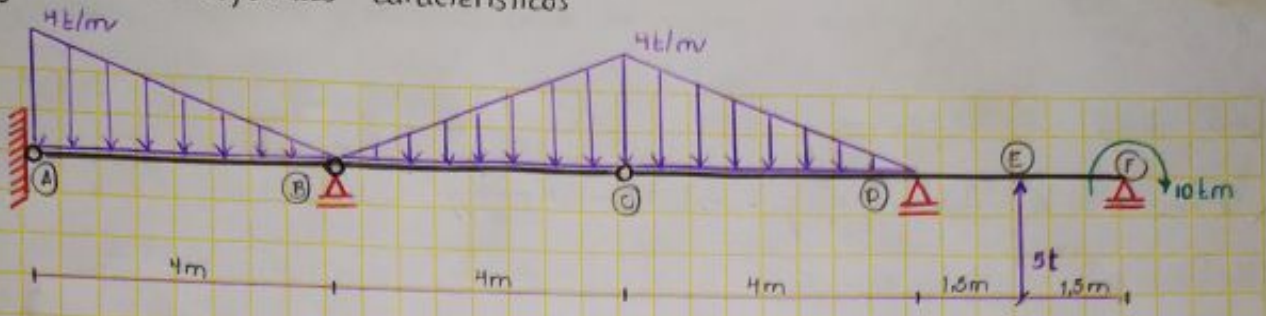


Cortante

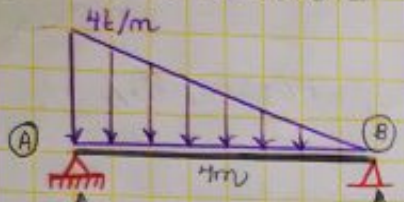
Escala = 2t/cm

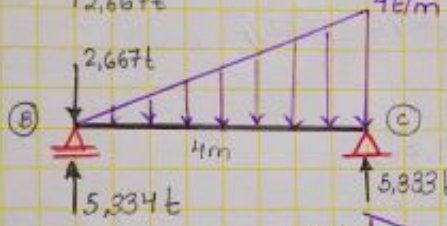


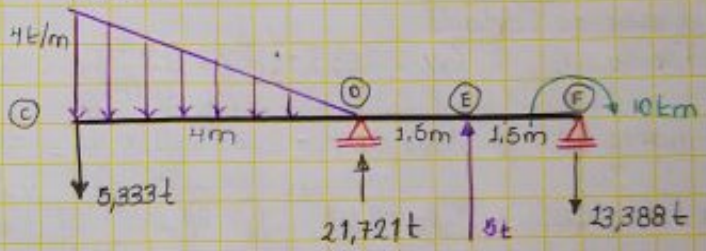
42 Diagramar los esfuerzos caracteristicos



1.- Cálculo de Reacciones







* Barra AB

$\sum M_{A=0}$ ↻ (+)

$\frac{4(4)}{2} \cdot \left[\frac{1}{3}(4)\right] - V_B(4) = 0$

$V_B = 2,667\,t$ //

$\sum F_V = 0$ ↑ (+)

$V_A - \frac{4(4)}{2} + 2,667 = 0$

$V_A = 5,333\,t$ //

* Barra BC

$\sum M_{C} = 0$ ↻ (+)

$\frac{4(4)}{2}\left[\frac{2}{3} \cdot 4\right] - V_C(4) = 0$

$V_C = 5,333\,t$ //

$\sum F_V = 0$ ↑ (+)

$-2,667 - \frac{4(4)}{2} + 5,333 + V_B = 0$

$V_B = 5,334\,t$

* Barra CF

$\sum M_{D} = 0$ ↻ (+)

$-5,333(4) - \frac{4(4)}{2} \cdot \left[\frac{2}{3}(4)\right] - 5(1,5) + 10 + V_F(3) = 0$

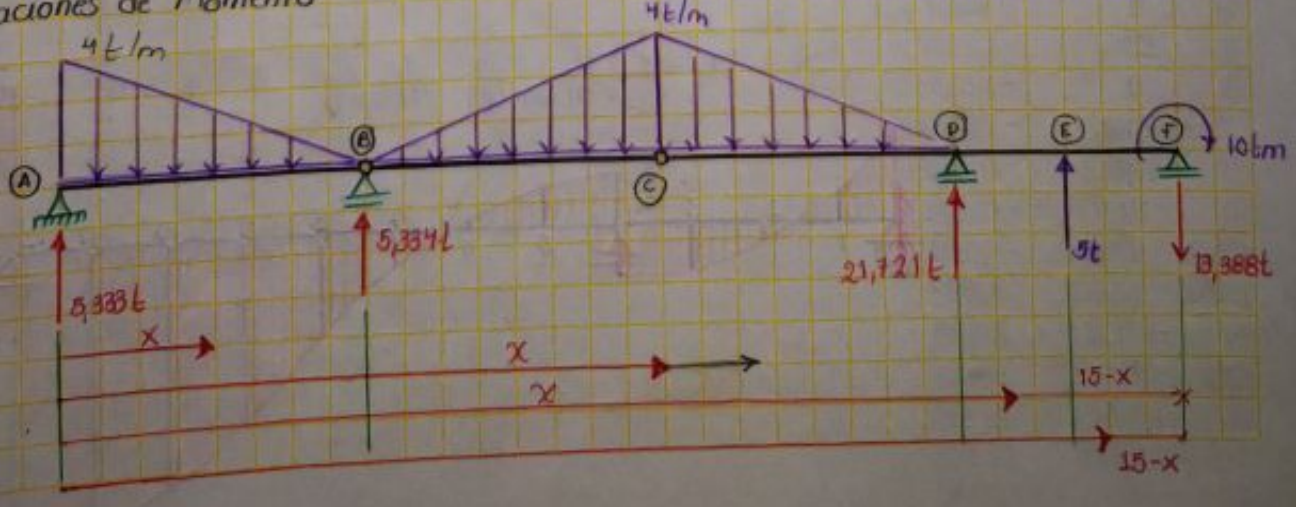
$V_F = 13,3884\,t$ //

$\sum F_V = 0$ ↑ (+)

$-5,333 - \frac{4(4)}{2} + V_D + 5 - 13,388 = 0$

$V_D = 21,721\,t$ //

2.- Ecuaciones de Momento



* Tramo AB

$$M_x = 5{,}333\,x - \left(4\frac{x^2}{2} - \frac{4x^3}{6(4)}\right) = 5{,}333\,X - 2X^2 + \frac{X^3}{6}\ //$$

* Tramo BC

$$M_x = (5{,}333 - 2{,}667)(x-4) - \frac{4(x-4)^3}{6(4)} = -0{,}1667\,x^3 + 2X^2 - 5{,}334X + 0{,}003\ //$$

* Tramo CD

$$M_x = -5{,}333\,(x-8) - \left[\frac{4(x-8)^2}{2} - \frac{4(x-8)^3}{6(4)}\right] = 0{,}167\,X^3 - 6X^2 + 58{,}667X - 170{,}669\ //$$

* Tramo DE

$$M_x = -10 - 13{,}388\,(15-x) + 5(13{,}5-x) = 8{,}388X - 143{,}32\ //$$

* Tramo EF

$$M_x = -10 - 13{,}388\,(15-x) = 13{,}388X - 210{,}82\ //$$

3- Ecuaciones Cortante

* Tramo AB $\quad Q_x = 5{,}333 - \left(4\cdot x - \frac{4x^2}{2(4)}\right) = 0{,}5X^2 - 4X + 5{,}333\ //$

* Tramo BC $\quad Q_x = 5{,}334 - 2{,}667 - \frac{4(x-4)^2}{2(4)} = -0{,}5X^2 + 4X - 5{,}333\ //$

* Tramo CD $\quad Q_x = -5{,}333 - \left[4(x-8) - \frac{4(x-8)^2}{2(4)}\right] = 0{,}5X^2 - 12X + 58{,}667\ //$

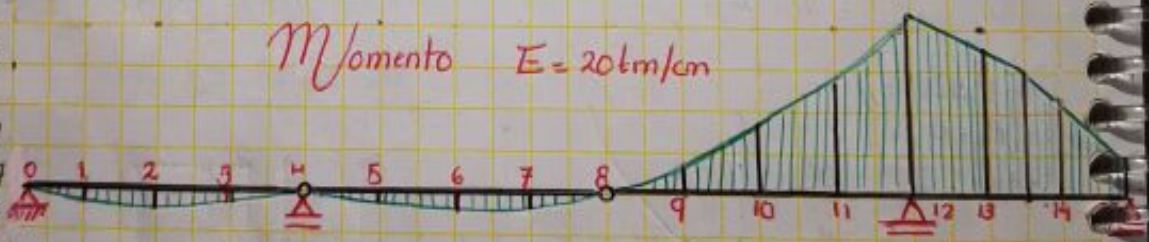
* Tramo DE $\quad Q_x = +13{,}388 - 5 = 8{,}388\ t\ //$

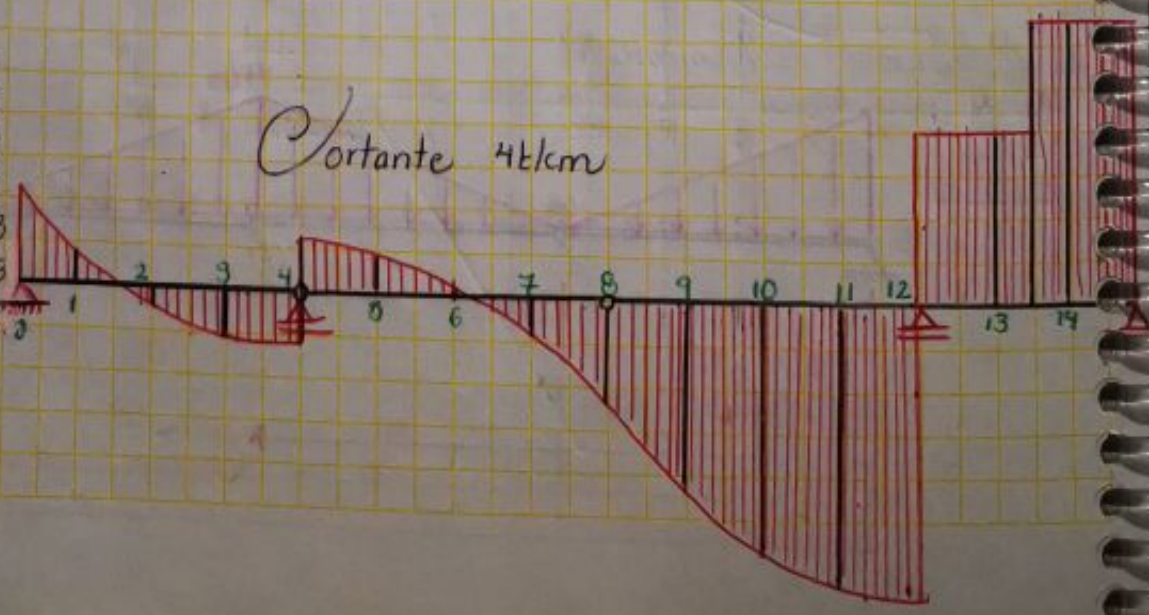
* TramoEF $\quad Q_x = 13{,}388\ t =$

20 tm/cm

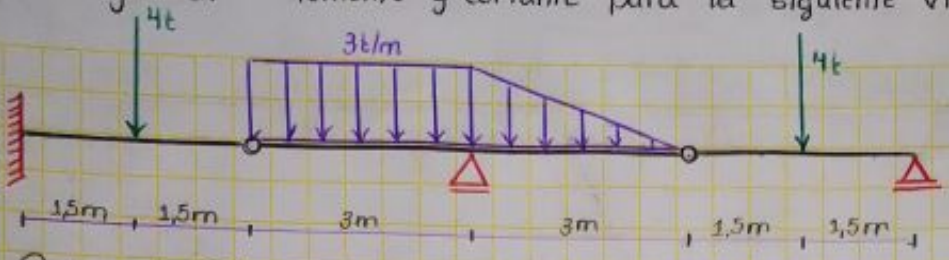
4.- Diagramas

x	Momento	Q
0	0	5,8333
1	3,5	18333
2	4	-0,6667
3	2,5	-2,1667
4(I)	0	-2,6667
4(D)	0	2,6667
5	2,5	2,1667
6	4	0,6667
7	3,5	-1,8383
8	0	-5,3333
9	-7,166	-8,8833
10	-17,333	-11,3333
11	-29,499	-12,8833
12(I)	-42,666	-13,3333
12(D)	-42,660	8,388
13	-34,266	8,388
13,5	-30,0712	8,388
13,5	-30,082	13,388
14	-23,388	13,388
15	-10	13,388

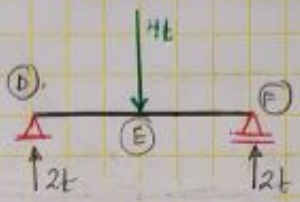


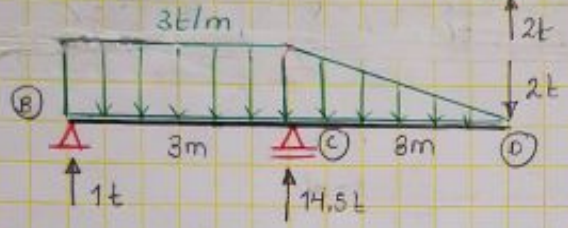


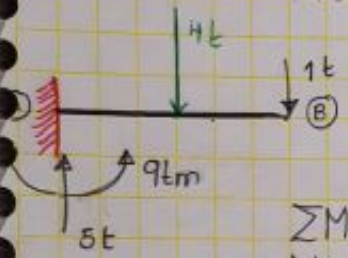
43 Diagramar Momento y cortante para la siguiente viga.



1.- Calculo de Reacciones







$\sum M_A = 0$

$M_A = 4(1,5) + 1(3) = 9tm$

$\sum F_V = 0$

$V_A = 5t$ //

* Barra BD

$\sum M_C = 0$ ↻ ⊕

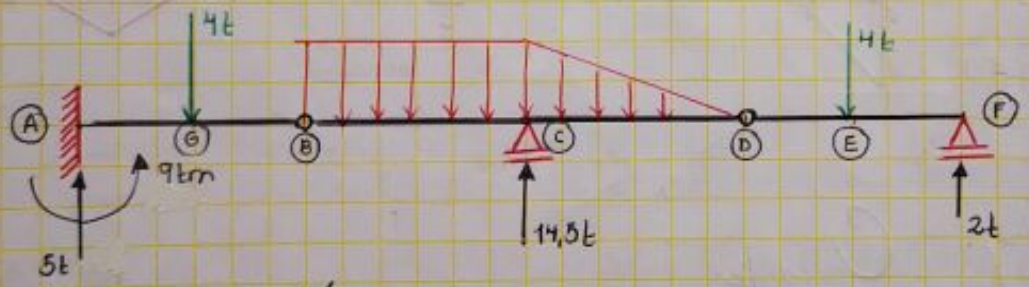
$2(3) + \frac{3(3)}{2}.(1) - 3(3)(1,5) + V_B(3) = 0$

$V_B = 1t$ //

$\sum F_V = 0$ ↑ ⊕

$1 - 3(3) - \frac{3(3)}{2} - 2 + V_C = 0$

$V_C = 14,5t$ //



2.- Ecuaciones de Momento

* Tramo AG $M_X = 5 \cdot X - 9$ //

* Tramo GB $M_X = -1(3-X) = -3 + X$

* Tramo BC $M_X = 1 \cdot (x-3) - 3\frac{(x-3)^2}{2} = -1,5x^2 + 10x - 16,5$

* Tramo CD $M_X = 1(x-3) - 9(x-4,5) + 14,5(x-6) - \left[\frac{3 \cdot (x-6)^2}{2} - \frac{3(x-6)^3}{6(3)}\right]$

$M_X = 0,1667\,x^3 - 4,5x^2 + 42,5\,x - 139,5$ //

* Tramo DE $M_X = 2 \cdot (x-9) = 2x - 18$ //

* Tramo EF $M_X = 2(12-x) = 24 - 2x$ //

3- Ecuaciones de Cortante

* Tramo AG

$Q_x = 5t$

* Tramo GB

$Q_x = 5 - 4 = 1t$

* Tramo BC

$Q_x = 1 - 3(x-3) = -3x + 10$

* Tramo CD

$Q_x = 1 - 3(3) + 14,5 - \left(3(x-6) - \frac{3(x-6)^2}{2(3)} \right) = 0,5x^2 - 9x + 42,5$ //

* Tramo DE

$Q_x = 2t$

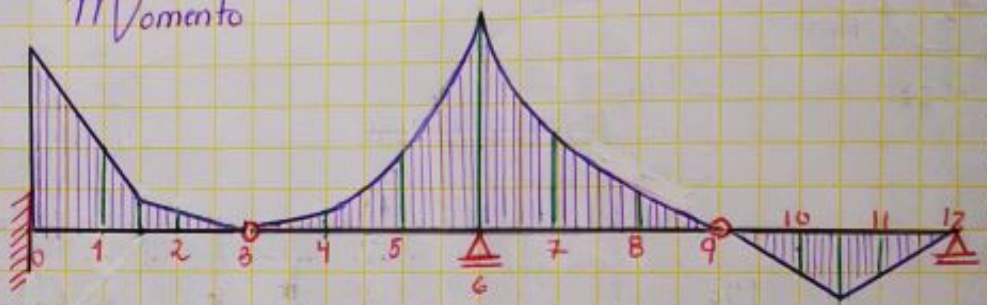
* Tramo EF

$Q_x = -2t$

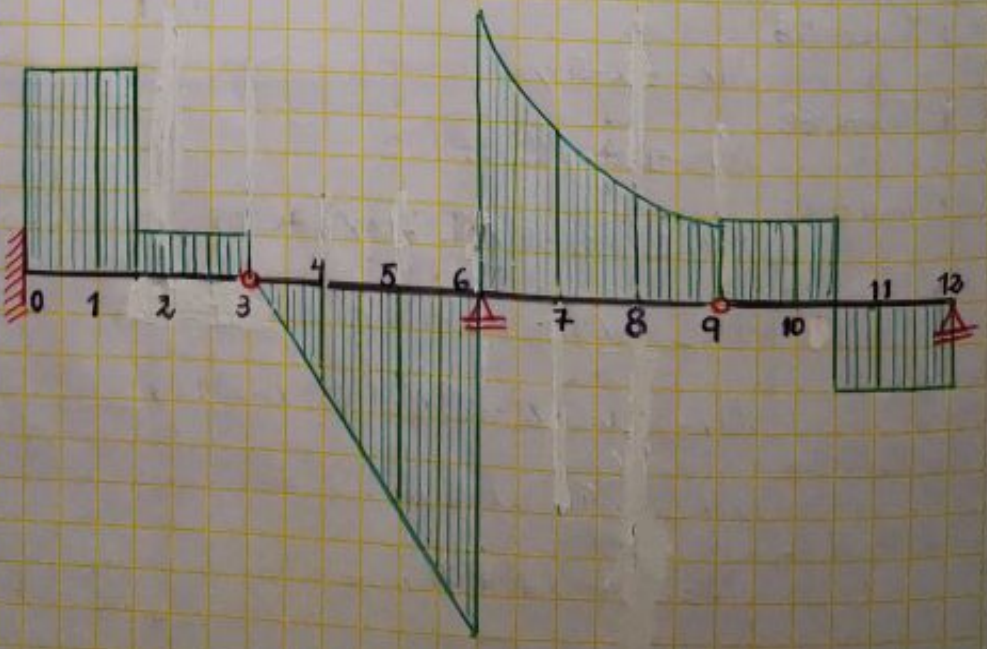
4- Diagramas

x	Momento	Cortante
0	-9	5
1	-4	5
1,5	-1,5	5/1
2	-1	1
3	0	1
4	-0,5	-2
5	-4	-5
6	-10,5	-8 / 6,5
7	-5,333	4
8	-2,166	2,5
9	0	2
10	2	2
10,5	3	2/-2
11	2	-2
12	0	-2

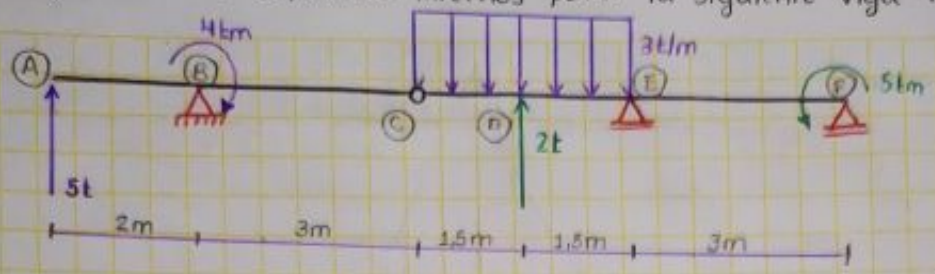
Momento



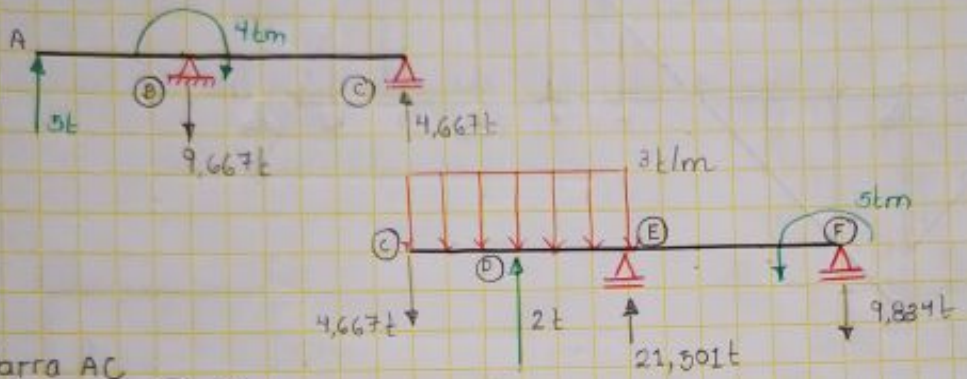
Cortante



44 Diagramar los esfuerzos internos para la siguiente viga Herber



1.- Reacciones



* Barra AC

$\sum M_B = 0$ ↷ ⊕

$5(2) + 4 - V_C(3) = 0$

$V_C = 4,667t$ //

$\sum F_V = 0$ ↑ ⊕

$5 - V_B + 4,667 = 0$

$V_B = 9,667t$ //

* Barra CF

$\sum M_E = 0$ ↶ ⊕

$-4,667(3) + 2(1,5) - 3(3)(1,5) - 5 + V_F(3) = 0$

$V_F = 9,834t$ //

$\sum F_V = 0$ ↑ ⊕

$-4,667 + 2 - 3(3) + V_E - 9,834 = 0$

$V_E = 21,301t$ //

2.- Ecuaciones de Momento

* Tramo AB $M_x = 5 \cdot x$
* Tramo BC $M_x = 4,667(5-x) = -4,667x + 23,335$ //
* Tramo CD $M_x = -4,667(x-5) - \frac{3(x-5)^2}{2} = -1,5x^2 + 10,333x - 14,165$ //
* Tramo DE $M_x = -4,667(x-5) - \frac{3(x-5)^2}{2} + 2(x-6,5) = -1,5x^2 + 12,333x - 27,165$ //
* Tramo EF $M_x = 5 - 9,834(11-x) = 9,834x - 103,174$ //

3.- Ecuaciones de Cortante

* Tramo AB $Q_x = 5t$
* Tramo BC $Q_x = -4,667t$
* Tramo CD $Q_x = -4,667 - 3(x-5) = -3x + 10,333$ //
* Tramo DE $Q_x = -4,667 + 2 - 3(x-5) = -3x + 12,333$ //
* Tramo EF $Q_x = 9,834t$ //

4.- Diagramas

x	Momento	Cortante
0	0	5
1	5	5
2	10/14	5/-4,667
3	9,333	-4,667
4	4,667	-4,667
5	0	-4,667
6	-6,165	-7,667

x	Momento	Cortante
6,5	-10,373	-9,167/-7,167
7	-14,332	-8,667
8	-24,498	-11,667/9,834
9	-14,668	9,834
10	-4,834	9,834
11	5	9,834

Diagrama de Momento

Esc = 5tm/cm

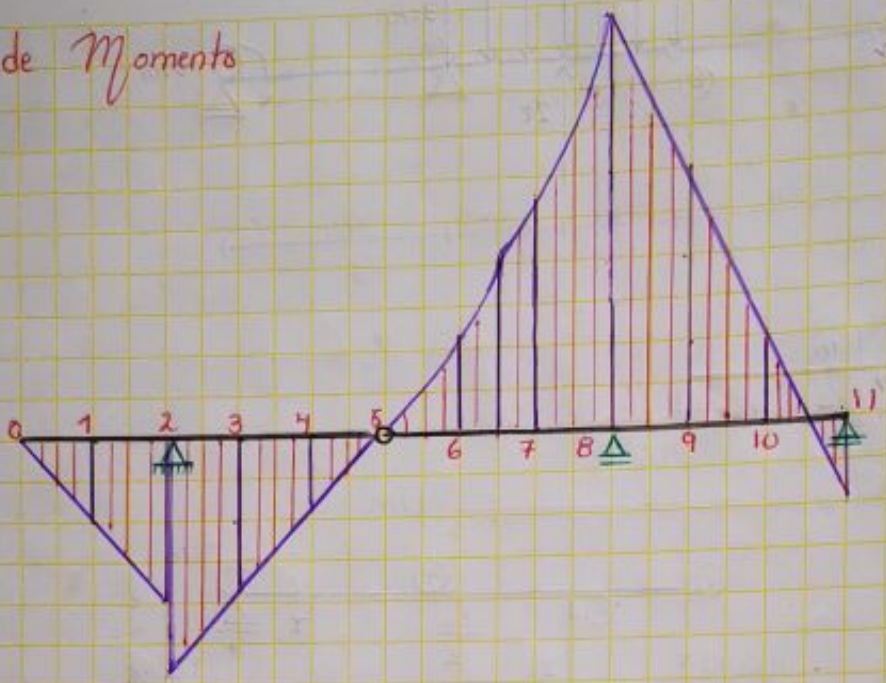
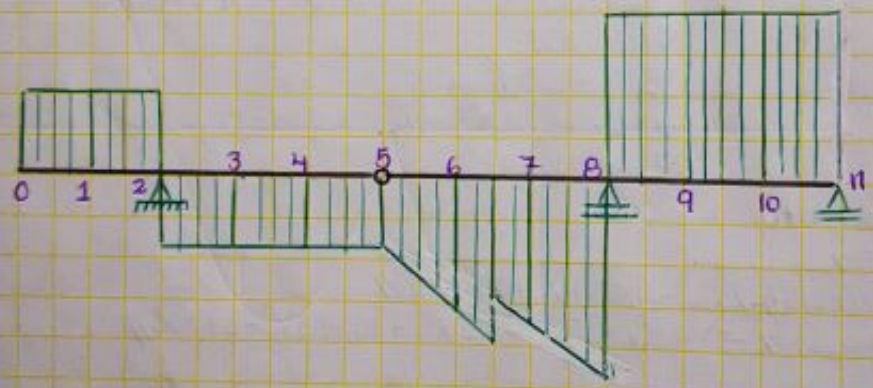
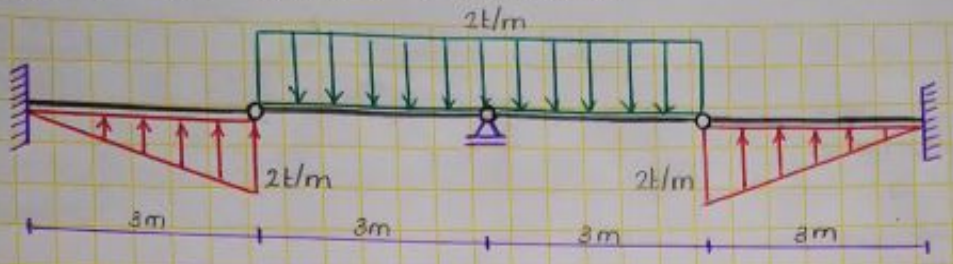


Diagrama de Cortante

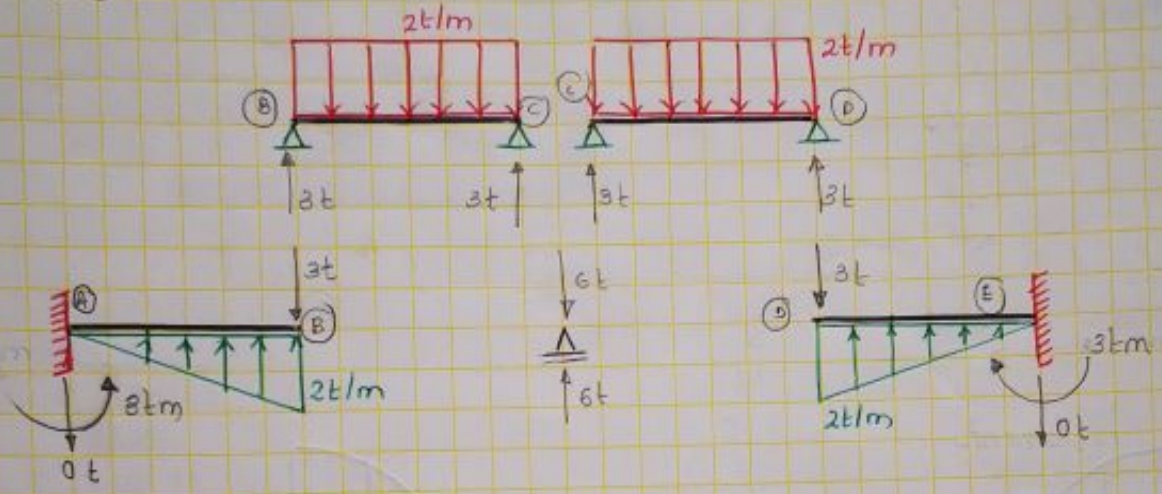
Esc = 5tm/cm



45 Diagramar los esfuerzos característicos



1 Reacciones



* Tramo AB = DE

$\sum M_A = 0$ ↻ ⊕

$-M_A - \frac{2(3)}{2}(2) + 3(3) = 0$

$M_A = 3\ tm$ //

$\sum F_V = 0$ ↑ ⊕

$-V_A + \frac{2(3)}{2} - 3 = 0 \quad \therefore V_A = 0\ t$ //

2 - Ecuaciones de Momento

* Tramo AB $\quad M_x = -3 + \frac{2x^3}{6(3)} = -3 + \frac{x^3}{9}$ //

* Tramo BC $\quad M_x = 3(x-3) - \frac{2(x-3)^2}{2} = -x^2 + 9x - 18$ //

* Tramo CD $\quad M_x = 3(x-6) - \frac{2(x-6)^2}{2} = -x^2 + 15x - 54$ //

* Tramo DE $\quad M_x = -3(x-9) + \left[\frac{2(x-9)^2}{2} - \frac{2(x-9)^3}{6(3)}\right] = \frac{-x^3 + 36x^2 - 432x + 1701}{9}$ //

3.- Ecuaciones de Cortante

* Tramo AB $\quad Q_x = \frac{2x^2}{2(3)} = \frac{x^2}{3}$ //

* Tramo BC $\quad Q_x = 3 - 2(x-3) = 9 - 2x$ //

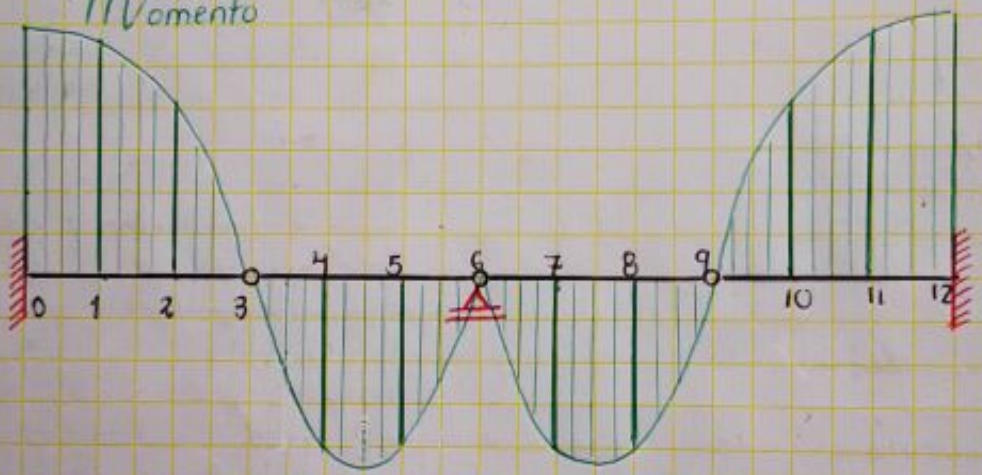
* Tramo CD $\quad Q_x = 3 - 2(x-6) = 15 - 2x$ //

* Tramo DE $\quad Q_x = -3 + \left[2(x-9) - \frac{2(x-9)^2}{2(3)}\right] = -0{,}333x^2 + 8x - 48$ //

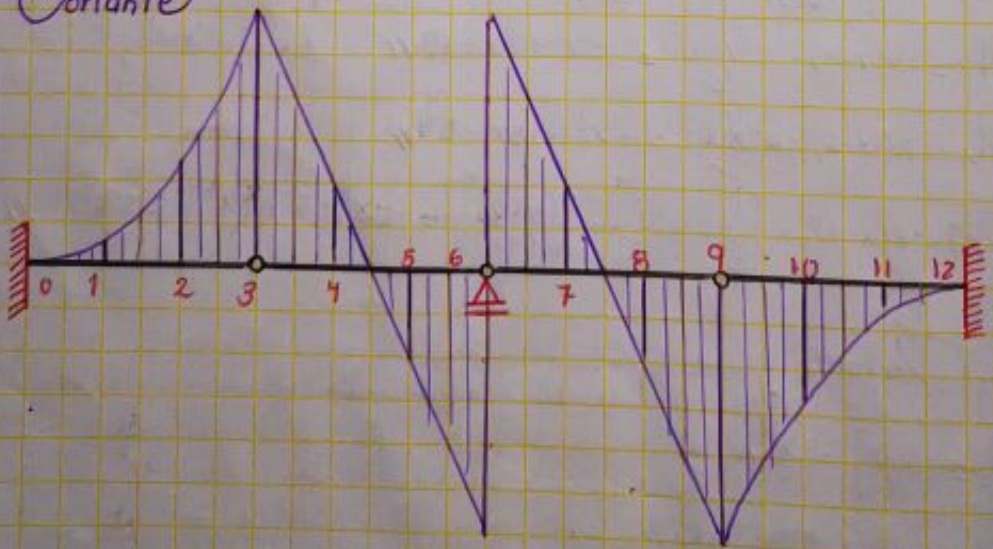
1.-Diagramas

x	Momento	Cortante
0	-3	0
1	-2,889	0,333
2	-2,111	1,3333
3	0	3
4	2	1
5	2	-1
6	0	-3 /3
7	2	1
8	2	-1
9	0	-3
10	-2,111	-1,333
11	-2,888	-0,333
12	- 3	0

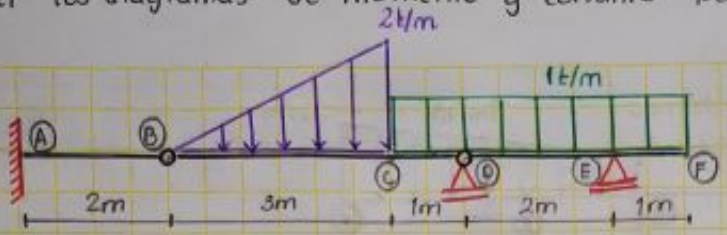
Momento



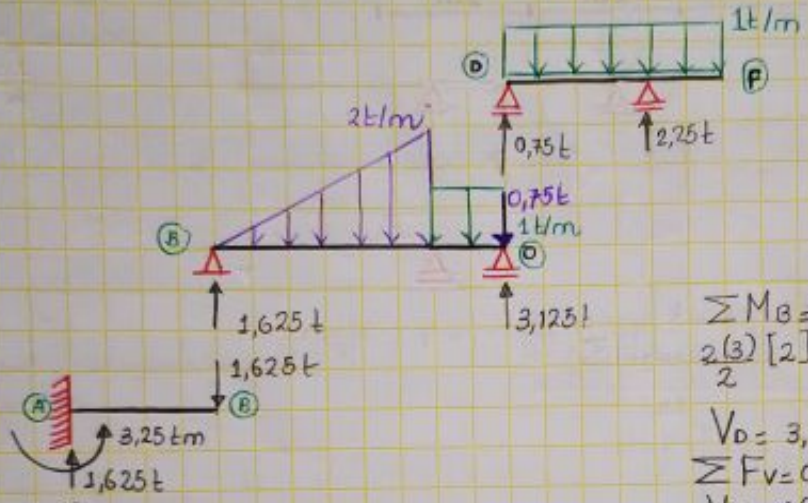
Cortante



46 Obtener los diagramas de momento y cortante para la siguiente viga



1.-Reacciones



* Barra DF

$\sum M_{(D)}=0$ ↺ (+)

$1(3)(1,5) - V_E(2)=0$

$V_E = 2,25t$ //

$\sum F_V=0$ ↑ (+)

$V_C - 1(3) + 2,25 = 0$

$V_C = 0,75t$

$\sum M_B=0$ (Barra BD)

$\frac{2(3)}{2}[2] + 1(1)(3,5) + 0,75(4) - V_D(4)=0$

$V_D = 3,125t$

$\sum F_V=0$ ↑ (+)

$V_B - \frac{2(3)}{2} - 1(1) - 0,75 + 3,125 = 0$

$V_B = 1,625t$ //

* Barra AB

$\sum F_V=0$ ↑ (+)

$V_A = 1,625t$

$\sum M_A=0$ ↺ (+)

$M_A = 1,625(2) = 3,25\ tm$ //

2.- Momentos

$M_A = -3,25tm$

$M_B = 0\ tm$

$M_C = 1,625(3) - \frac{2(3)}{2}.[1] = 1,875tm$

$M_D = 0$

$M_E = -1(1)(0,5) = -0,5tm$

3.- Cortantes

$Q_A = 1,625t$ //

$Q_B = 1,625t$ //

$Q_C = 1,625 - \frac{2(3)}{2} = -1,375t$ //

$Q_{D\,(IZQ)} = 1,625 - \frac{2(3)}{2} - 1(1) = -2,375t$ //

$Q_{D\,(DER)} = 0,75t$ //

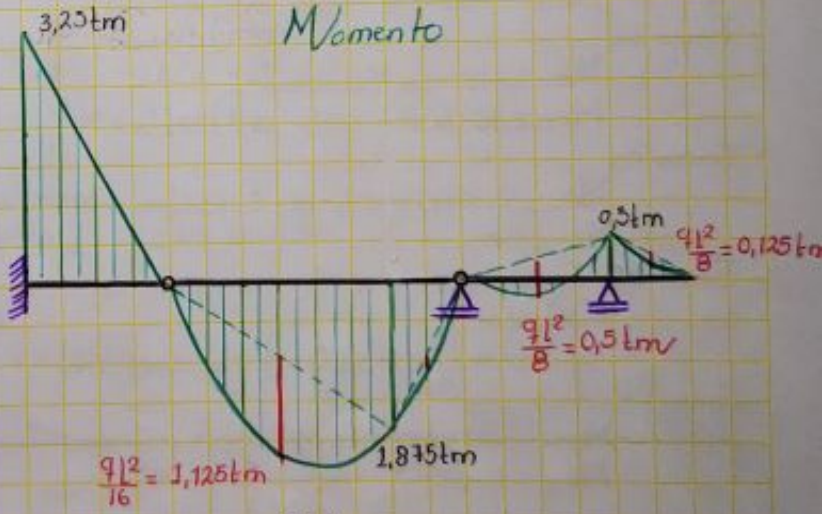
$Q_{E\,(IZQ)} = 0,75 - 1(2) = -1,25t$ //

$Q_{E\,(DER)} = 1(1) = 1t$ //

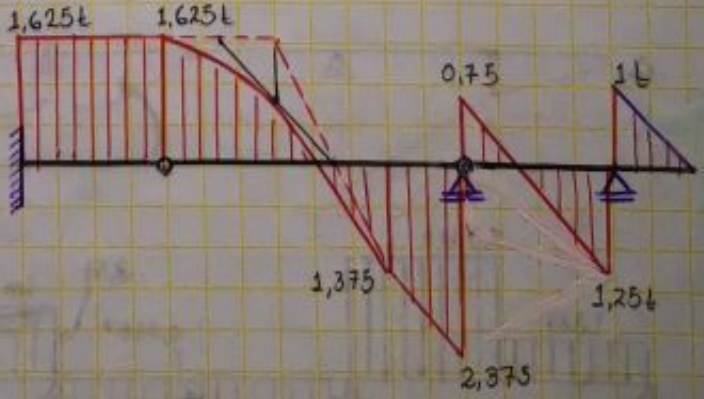
$Q_F = 0t$ //

4.- Diagramas

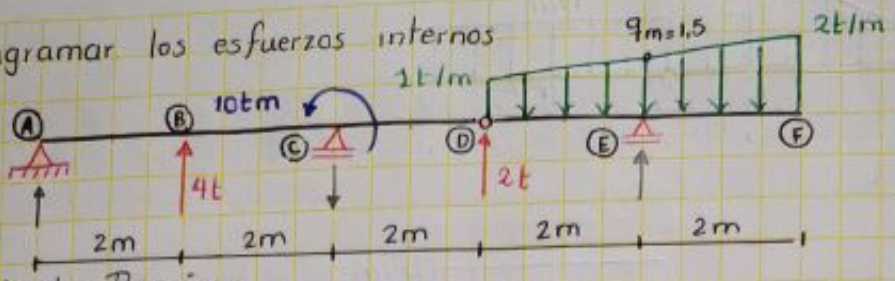
Momento



Cortante



47 Diagramar los esfuerzos internos



1.- Cálculo de Reacciones

$\sum M_{(D)}=0$ ↻ ⊕ (a la derecha de la articulación)

$1(4)(2) + \frac{1(4)}{2}\cdot\frac{2}{3}(4) - V_E(2)=0$

$V_E = 6,6667\,t$

$\sum M_A=0$ ↻ ⊕

$-4(2) - 10 + V_C(4) - 2(6) - 6,6667(8) + 1(4)8 + \frac{1(4)}{2}\cdot\left(6+\frac{2}{3}(4)\right)=0$

$V_C = 8,5\,t$

$\sum F_V=0$ ↑ ⊕

$V_A + 4 - 8,5 + 2 + 6,6667 - \left(\frac{1+2}{2}\right)\cdot 4 = 0$

$V_A = 1,8333\,t$ //

2.- Momentos

Tramo AB

$M_A = 0$

$M_B = 1,8333(2) = 3,6666\,tm$

Tramo BC

$M_B = 1,8333(2) = 3,6666\,tm$

$M_C = 1,8333(4) + 4(2) = 15,3332\,tm$

Tramo CD

$M_C = 1,8333(4) + 4(2) - 10 = 5,3332\,tm$

$M_D = 0\,tm$

Tramo DE

$M_D = 0\,tm$

$M_E = -(1,5\cdot 2)\cdot 1 - \frac{0,5(2)}{2}\cdot\frac{2}{3}(2) = 3,666\,tm$

Tramo EF

$M_E = 3,666\,tm$

$M_F = 0\,tm$

3.- Cortante

Tramo AB

$Q_A = 1,8333\,t$

$Q_B = 1,8333\,t$

Tramo BC

$Q_{B'} = 1,8333 + 4 = 5,8333\,t$

$Q_C = 1,8333 + 4 = 5,8333\,t$

Tramo CD

$Q_C = 1,8333 + 4 - 8,5 = -2,6667\,t$

$Q_D = -2,6667\,t$

Tramo DE

$Q_D = 1,8333 + 4 - 8,5 + 2 = -0,6667\,t$

$Q_E = 1,8333 + 4 - 8,5 + 2 - \left(\frac{1+1,5}{2}\right)\cdot 2$

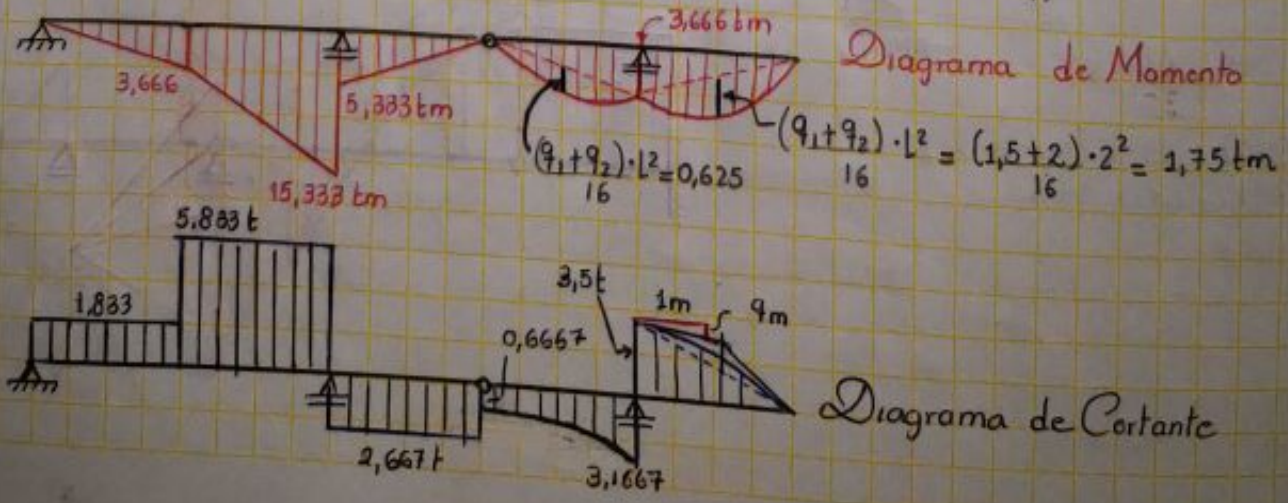
$Q_E = -3,1667\,t$ //

Tramo EF

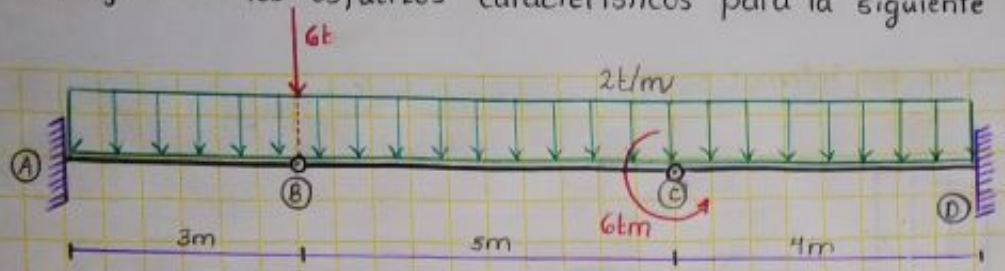
$Q_E = \left(\frac{1,5+2}{2}\right)\cdot 2 = 3,5\,t$ //

$Q_F = 0\,t$ //

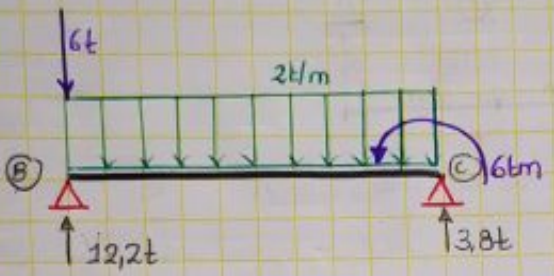
4.- Diagramas



48 Diagramar los esfuerzos característicos para la siguiente viga compuesta



1-Reacciones



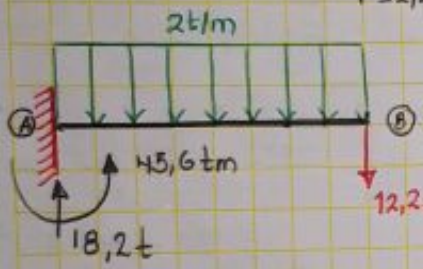
* Barra BC

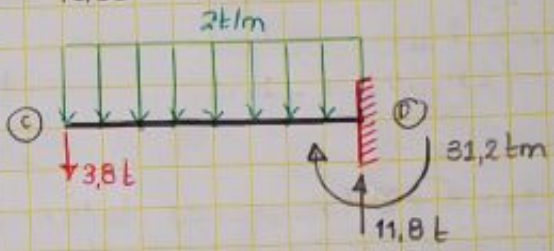
$\sum M_B = 0$ ↻ (+)

$2(5)(2,5) - 6 - V_C(5) = 0 \quad \therefore V_C = 3,8t$ //

$\sum F_V = 0$ ↑ (+)

$V_B - 6 - 2(5) + 3,8 = 0 \quad \therefore V_B = 12,2t$ //





* Barra AB

$\sum F_V = 0$ ↑ (+)

$V_A - 2(3) - 12,2 = 0$

$V_A = 18,2t$ //

$\sum M_A = 0$ ↻ (+)

$M_A = 2(3)(1,5) + 12,2(3)$

$M_A = 45,6tm$ //

* Barra CD

$\sum F_V = 0$ ↑ (+)

$-3,8 - 2(4) + V_D = 0$

$V_D = 11,8t$ //

$\sum M_D = 0$ ↻ (+)

$M_D - 3,8(4) - 2(4)(2) = 0$

$M_D = 31,2tm$ //

2 Momentos

$M_A = -45,6tm$

$M_B = 0tm$

$M_{C(IZQ)} = 6tm$

$M_{C(DER)} = 0tm$

$M_D = -31,2tm$

3-Cortantes

$Q_A = 18,2t$

$Q_{B(IZQ)} = 18,2 - 2(3) = 12,2t$

$Q_{B(DER)} = 18,2 - 2(3) - 6 = 6,2t$

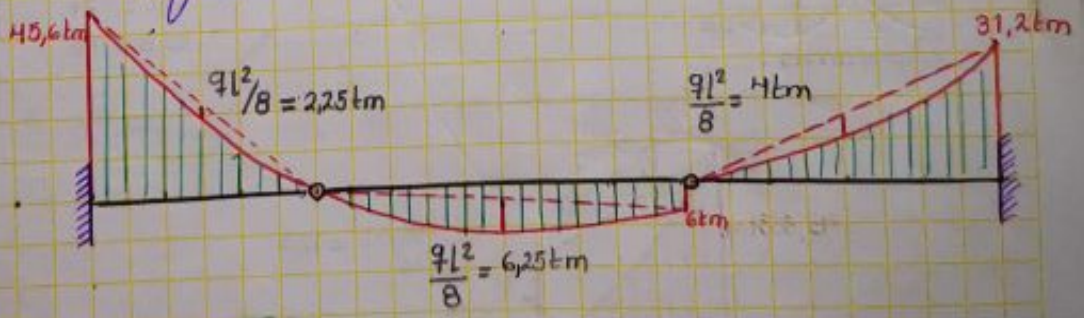
$Q_{C(IZQ)} = -3,8t$

$Q_{C(DER)} = -3,8t$

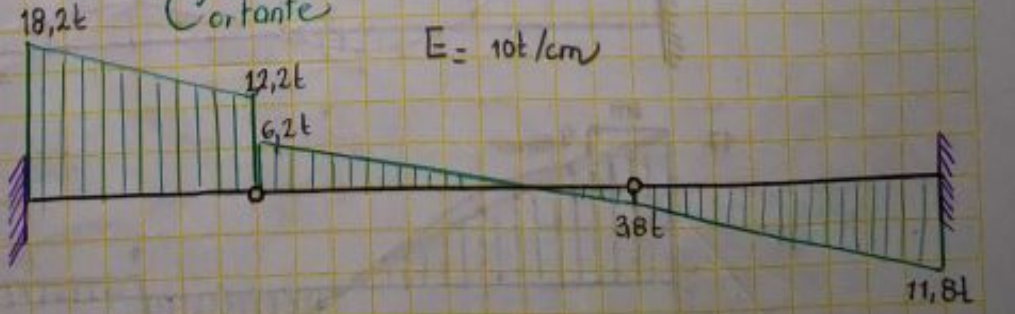
$Q_D = -11,8t$

4-Diagramas

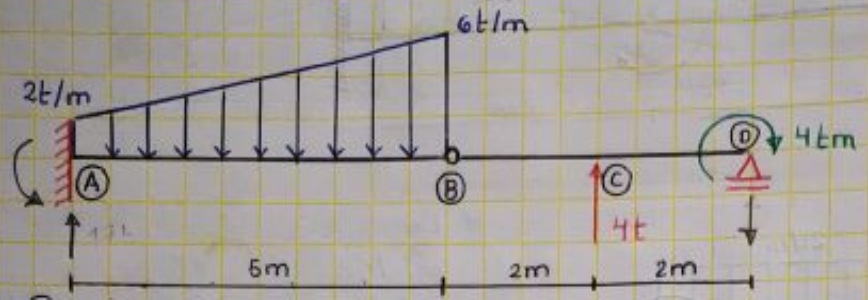
Momento Esc = 20tm/cm



Cortante E = 10t/cm



49 Diagramar los esfuerzos internos



1.- Reacciones

$\sum M_B = 0$ ↻⊕ (a la derecha de la articulación)

$-4(2) + 4 + V_D(4) = 0$

$V_D = 1t$ //

$\sum F_V = 0$ ↑⊕

$V_A - \left(\frac{2+6}{2}\right).5 + 4 - 1 = 0$

$V_A = 17t$ //

$\sum M_B = 0$ ↻⊕ (a la izquierda de la articulación)

$17(5) - 2(5)(2,5) - \frac{4(5)}{2}.\left[\frac{1}{3}(5)\right] - M_A = 0$

$M_A = 43,333\,tm$ //

2.- Momentos

Tramo AB

$M_A = -43,333\,tm$

$M_B = 0\,tm$

Tramo BC

$M_B = 0\,tm$

$M_C = -4 - 1(2) = -6\,tm$

TramoCD

$M_C = -6\,tm$

$M_D = -4\,tm$

3.- Cortante

Tramo AB

$Q_A = 17t$

$Q_B = 17 - \left(\frac{2+6}{2}\right).5 = -3t$

Tramo BC

$Q_B = 17 - \left(\frac{2+6}{2}\right)5 = -3t$

$Q_C = -3t$

Tramo CD

$Q_C = 1t$

$Q_D = 1t$

3.- Diagramas

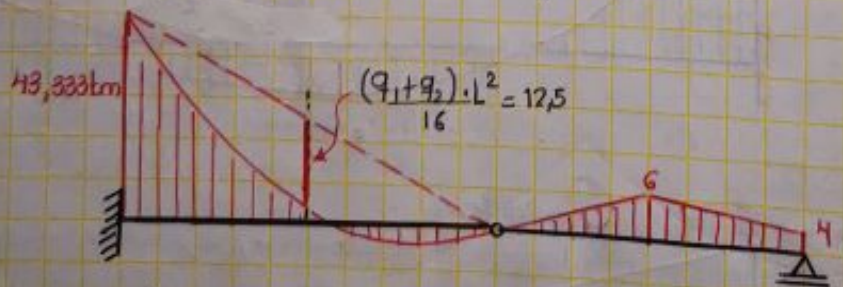
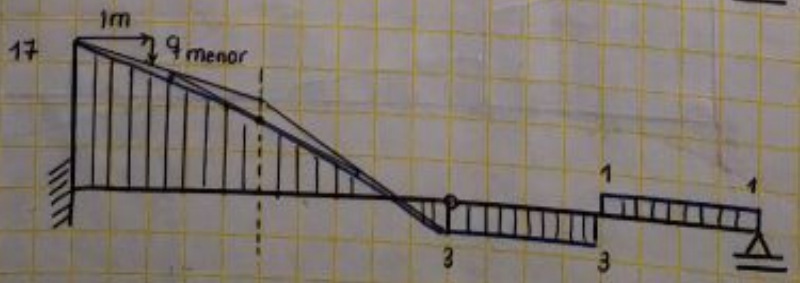
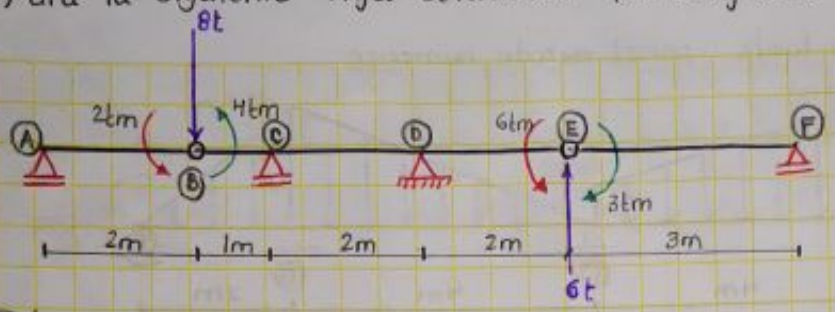


Diagrama de Momento

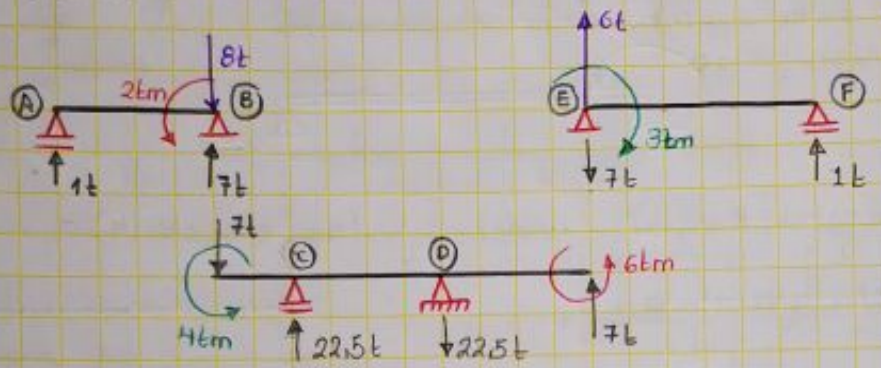


Diagroma de Cortante

50. Para la siguiente viga determinar los diagrama de Momento y Cortante



1- Reacciones



* Barra AB

$\sum M_A = 0$ ↺ (+)

$8(2) - 2 - V_B(2) = 0$

$V_B = 7t$ //

$\sum F_V = 0$ ↑ (+)

$V_A - 8 + 7 = 0$

$V_A = 1t$

* Barra EF

$\sum M_E = 0$ ↻ (+)

$3 - V_F(3) = 0$

$V_F = 1t$ //

$\sum F_V = 0$ ↑ (+) //

$6 + 1 - V_E = 0$

$V_E = 7t$

* Barra CD

$\sum M_C = 0$ ↻ (+)

$-4 - 6 - 7(1) - 7(4) + V_D(2) = 0$

$V_D = 22,5t$ //

$\sum F_V = 0$ ↑ (+)

$V_C = 22,5t$

2. Momentos

$M_A = 0\,tm$

$M_{B(IZQ)} = 2tm$

$M_{B(DER)} = -4tm$

$M_C = -7(1) - 4 = -11tm$

$M_D = 6 + 7(2) = 20tm$

$M_{E(IZQ)} = 6tm$

$M_{E(DER)} = 3tm$

$M_F = 0tm$

3. Cortante

$Q_A = 1t$

$Q_{B(IZQ)} = 1t$

$Q_{B(DER)} = -7t$

$Q_{C(IZQ)} = -7t$

$Q_{C(DER)} = -7 + 22,5 = 15,5t$

$Q_{D(IZQ)} = 15,5t$

$Q_{D(DER)} = 15,5 - 22,5 = -7t$

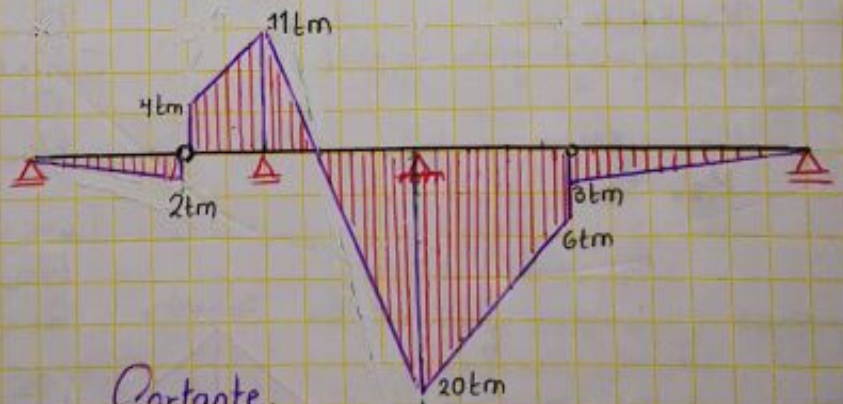
$Q_{E(IZQ)} = -7t$

$Q_{E(DER)} = -1t$

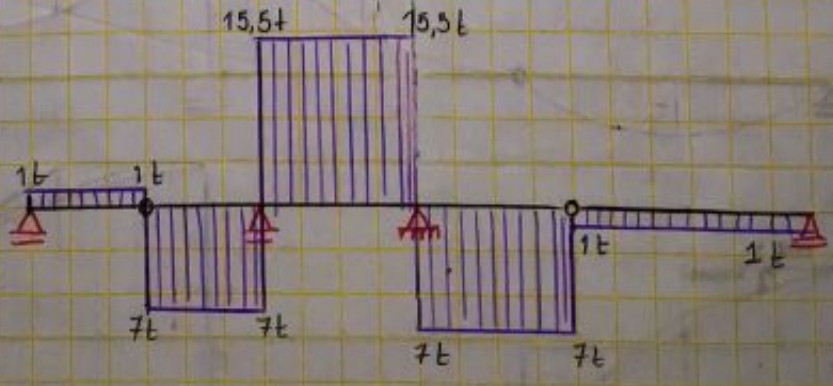
$Q_F = -1t$

4.- Diagramas.

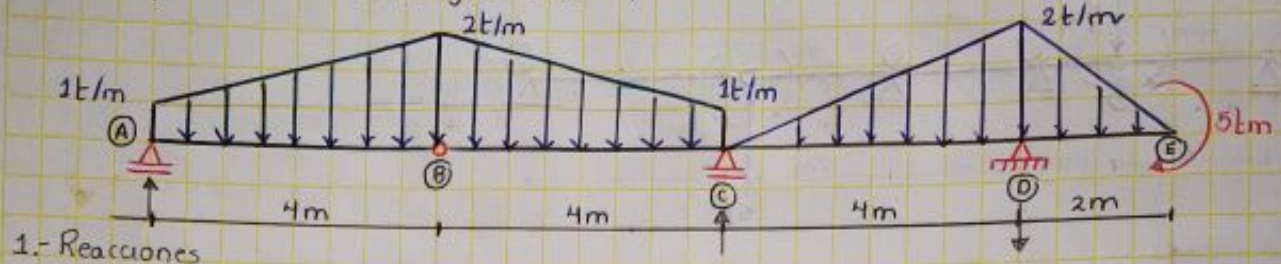
Momento



Cortante



51 Diagramar Momento y cortante por el método numérico



1.- Reacciones

$\Sigma M_B = 0$ ↻ ⊕ (a la izquierda de la articulación)

$V_A(4) - 1(4)(2) - \frac{1(4)}{2}\cdot\frac{1}{3}(4) = 0$

$V_A = 2{,}667\,t$ //

$\Sigma M_D = 0$ ↻ ⊕

$2{,}667(12) - 1(4)(10) - \frac{1(4)}{2}\left[8 + \frac{1}{3}(4)\right] - 1(4)(6) - \frac{1(4)}{2}\cdot\left[4 + \frac{2}{3}(4)\right] + V_C(4) -$

$- \frac{2(4)}{2}\cdot\frac{1}{3}(4) + \frac{2(2)}{2}\cdot\frac{1}{3}(2) + 5 = 0$

$V_C = 15{,}749\,t$ //

$\Sigma F_V = 0$ ↑ ⊕

$2{,}667 + \left(\frac{1+2}{2}\right)\cdot 4 - \left(\frac{2+1}{2}\right)\cdot 4 + 15{,}749 - \frac{2(6)}{2} - V_D = 0$

$V_D = 0{,}416\,t$ //

2.- Momento

Tramo AB
$M_A = 0$
$M_B = 0$

Tramo BC
$M_B = 0$
$M_C = 2{,}667(8) - 1(8)(4) - \frac{1(8)}{2}\cdot 4$
$M_C = -26{,}664\,tm$

Tramo CD
$M_C = -26{,}664\,tm$
$M_D = -5 - \frac{2(2)}{2}\cdot\frac{1}{3}(2) = -6{,}333\,tm$

Tramo DE
$M_D = -6{,}333\,tm$
$M_E = -5\,tm$

3.- Cortante

Tramo AB
$Q_A = 2{,}667\,t$
$Q_B = 2{,}667 - \left(\frac{1+2}{2}\right)\cdot 4 = -3{,}333\,t$

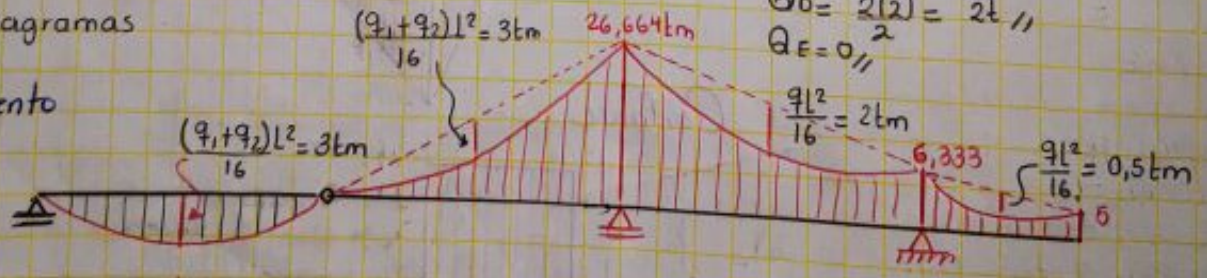
Tramo BC
$Q_B = -3{,}333\,t$ //
$Q_C = 2{,}667 - 1(8) - \frac{1(8)}{2} = -9{,}333\,t$ //

Tramo CD
$Q_C = -9{,}333 + 15{,}749 = 6{,}416\,t$ //
$Q_D = +\frac{2(2)}{2} + 0{,}416 = +2{,}416\,t$ //

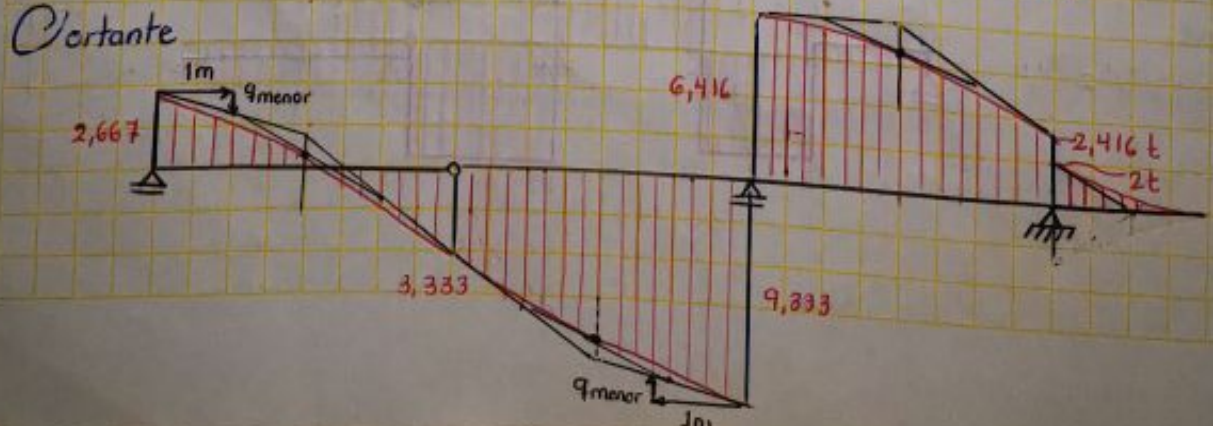
Tramo DE
$Q_D = \frac{2(2)}{2} = 2\,t$ //
$Q_E = 0$ //

4.- Diagramas

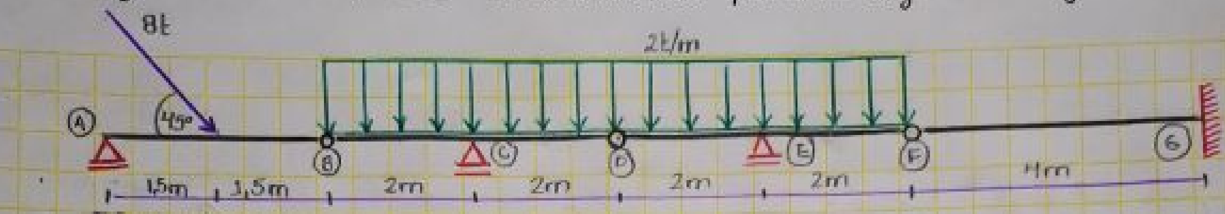
Momento

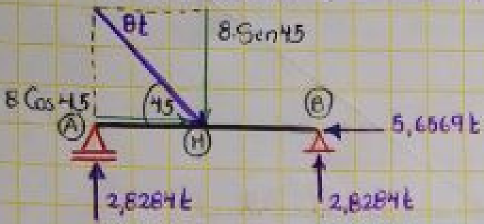


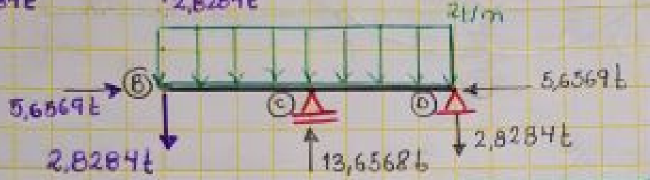
Cortante

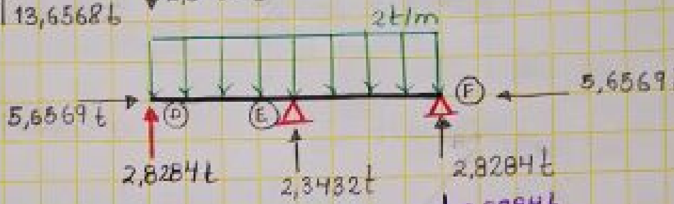


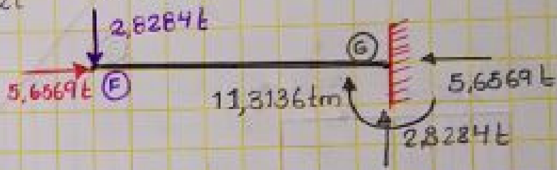
52. Diagramar los esfuerzos característicos para la siguiente viga











1- Reacciones

* Barra AB

$\Sigma M_A = 0$ ↻ ⊕

$8\,Sen45\,(1,5) - V_B(3) = 0$

$V_B = 2,8284\,t$ //

$\Sigma F_V = 0$ ↑⊕

$V_A - 8 \cdot Sen45 + 2,8284 = 0$

$V_A = 2,8284\,t$ //

$\Sigma F_H = 0$ →⊕

$H_B = 8\,Cos45 = 5,6569\,t$ //

* Barra BD

$\Sigma M_C = 0$ ↻ ⊕

$V_D(2) - 2,8284(2) = 0$

$V_D = 2,8284\,t$ //

$\Sigma F_H = 0$ →⊕

$H_D = 5,6569\,t$

$\Sigma F_V = 0$ ↑⊕

$-2(4) - 2,8284 - 2,8284 + V_C = 0$

$V_C = 13,6568\,t$

* Barra DF

$\Sigma M_E = 0$ ↻ ⊕

$2,8284(2) - V_F(2) = 0$

$V_F = 2,8284\,t$ //

$\Sigma F_V = 0$ ↑⊕

$2,8284 - 2(4) + 2,8284 + V_E = 0$

$V_E = 2,3432\,t$ //

$\Sigma F_H = 0$ →⊕

$H_F = 5,6569\,t$ //

* Barra FG

$\Sigma F_V = 0$ ↑⊕

$V_G = 2,8284\,t$ //

$\Sigma F_H = 0$ →⊕

$H_G = 5,6569\,t$ //

$\Sigma M_G = 0$ ↻ ⊕

$-2,8284(4) + M_G = 0$

$M_G = 11,3136\,tm$

2- Momentos

$M_A = 0\,tm$

$M_H = 2,8284(1,5) = 4,2426\,t$

$M_B = 0\,tm$

$M_C = -2,8284(2) - 2(2)(1) = -9,6568\,tm$

$M_D = 0\,tm$

$M_E = 2,8284(2) - 2(2)(1) = 1,6568\,tm$

$M_F = 0\,tm$

$M_G = -11,3136\,tm$

3- Cortantes

$Q_A = 2,8284\,t$ //

$Q_H(IZQ) = 2,8284\,t$ //

$Q_H(DER) = 2,8284 - 8\,Sen45 = -2,8285\,t$ //

$Q_B = -2,8284\,t$ //

$Q_C(IZQ) = -2,8284 - 2(2) = -6,8284\,t$ //

$Q_C(DER) = -2,8284 - 2(2) + 13,6569 = 6,8285\,t$,

$Q_D = 2,8284\,t$ //

$Q_E(IZQ) = 2,8284 - 2(2) = -1,1716\,t$ //

$Q_E(DER) = 2,8284 - 2(2) + 2,3432 = 1,1716\,t$,

$Q_F = -2,8284\,t$ //

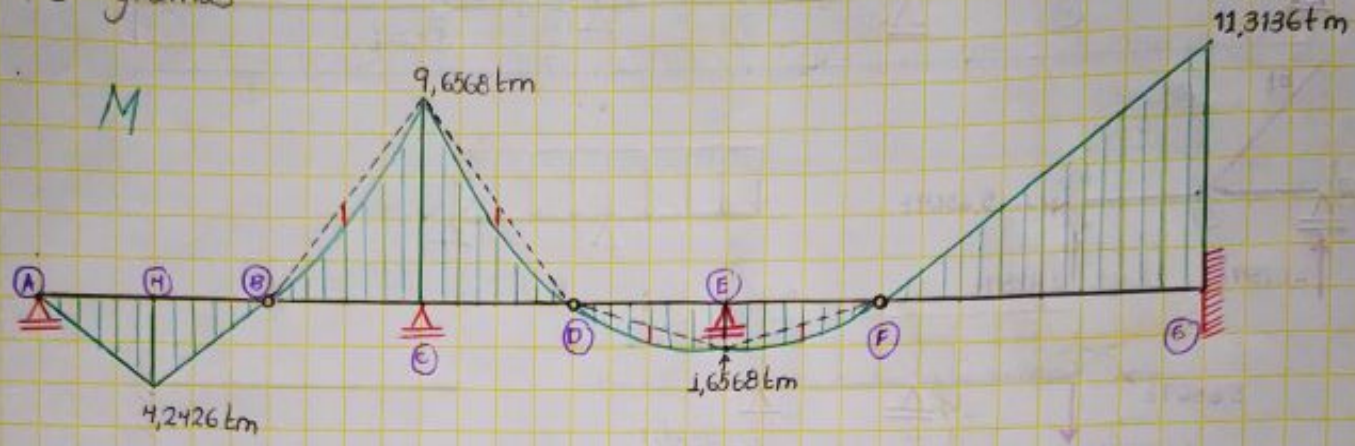
$Q_G = -2,8284\,t$ //

4- Normal

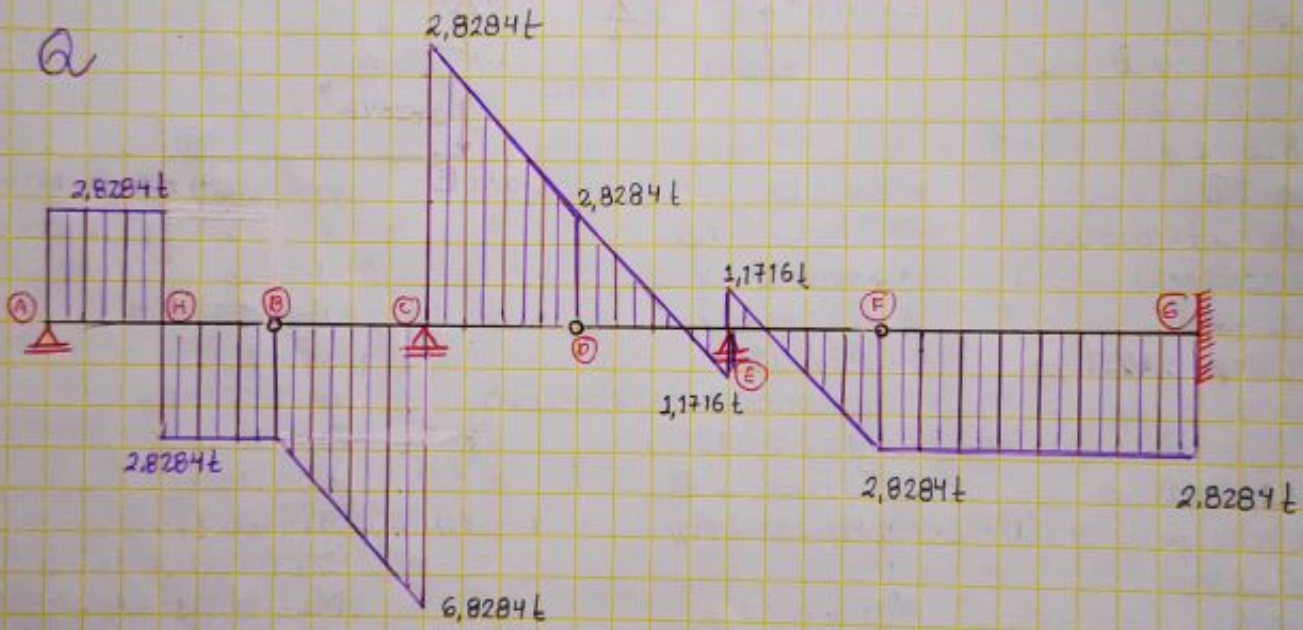
$N_{HG} = -5,6569\,t$ //

5.- Diagramas

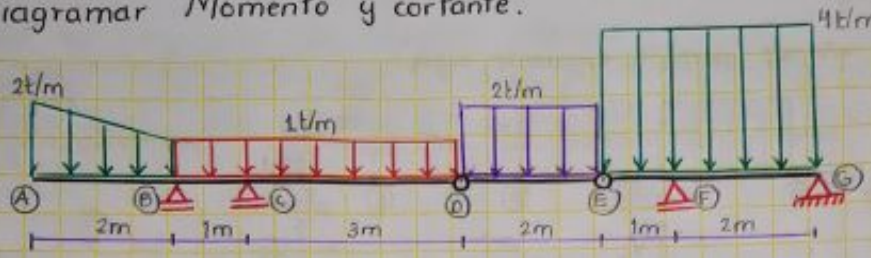
M



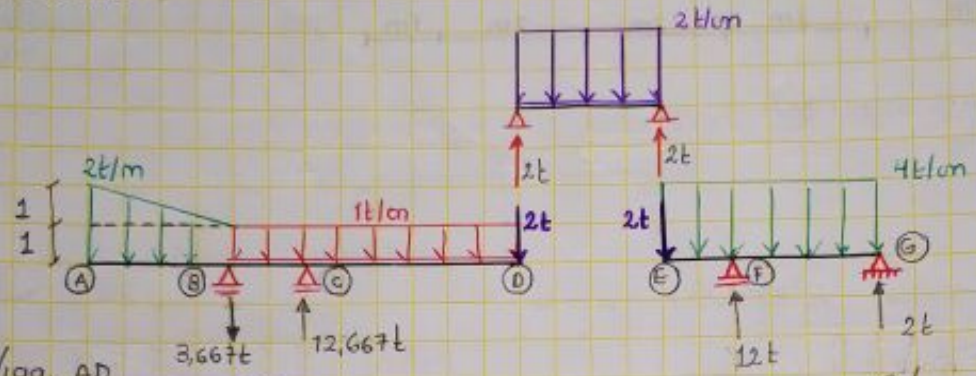
Q



53 Diagramar Momento y cortante.



1 Reacciones



Viga AD

$\sum M_B = 0$ ↻ (+)

$-1(2)(1) - \frac{1(2)}{2}\left[\frac{2}{3}.2\right] + 1(4)(2) + 2(4) - V_C(1) = 0$

$V_C = 12{,}667t$

$\sum F_V = 0$ ↑ (+)

$-\frac{(2+1)}{2}.2 - 1(4) - 2 + 12{,}667 - V_B = 0$

$V_B = 3{,}667t$

Viga EG

$\sum M_G = 0$ ↻ (+)

$-2(3) - 4(3)(1{,}5) + V_F(2) = 0$

$V_F = 12t$

$\sum F_V = 0$ ↑ (+)

$-2 - 4(3) + 12 + V_G = 0$

$V_G =$

2. Momentos

$M_A = 0$

$M_B = -1(2)(1) - \frac{1(2)}{2}\left[\frac{2}{3}.2\right] = -3{,}333\,tm$

$M_C = -2(3) - 1(3)(1{,}5) = -10{,}5\,tm$

$M_D = 0t$

$M_E = 0t$

$M_F = -2(1) - 4(1)(0{,}5) = -4\,tm$

$M_G = 0\,tm$

3-Cortantes

$Q_A = 0t$

$Q_{B(Izq)} = -\left(\frac{2+1}{2}\right).2 = -3t$

$Q_{B(der)} = -\frac{(2+1)}{2}.2 + 3{,}667 = -6{,}667t$

$Q_{C(Izq)} = 2 + (1)(3) - 12{,}667 = -7{,}667t$

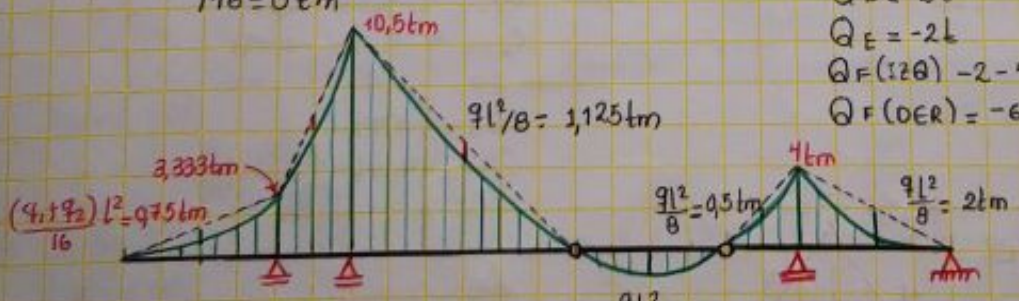
$Q_{C(der)} = 2 + (1)(3) = 5t$

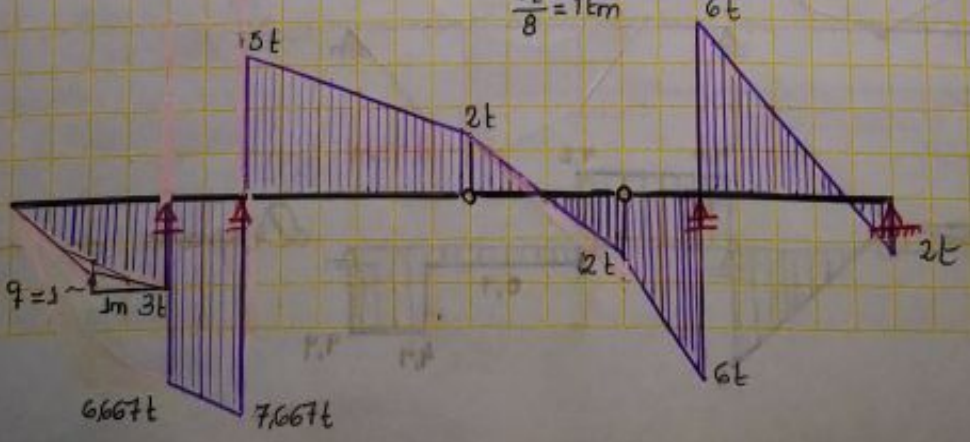
$Q_D = 2t$

$Q_E = -2t$

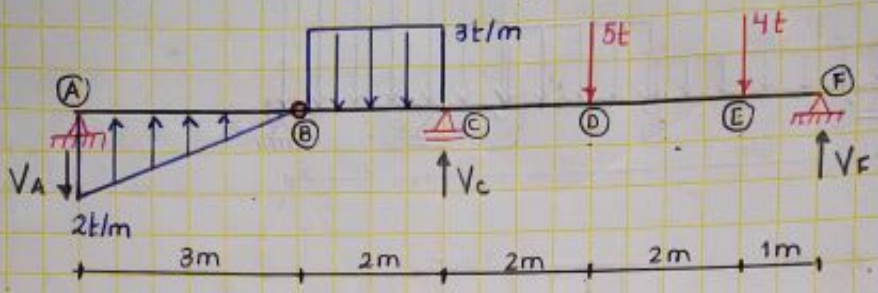
$Q_{F(IZQ)} = -2 - 4(1) = -6t$

$Q_{F(DER)} = -6 + 12 = 6t$; $Q_G = -2t$





Obtener las esfuerzos internos para la siguiente viga



1.- Reacciones

$\Sigma M_B = 0$ ↻⊕ (a la izquierda)

$\frac{2(3)}{2}(2) - V_A(3) = 0$

$V_A = 2t$ //

$\Sigma M_F = 0$ ↻⊕

$-2(10) + \frac{2(3)}{2}.(9) - 3(2)(6) + V_C(5) - 5(3) - 4(1) = 0$

$V_C = 9,6t$ //

$\Sigma F_V = 0$ ↑⊕

$-2 + \frac{2(3)}{2} - 3(2) + 9,6 - 5 - 4 + V_F = 0$

$V_F = 4,4t$ //

2.- Momentos

Tramo AB

$M_A = 0$

$M_B = -2(3) + \frac{2(3)}{2}(2) = 0$

Tramo BC

$M_B = 0$

$M_C = -5(2) - 4(4) + 4,4(5) = -4tm$

Tramo CD

$M_C = -4tm$

$M_D = -4(2) + 4,4(3) = 5,2tm$

Tramo DE

$M_D = 5,2tm$

$M_E = 4,4(1) = 4,4tm$

Tramo EF

$M_E = 4,4tm$

$M_F = 0tm$

3.- Cortante

Tramo AB

$Q_A = -2t$

$Q_B = -2 + \frac{2(3)}{2} = 1t$

Tramo BC

$Q_B = 1t$

$Q_C = -2 + \frac{2(3)}{2} - 3(2) = -5t$

Tramo CD

$Q_C = 5 + 4 - 4,4 = 4,6t$

$Q_D = 5 + 4 - 4,4 = 4,6t$

Tramo DE

$Q_D = 4 - 4,4 = -0,4t$

$Q_E = 4 - 4,4 = -0,4t$

Tramo EF

$Q_E = -4,4t$

$Q_F = -4,4t$

4.- Diagramas

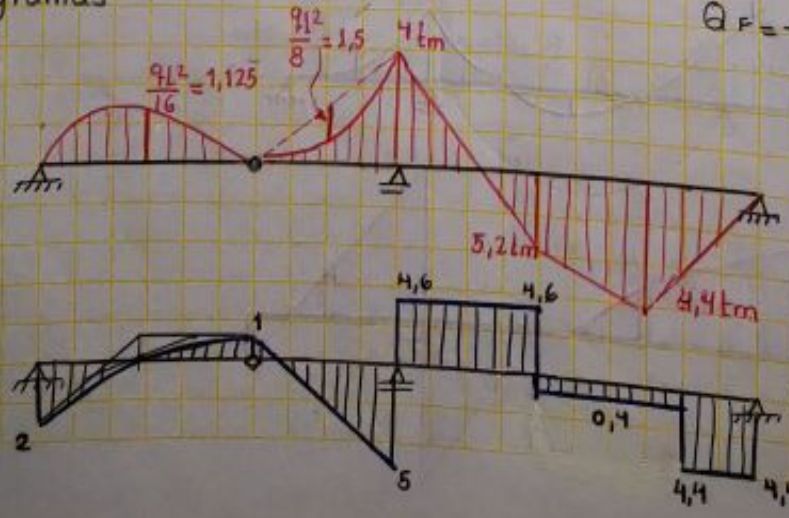
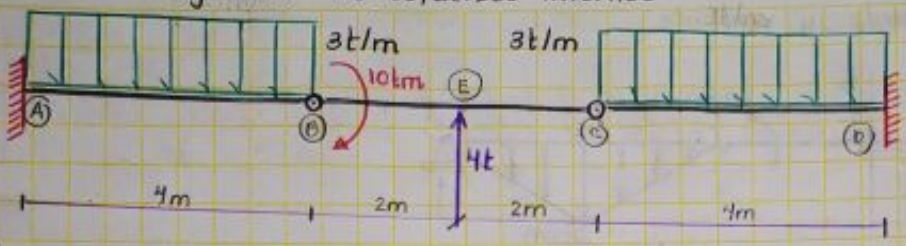


Diagrama de Momento

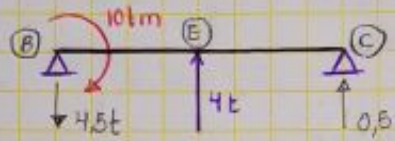
Diagrama de Cortante.

Obtener los diagramas de esfuerzos internos



1-Cálculo de Reacciones

Barra $\overline{BC}$



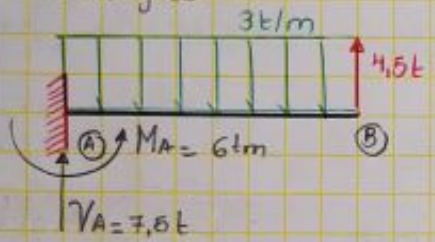
$\sum M_B = 0 \curvearrowright +$

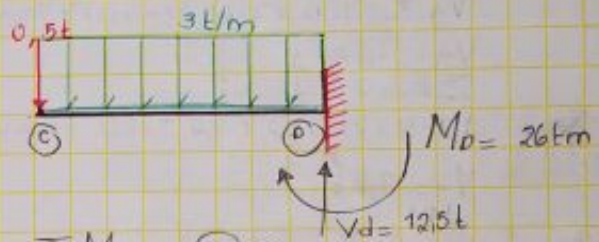
$10 - 4(2) - V_C(4) = 0 \therefore V_C = 0,5t$

$\sum F_V = 0 \uparrow +$

$-V_B + 4 + 0,5 = 0 \therefore V_B = 4,0t$

Barras AB y CD





$\sum M_A = 0 \curvearrowright +$

$-M_A - 4,5(4) + 3(4)(2)$

$M_A = 6tm$

$\sum F_V = 0 \uparrow +$

$V_A - 3(4) + 4,5 = 0$

$V_A = 7,5t$ //

$\sum M_D = 0 \curvearrowright +$

$M_D - 3(4)(2) - 0,5(4) = 0$

$M_D = 26tm$

$\sum F_V = 0 \uparrow +$

$-0,5 - 3(4) + V_d = 0$

$V_d = 12,5t$ //

2-Momentos

Barra AB

$M_A = -6tm$

$M_B = 0tm$

Barra BC

$M_B = 10tm$

$M_C = 0tm$

$M_E = -4,5(2) + 10 = 1tm$

Barra CD

$M_C = 0tm$

$M_D = -26tm$

3-Cortantes

Barra AB

$Q_A = 7,5t$

$Q_B = -4,5t$

Barra BC

$Q_B = -4,5t$

$Q_E = -4,5 + 4 = -0,5t$

$Q_C = -0,5t$

Barra CD

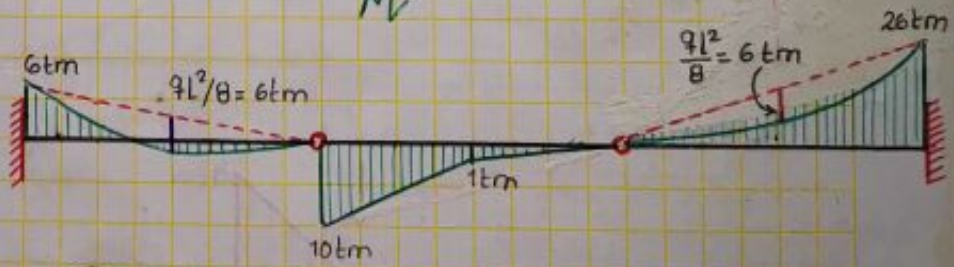
$Q_C = -0,5$

$Q_D = -12,5t$

4.- Diagramas

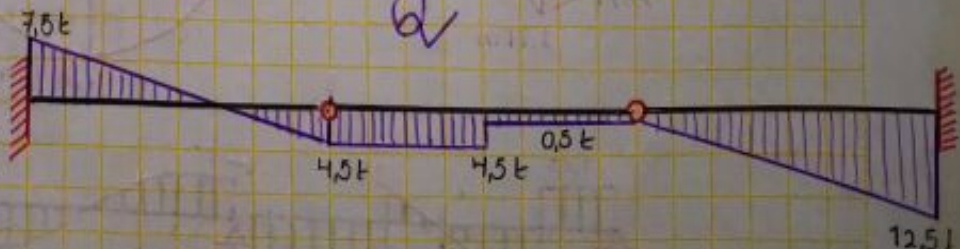
Momento · Escala = 10tm/cm

M

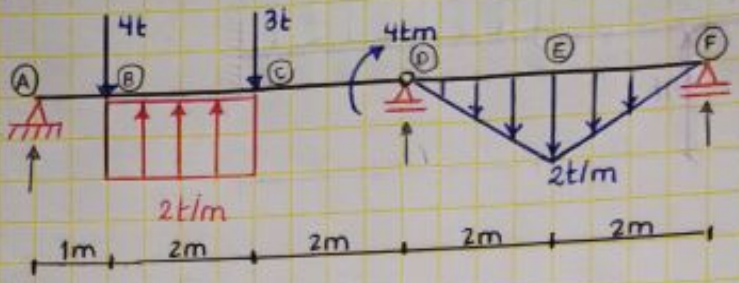


Cortante Escala = 10t/cm

Q



Diagramar Momento y cortante



1. Reacciones

$\sum M_D = 0$ ↺ ⊕ (a la derecha)

$\frac{2(4)}{2} \cdot 2 - V_F(4) = 0$

$V_F = 2t$

$\sum M_D = 0$ ↺ ⊕ (a la izquierda)

$V_A(5) - 4(4) + 2(2)(3) - 3(2) + 4 = 0$

$V_A = 1,2t$

$\sum F_V = 0$ ↑ ⊕

$1,2 - 4 + 2(2) - 3 + V_D - \frac{2(4)}{2} + 2 = 0$

$V_D = 3,8t$

2. Momentos

Tramo AB $M_A = 0$

$M_B = 1,2(1) = 1,2tm$

Tramo BC $M_B = 1,2tm$

$M_C = 1,2(3) + 2(2)(1) - 4(2) = -0,4tm$

Tramo CD $M_C = -0,4tm$

$M_D = 4tm$

Tramo DE $M_D = 0tm$

$M_E = 2(2) - \frac{2(2)}{2} \cdot \frac{1}{3}(2) = 2,667tm$

Tramo EF $M_E = 2,667tm$

$M_F = 0$

3. Cortante

Tramo AB $Q_A = 1,2t$

$Q_B = 1,2t$

Tramo BC

$Q_B = 1,2 - 4 = -2,8t$

$Q_C = 1,2 - 4 + 2(2) = 1,2t$

Tramo CD

$Q_C = 1,2 - 4 + 2(2) - 3 = -1,8t$

$Q_D = -1,8t$

Tramo DE

$Q_D = -1,8 + 3,8 = 2t$

$Q_E = -2 + \frac{2(2)}{2} = 0$

Tramo EF

$Q_E = 0$

$Q_F = 0$

4. Diagramas

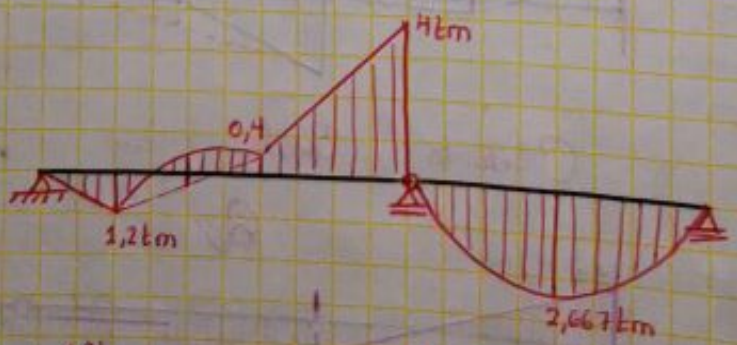


Diagrama de Momento

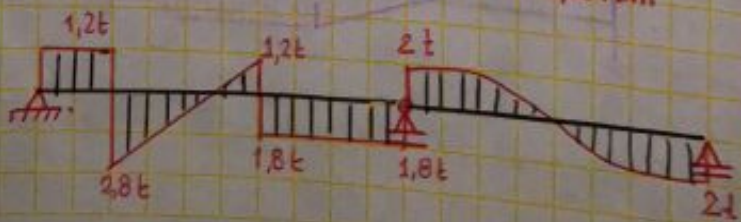
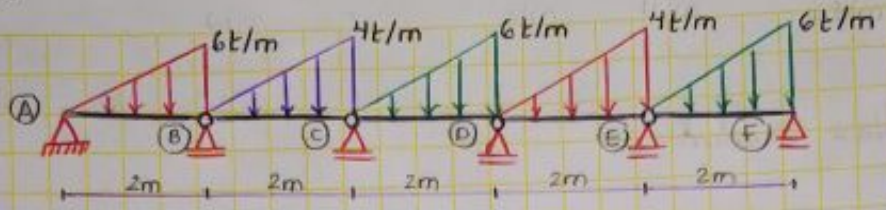
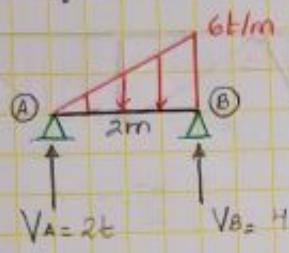


Diagrama de Cortante

Diagramar Momento y Cortante para la siguiente viga



1- Reacciones

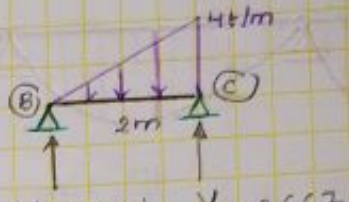


$\sum M_A = 0$ ↻ ⊕

$\frac{6(2)}{2}\left[\frac{2}{3}.2\right] - V_B(2) = 0 \quad \therefore V_B = 4t$ //

$\sum F_V = 0$ ↑ ⊕

$V_A - \frac{6(2)}{2} + 4 = 0 \quad \therefore V_A = 2t$ //



$\sum M_B = 0$ ↻ ⊕

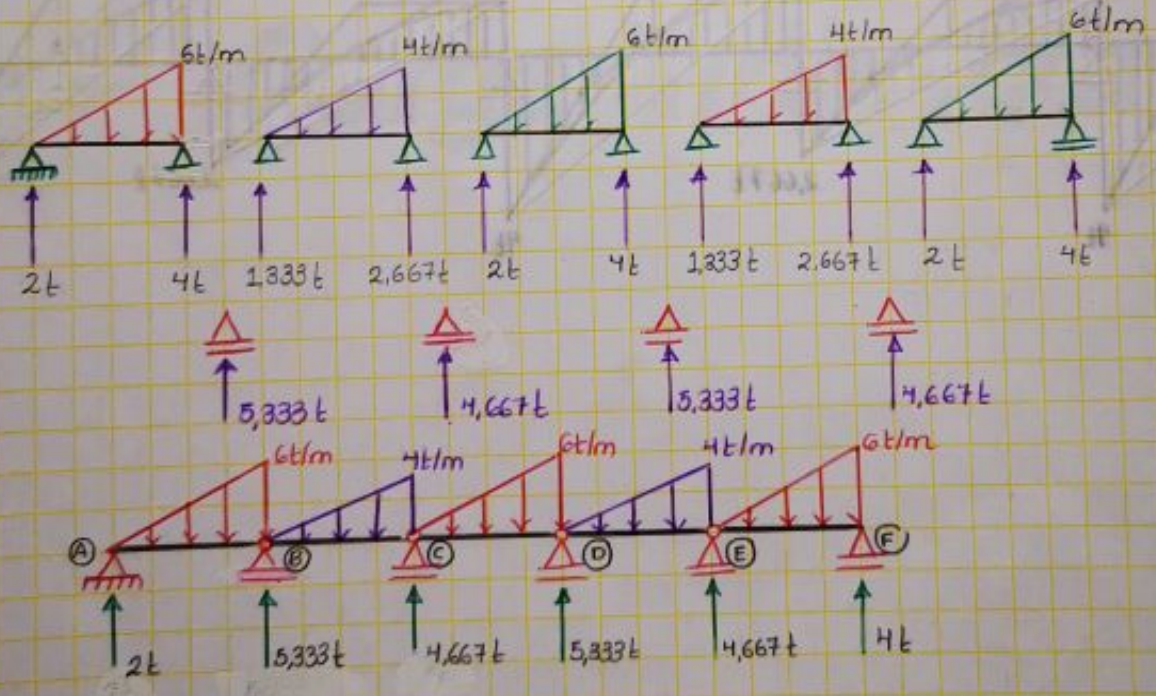
$\frac{4(2)}{2}\left[\frac{2}{3}.2\right] - V_C(2) = 0$

$V_C = 2{,}667t$ //

$\sum F_V = 0$ ↑ ⊕

$V_B - \frac{4(2)}{2} + 2{,}667 = 0$

$V_B = 1{,}333t$ //



2- Cortantes

Barra AB

$Q_A = 2t$

$Q_B = -4t$

Barra BC

$Q_B = 1{,}333t$

$Q_C = -2{,}667t$

Barra CD

$Q_C = 2t$

$Q_D = 4t$

Barra DE

$Q_D = 1{,}333t$

$Q_E = -2{,}667t$

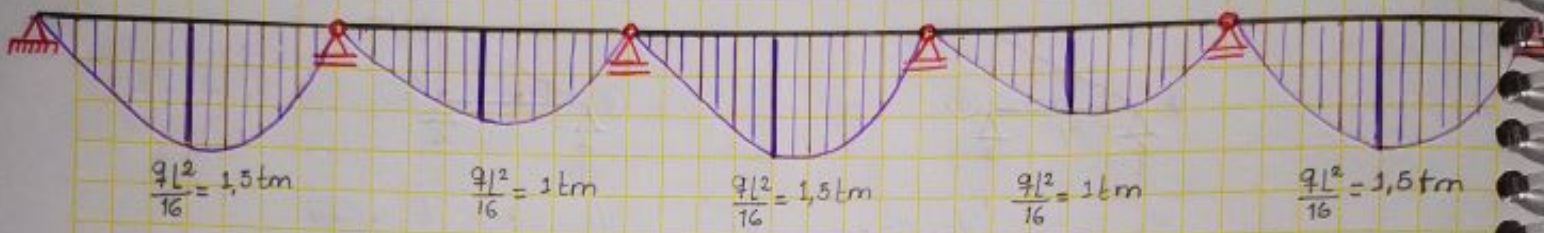
Barra EF

$Q_E = 2t$

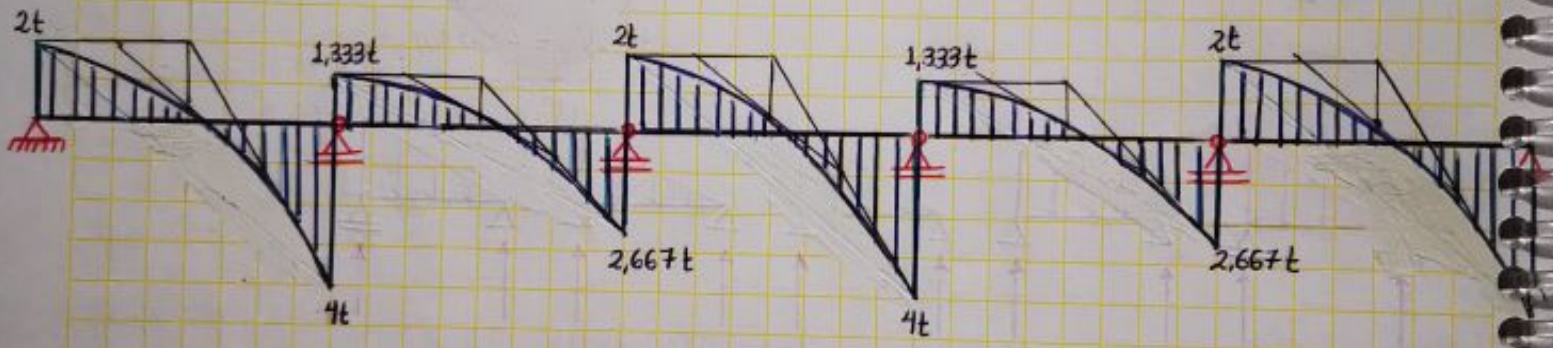
$Q_F = -4t$

3.- Diagramas

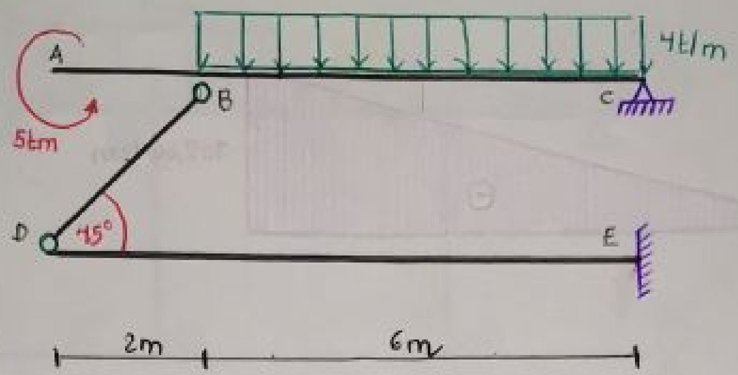
Momento Escala = 1tm/cm



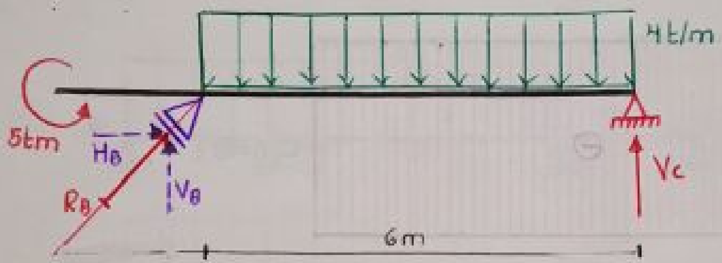
Cortante
Escala = 2t/cm



Diagramar los esfuerzos cararacterísticos para la viga D-E



1.- Barra $\overline{AC}$



$\sum M_{Ⓒ} = 0$ ↷ ⊕

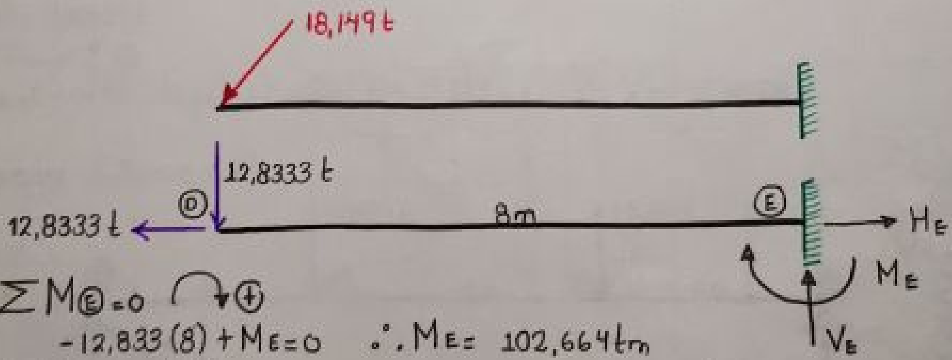
$-5 + V_B(6) - 4(6)(3) = 0$

$V_B = 12{,}8333\ t$

$\cos 45 = \frac{12{,}8333}{R_B}$

$R_B = \frac{12{,}8333}{\cos 45} = 18{,}149\ t$

2.- Reacciones barra $\overline{DE}$



$\sum M_{Ⓔ} = 0$ ↷ ⊕

$-12{,}833(8) + M_E = 0 \quad \therefore M_E = 102{,}664\ tm$

$\sum F_V = 0$ ↑ ⊕

$V_E = 12{,}8333\ t$

$\sum F_H = 0$ → ⊕

$H_E = 12{,}8333\ t$

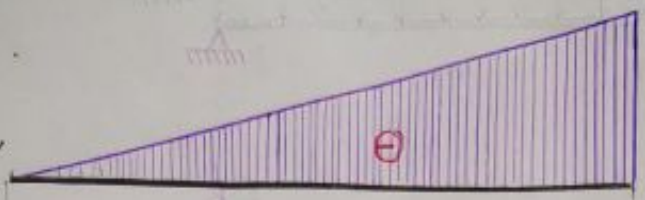
3.- Ecuaciones

- Momento $M_x = -12{,}8333 \cdot x$
- Cortante $Q_x = -12{,}8333\ t$
- Normal $N_x = 12{,}8333\ t$

4.- Diagramas

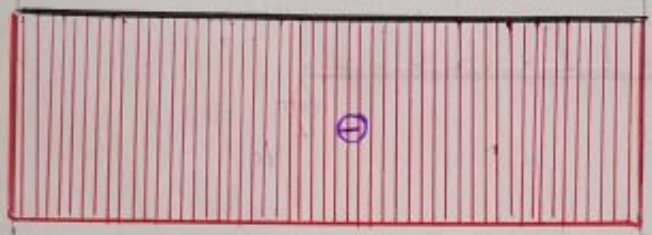
M

Esc = 50tm/cm



Q-

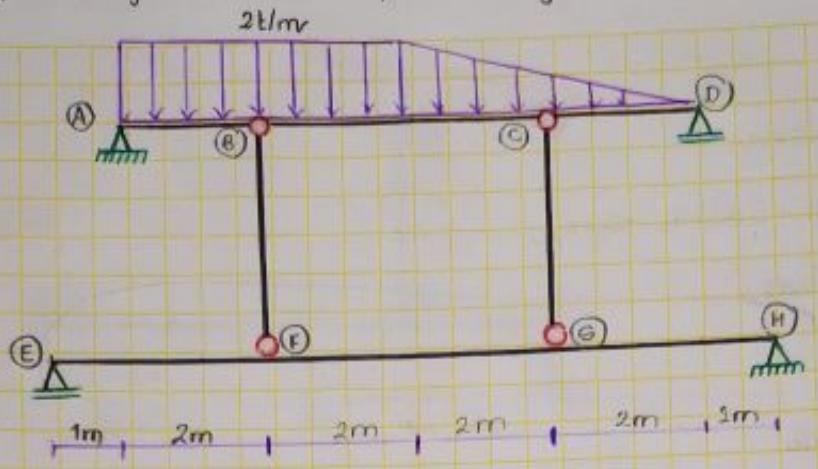
Esc = 5t/cm



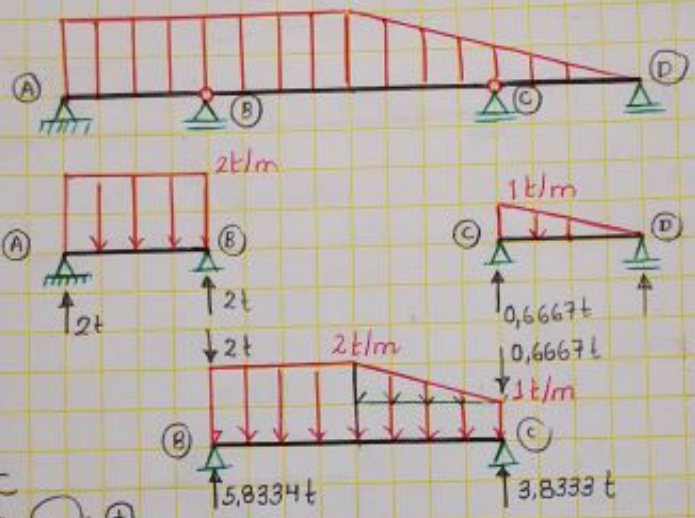
N

Esc = 5t/cm

Obtener los diagramas internos para la viga E-H



1 Barra AD



* Barra AB

$V_A = V_B = \frac{2(2)}{2} = 2t$ //

* Barra CD

$\sum M_D = 0$ ↷ (+)

$-\frac{1(2)}{2}\left(\frac{2}{3}2\right) + V_C(2) = 0$

$V_C = 0,6667\,t$ //

* Barra BC

$\sum M_B = 0$ ↷ (+)

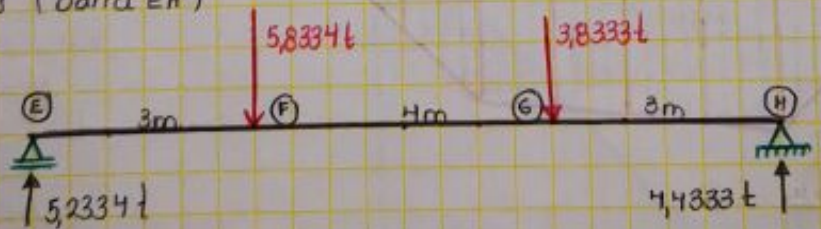
$2(2)(1) + 1(2)(3) + 0,6667(4) + \frac{1(2)}{2}\cdot\left[2 + \frac{1}{3}(2)\right] - V_C(4) = 0$

$V_C = 3,8333\,t$

$\sum F_V = 0$ ↑ (+)

$V_B - 2 - 2(2) - \frac{(2+1)}{2}(2) - 0,6667 + 3,8333 = 0 \quad \therefore V_B = 5,8334\,t$

2- Reacaones (Barra EH)



$\sum M_E = 0$ ↷ (+)

$5,8334(3) + 3,8333(7) - V_H(1$

$V_H = 4,4333\,t$ //

$\sum F_V = 0$ ↑ (+)

$V_E - 5,8334 - 3,8333 + 4$

$V_E = 5,2334\,t$ //

3.- Ecuaciones de Momento

* Tramo EF

$M_X = 5,2334 \cdot X$ //

* Tramo FG

$M_X = 5,2334\,X - 5,8334(X-3) = -0,6X + 17,5$ //

* Tramo GH

$M_X = 4,4333(10-X) = 44,333 - 4,4333X$

4.- Cortantes

* Tramo EF

$Q = 5,2334 t$ //

* Tramo FG

$Q = 5,2334 - 5,8334 = -0,6 t$ //

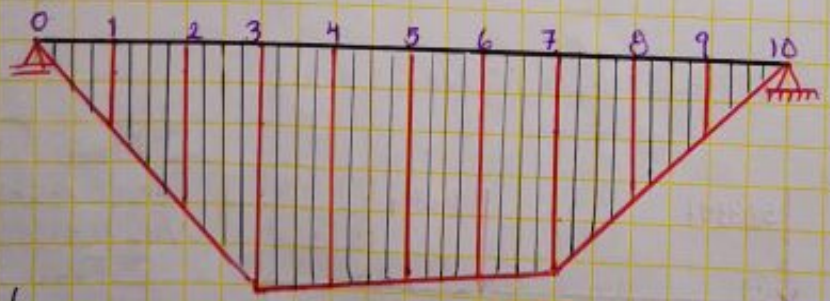
* Tramo GH

$Q = -4,4333 t$ //

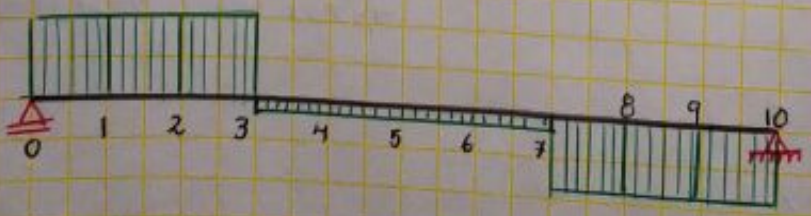
5.- Diagramas

x	Momento	Cortante
0	0	5,2334
1	5,2334	5,2334
2	10,4668	5,2334
3	15,7002	5,2334 / -0,6
4	15,1	-0,6
5	14,5	-0,6
6	13,9	-0,6
7	13,3	-0,6 / -4,4333
8	8,866	-4,4333
9	4,4333	-4,4333
10	0	-4,4333

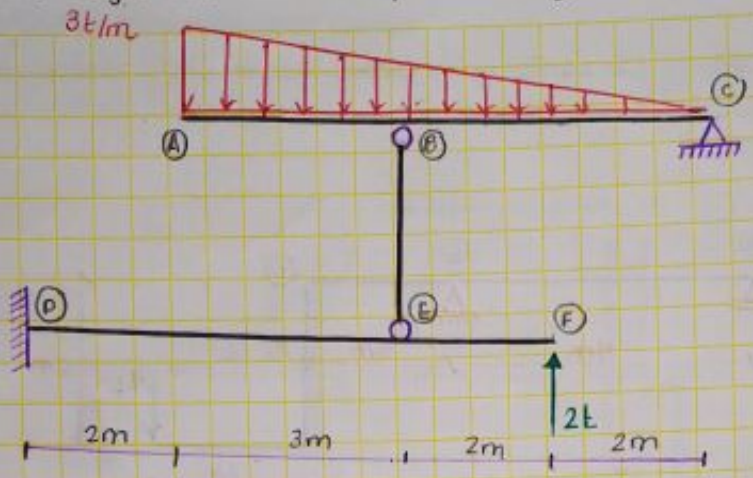
Momento



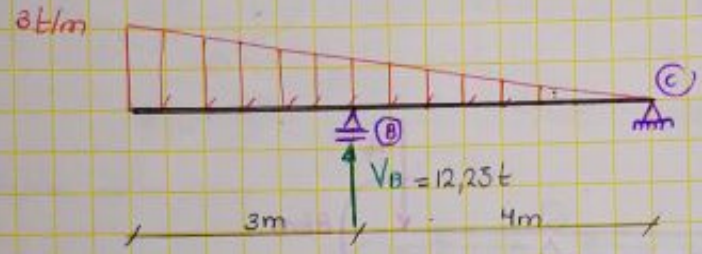
Cortante



Obtener los diagramas internos para la viga D-F



1.- Barra AC

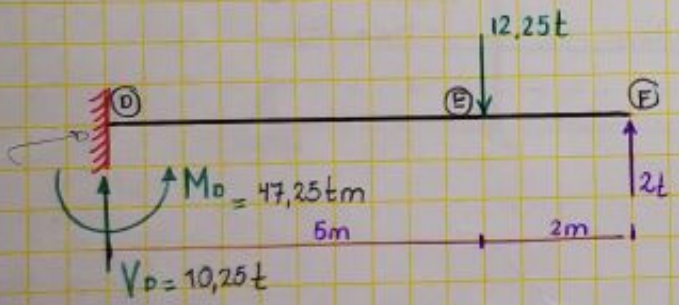


$\sum M_C = 0$ ↺ (+)

$V_B(4) - \frac{3(7)}{2}\left[\frac{2}{3}(7)\right] = 0$

$V_B = 12,25t$

2 - Barra DF



$\sum M_D = 0$ ↺ (+)

$-M_D + 12,25(5) - 2(7) = 0$

$M_D = 47,25tm$

$\sum F_V = 0$ ↑ (+)

$V_D - 12,25 + 2 = 0$

$V_D = 10,25t$

3 - Momentos

$M_D = -47,25tm$

$M_E = 2(2) = 4tm$

$M_F = 0tm$

4.- Cortantes

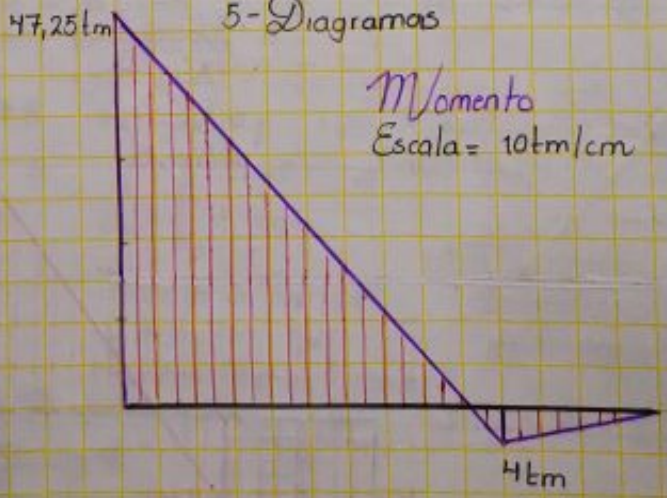
$Q_D = 10,25t$

$Q_E = 10,25t$ (Izq)

$Q_E = 10,25 - 12,25$

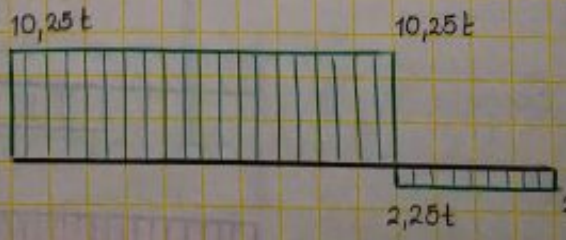
$Q_E = -2,25t$ (Der)

5 - Diagramas



Momento

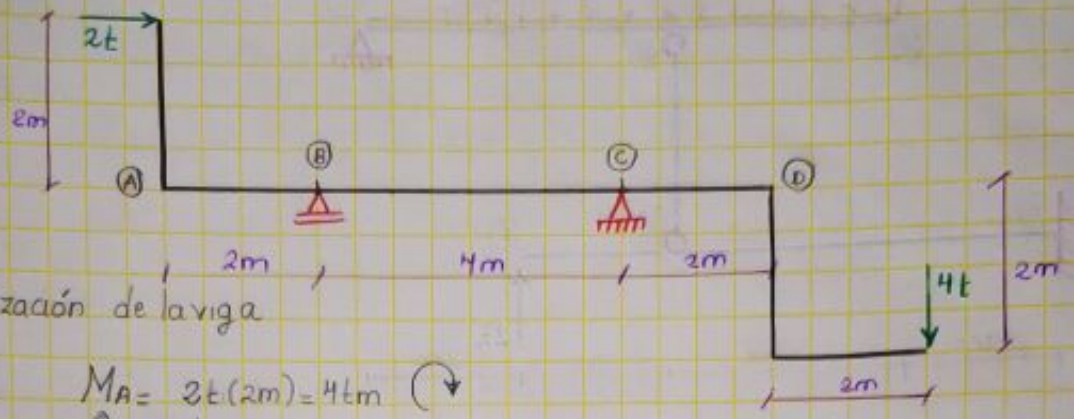
Escala = 10tm/cm



Cortante

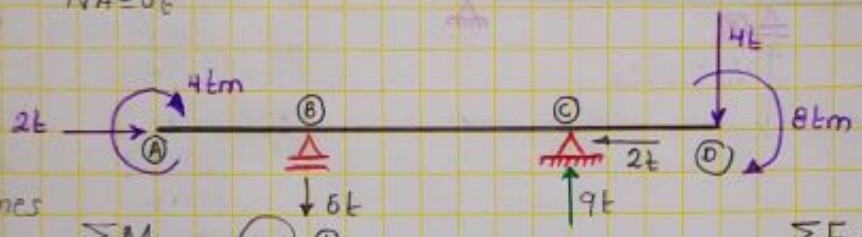
Escala = 10t/cm

Obtener las diagramas de Momento, Cortante y Normal de la viga AD



1- Idealización de la viga

$M_A = 2t(2m) = 4tm$ ↻
$Q_A = 0t$
$N_A = -2t$
$M_D = -4t(2m) = -8tm$
$Q_A = 4t$
$N_A = 0t$



2 Reacciones

$\Sigma M_B = 0$ ↻ ⊕
$4 + 8 + 4(6) - V_C(4) = 0 \quad \therefore V_C = 9t //$

$\Sigma F_H = 0 \rightarrow \oplus$
$H_C = 2t //$

$\Sigma F_V = 0$ ↑ ⊕
$-V_B + 9 - 4 = 0 \quad \therefore V_B = 5t //$

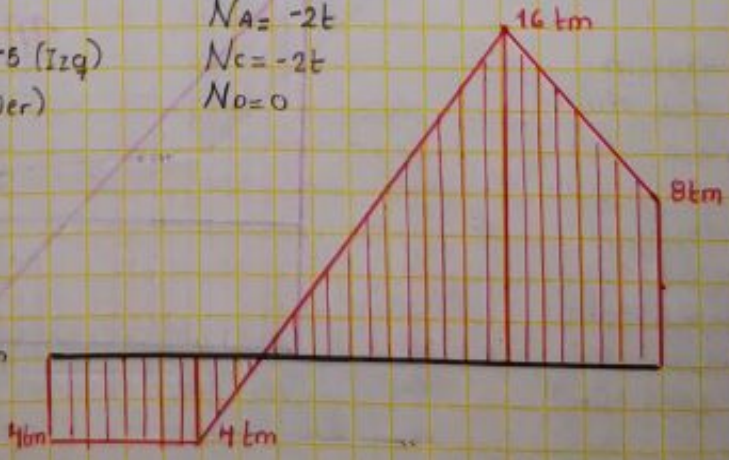
3- Momentos
$M_A = 4tm$
$M_B = 4tm$
$M_C = -8tm - 4(2) = -16tm$
$M_D = -8tm$

4- Cortante
$Q_A = 0$
$Q_B = -5t$; $Q_C = -5$ (Izq)
$Q_C = -5 + 9 = 4t$ (Der)
$Q_D = 4t$

5- Normal
$N_A = -2t$
$N_C = -2t$
$N_D = 0$

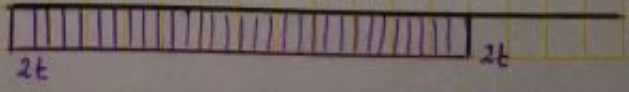
6- Diagramas

Momento Escala = 4tm/cm

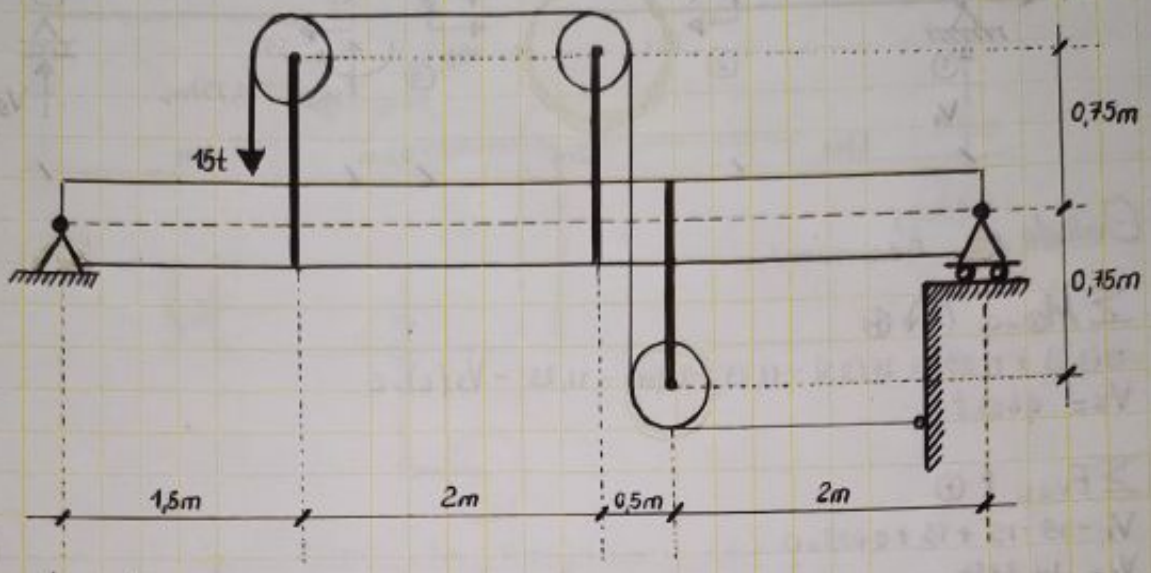


Cortante Escala = 10t/cm

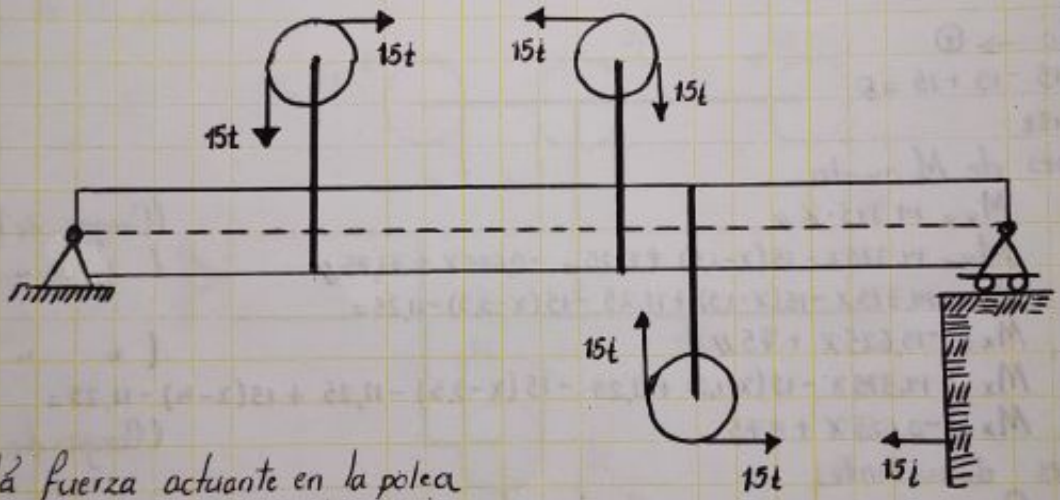
Normal Escala = 4t/cm



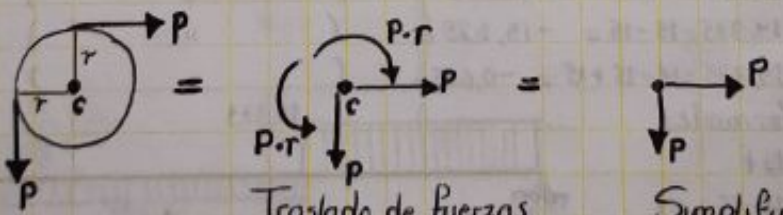
1.- Calcular reacciones y diagramar Momento



1.- Análisis en las poleas

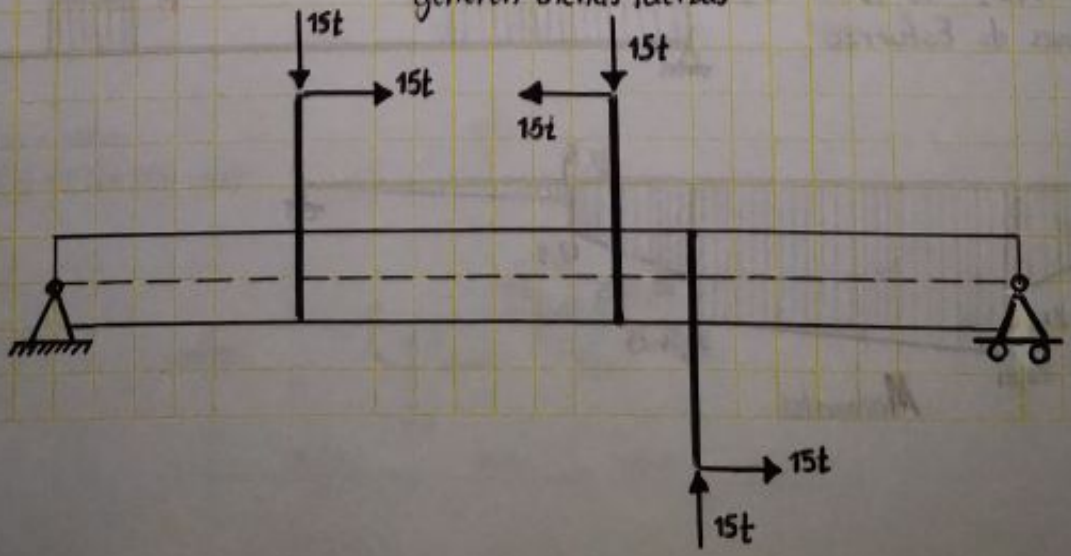


Cada fuerza actuante en la polea deberá trasladarse al centro de la misma

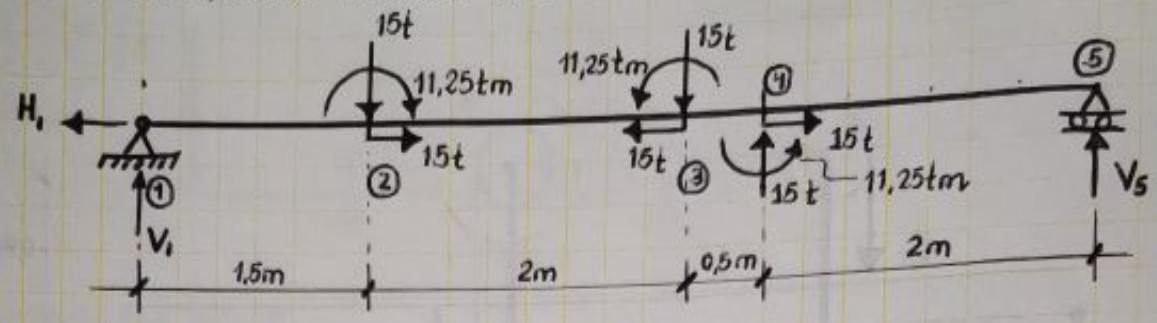


Traslado de fuerzas y momentos que generen dichas fuerzas

Simplificación de los momentos



Trasladamos las fuerzas ubicadas en el centro de las poleas al baricentro de la viga



2.- Cálculo de Reacciones

$\sum M_{①} = 0 \quad \curvearrowright \oplus$

$15(1,5) + 11,25 + 15(3,5) - 11,25 - 15(4) - 11,25 - V_5(6) = 0$

$V_5 = 0,625\ t$

$\sum F_V = 0 \quad \uparrow \oplus$

$V_1 - 15 - 15 + 15 + 0,625 = 0$

$V_1 = 14,375\ t$

$\sum F_H = 0 \quad \rightarrow \oplus$

$-H_1 + 15 - 15 + 15 = 0$

$H_1 = 15\ t$

3.- Ecuaciones de Momento

Tramo 1-2 $M_x = 14,375 - x$ // (Cargas de la izquierda)

Tramo 2-3 $M_x = 14,375x - 15(x-1,5) + 11,25 = -0,625x + 33,75$ // (" " " ")

Tramo 3-4 $M_x = 14,375x - 15(x-1,5) + 11,25 - 15(x-3,5) - 11,25 =$

$M_x = -15,625x + 75$ // (" " " ")

Tramo 4-5 $M_x = 14,375x - 15(x-1,5) + 11,25 - 15(x-3,5) - 11,25 + 15(x-4) - 11,25 =$

$M_x = -0,625x + 8,75$ (Cargas de la izquierda)

4.- Ecuaciones de cortantes

Tramo 1-2 $Q_x = 14,375\ t$ (cargas de la izquierda)

Tramo 2-3 $Q_x = 14,375 - 15 = -0,625\ t$ (" ")

Tramo 3-4 $Q_x = 14,375 - 15 - 15 = -15,625\ t$ (" ")

Tramo 4-5 $Q_x = 14,375 - 15 - 15 + 15 = -0,625\ t$ (" ")

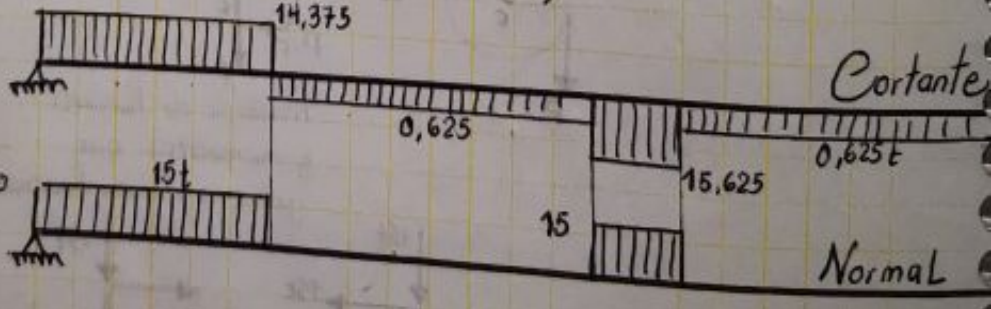
5.- Ecuaciones de Normales

Tramo 1-2 $N_x = 15\ t$

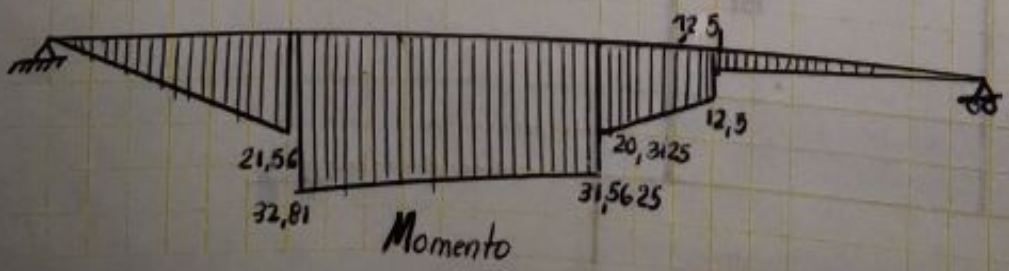
Tramo 2-3 $N_x = 15 - 15 = 0\ t$

Tramo 3-4 $N_x = 15 - 15 + 15 = +15\ t$

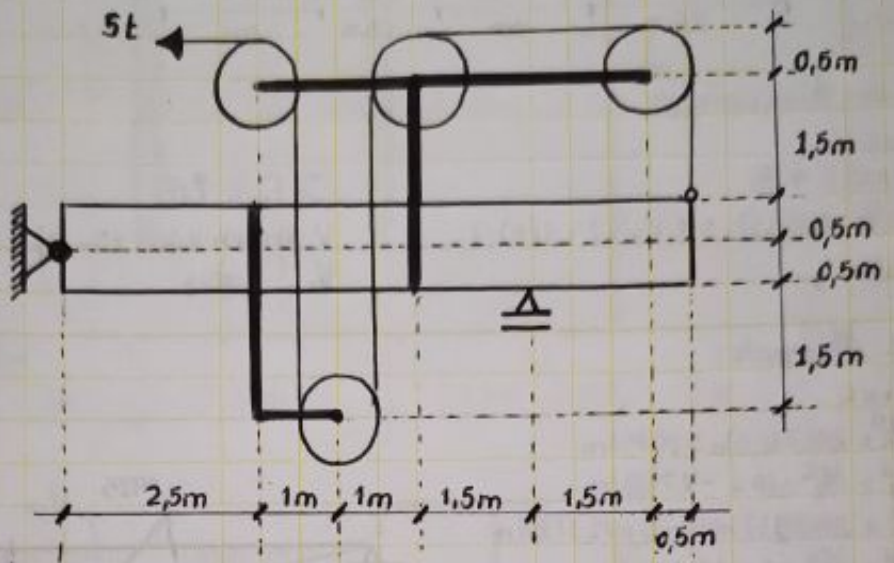
Tramo 4-5 $N_x = 15 - 15 + 15 - 15 = 0$



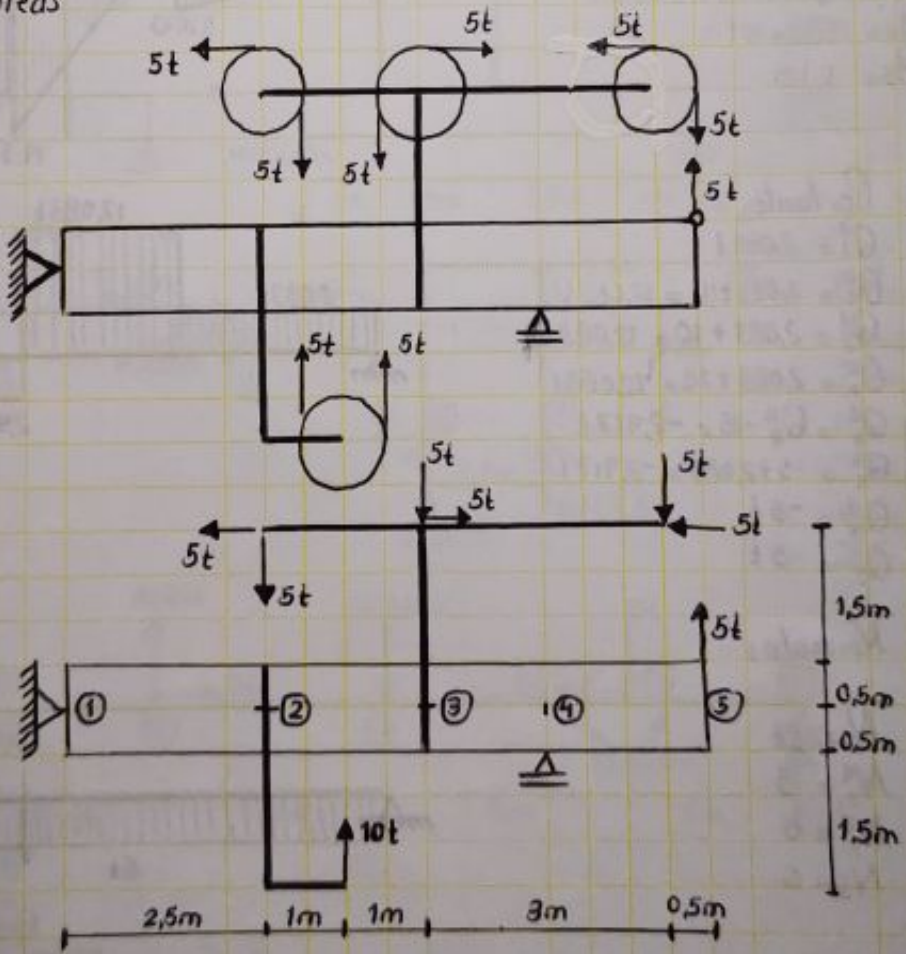
6.- Diagrama de Esfuerzo



2.- Calcular reacciones y diagramar momento, normal y cortante



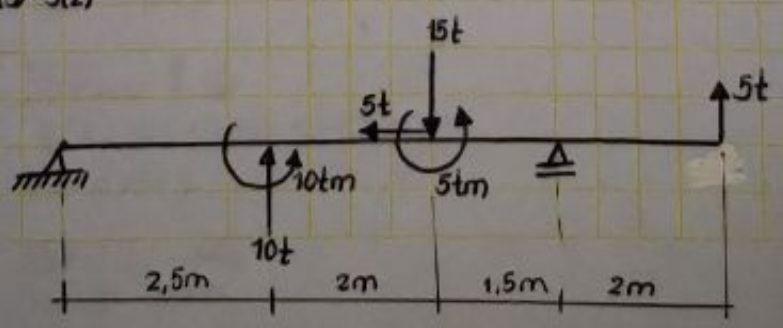
1. Análisis de las poleas

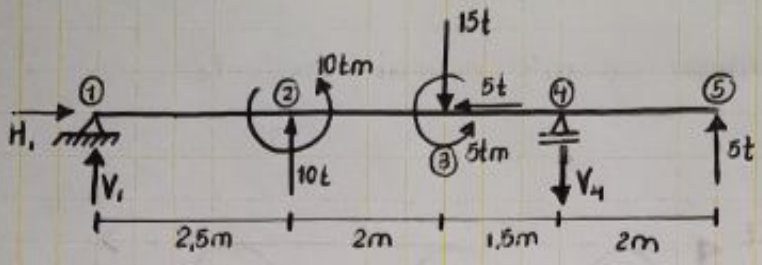


$M_2 = -10t \cdot 1m = -10tm$

$M_3 = -5(2) - 5(2) + 5(2) + 5(3) - 5(2)$

$M_3 = -5tm$





Calculo de Reacciones

$\sum M_1 = 0 \curvearrowright +$

$-10 - 10(2,5) + 15(4,5) - 5 + V_4(6) - 5(8) = 0$

$V_4 = 2,083\,t$

$\sum F_V = 0 \uparrow +$

$V_1 + 10 - 15 - 2,083 + 5 = 0$

$V_1 = 2,083\,t$

$\sum F_H = 0 \rightarrow +$

$H_1 - 5 = 0$

$H_1 = 5\,t$ //

Calculo de Momentos

(CI) $M_1 = 0$

(CI) $M_2^a = 2,083(2,5) = 5,2075\,tm$

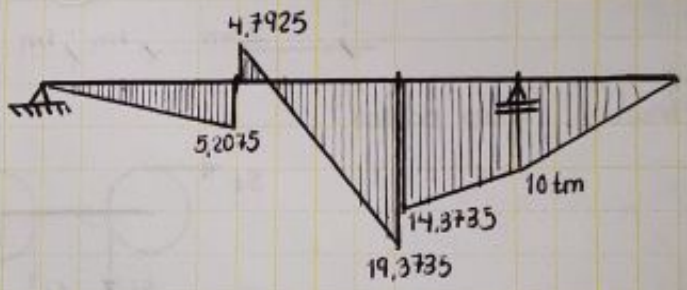
(CI) $M_2^d = M_2^a - 10 = -4,7925\,tm$

(CI) $M_3^a = 2,083(7,5) + 10(2) - 10 = 19,3735\,tm$

(CI) $M_3^d = M_3^a - 5 = 14,3735\,tm$

(CD) $M_4 = 5(2) = 10\,tm$

(CD) $M_5 = 0\,tm$



Cálculo de Cortante

CI $Q_1^d = 2,083\,t$

CI $Q_2^a = 2,083\,t$

CI $Q_2^d = 2,083 + 10 = 12,083\,t$

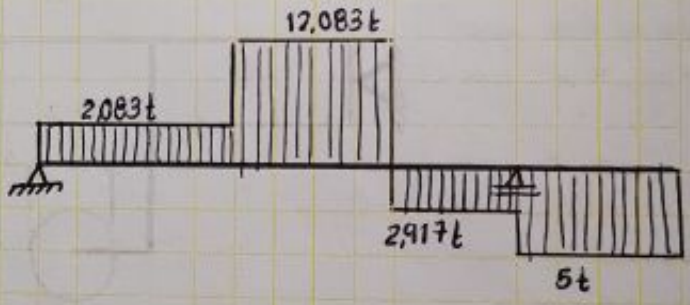
CI $Q_3^a = 2,083 + 10 = 12,083\,t$

CI $Q_3^d = Q_3^a - 15 = -2,917\,t$

CD $Q_4^a = -5 + 2,083 = -2,917\,t$

CD $Q_4^d = -5\,t$

CD $Q_5^a = -5\,t$



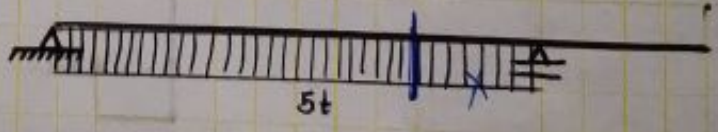
Cálculo de Normales

$N_1 = -5\,t$

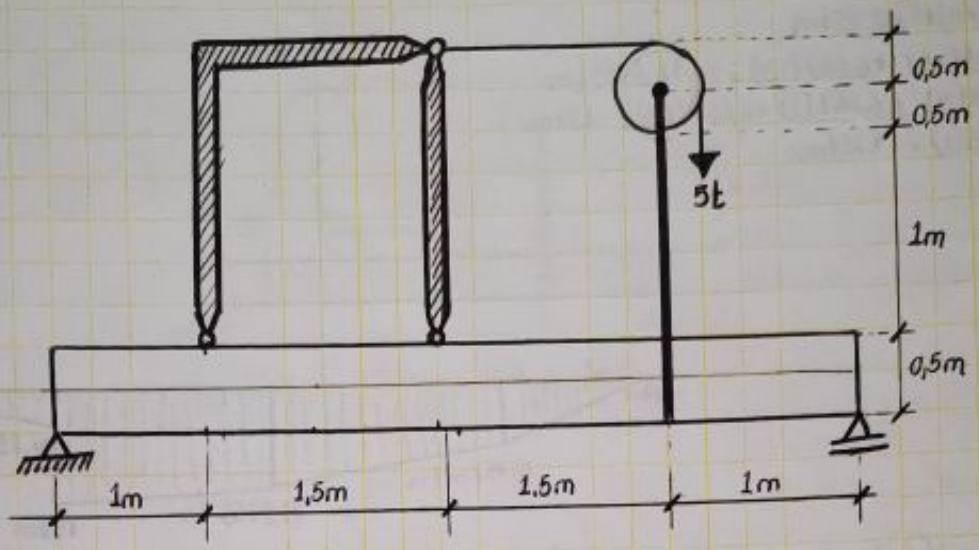
$N_3^a = -5$

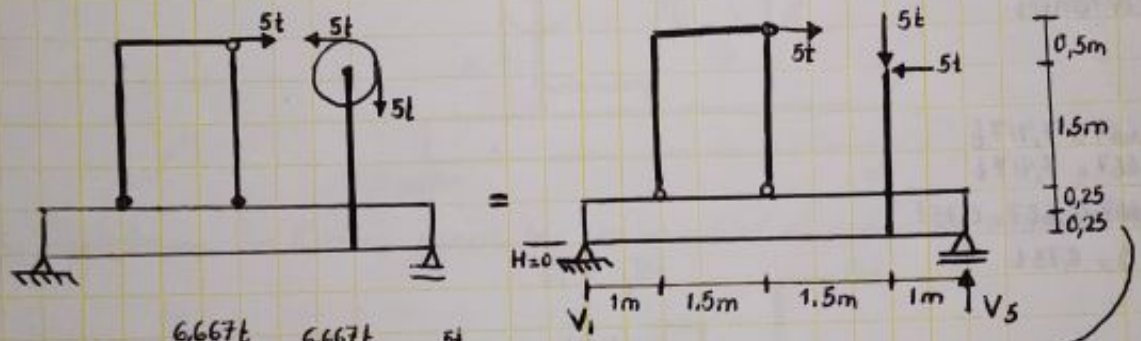
$N_3^d = 0$

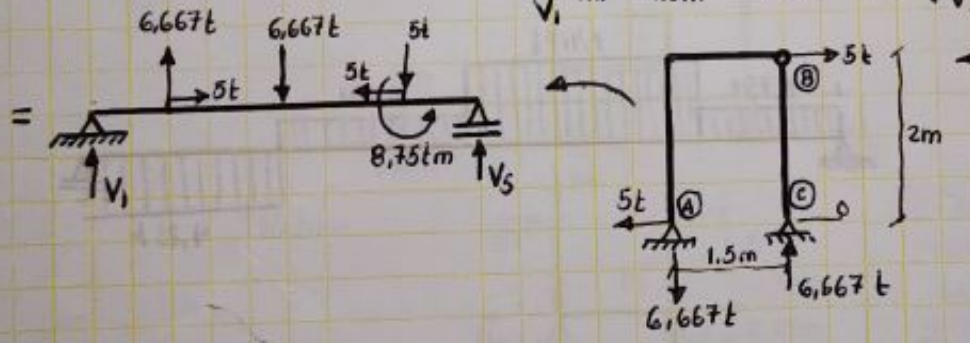
$N_5 = 0$



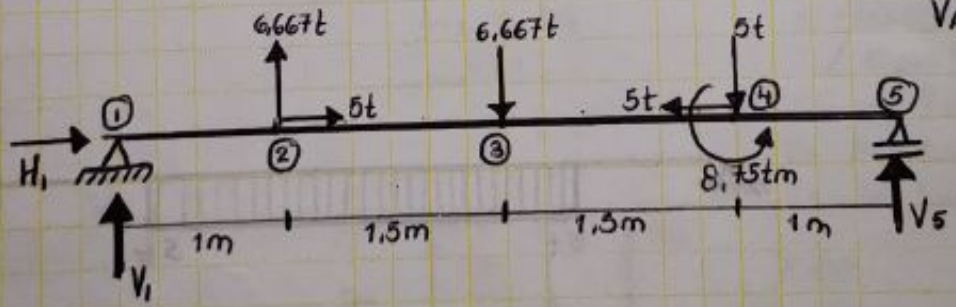
3 Analizar la siguiente estructura horizontal







$\Sigma M_{B=0}$ ↷⊕ (B-C)
$H_C = 0$
$\Sigma F_{H=0}$ →⊕
$H_A = 5t$
$\Sigma M_{A=0}$ ↷⊕
$V_C = 6,667t$
$\Sigma F_{V=0}$ ↑⊕
$V_A = -6,667t$



Calculo de Reacciones

$\Sigma M_{①=0}$ ↷⊕
$-6,667(1) + 6,667(2,5) + 5(4) - 8,75 - V_5(5) = 0$
$V_5 = 4,25t$ //

$\Sigma F_{V=0}$ ↑⊕
$V_1 + 6,667 - 6,667 - 5 + 4,25 = 0$
$V_1 = 0,75t$ //

$\Sigma F_{H=0}$ →⊕
$H_1 + 5 - 5 = 0$
$H_1 = 0$ //

Cálculo de Momentos

CI $M_1 = 0$

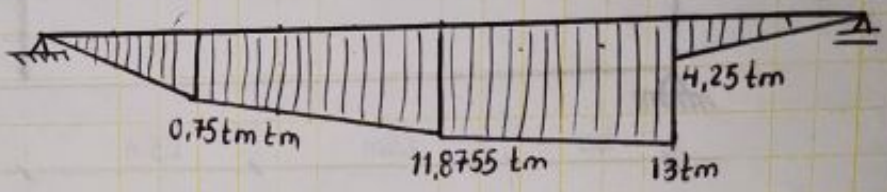
CI $M_2 = 0,75(1) = 0,75 tm$

CI $M_3 = 0,75(2,5) + 6,667(1,5) = 11,8755 tm$

CI $M_4^a = 0,75(4) + 6,667(3) - 6,667(1,5) = 13 tm$

CD $M_4^d = 4,25(1) = 4,25 tm$

CD $M_5 = (0)$



Cálculo de Cortantes

CI $Q_1^d = 0,75 t$

CI $Q_2^a = 0,75 t$

CI $Q_2^d = 0,75 + 6,667 = 7,417 t$

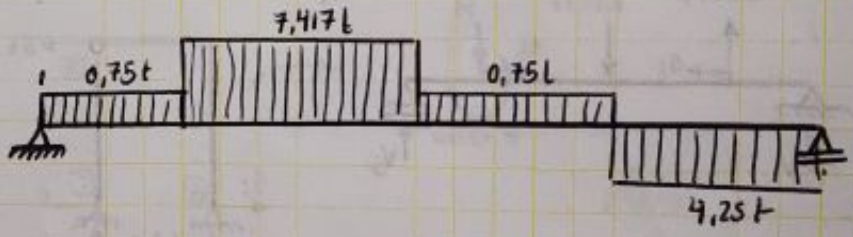
CI $Q_3^a = 0,75 + 6,667 = 7,417 t$

CI $Q_3^d = 0,75 + 6,667 - 6,667 = 0,75 t$

CD $Q_4^a = -4,25 + 5 = 0,75 t$

CD $Q_4^d = -4,25 t$

CD $Q_5^a = -4,25 t$



Cálculo de Normal

$N_1 = 0$

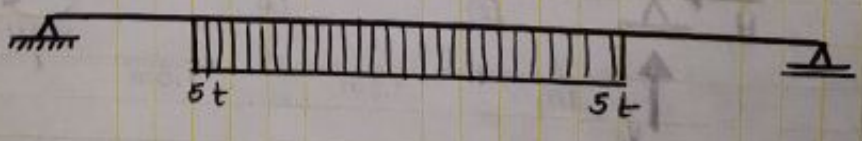
$N_2^a = 0$

$N_2^d = -5$

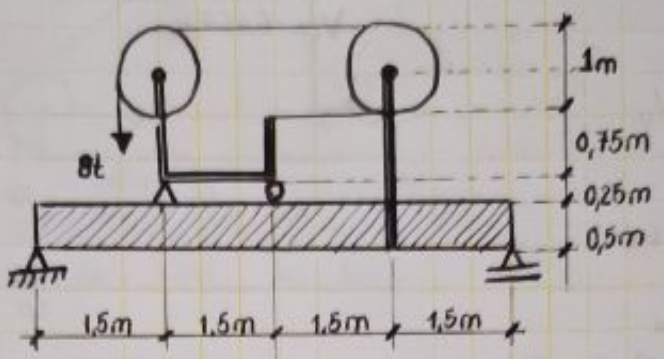
$N_4^a = -5$

$N_4^d = 0$

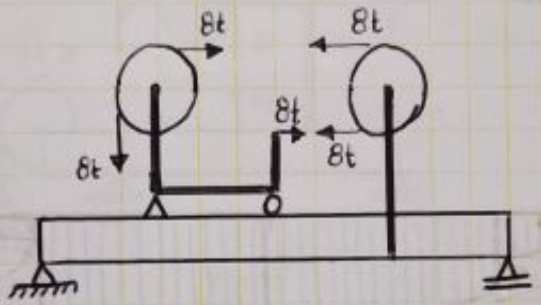
$N_5 = 0$



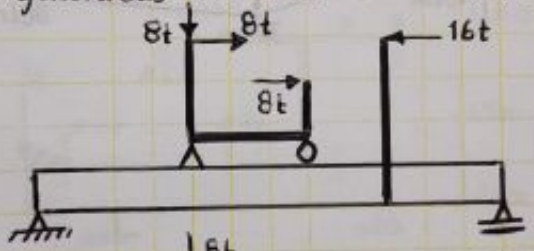
4 Analizar los diagramas de esfuerzos en la sgte viga



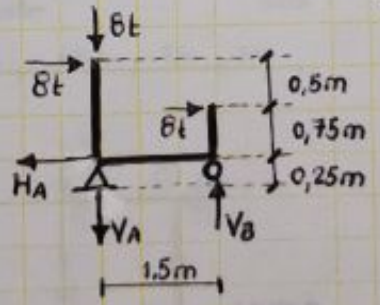
1.- Análisis de las poleas



Trasladamos las fuerzas generadas en los cables al centro de cada polea



Pórtico



$\Sigma F_{H=0} \rightarrow \oplus$

$-H_A + 8 + 8 = 0$

$H_A = 16t$ //

$\Sigma M_{A=0} \circlearrowright \oplus$

$8(1,25) + 8(0,75) - V_B(1,5) = 0$

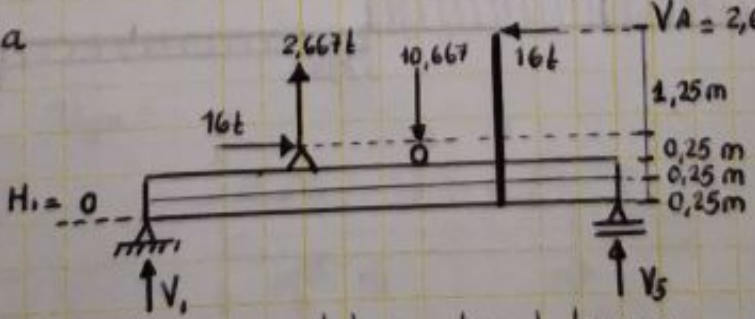
$V_B = 10,667t$ //

$\Sigma F_{V=0} \uparrow \oplus$

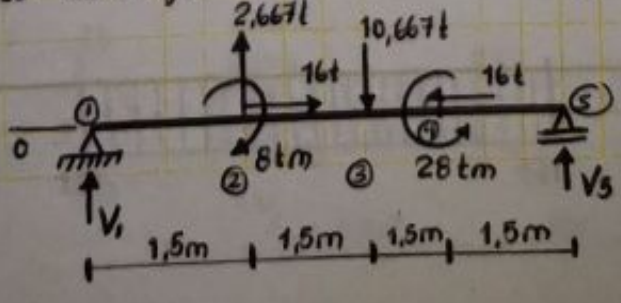
$-V_A - 8 + 10,6677$

$V_A = 2,667t$ //

Viga



Trasladamos las cargas al baricentro de la viga



Calculo de Reacciones

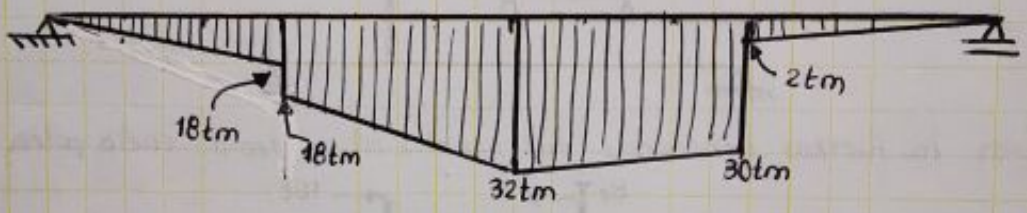
$\sum M_{①}=0$ ↻⊕
$-2,667(1,5)+8+10,667(3)-28-V_5(6)=0$
$V_5 = 1,333\,t$ //

$\sum F_V=0$ ↑⊕
$V_1+2,667-10,667+1,333=0$
$V_1 = 6,667\,t$ //

$\sum F_H=0$ →⊕
$H_1+16-16=0$
$H_1=0$ //

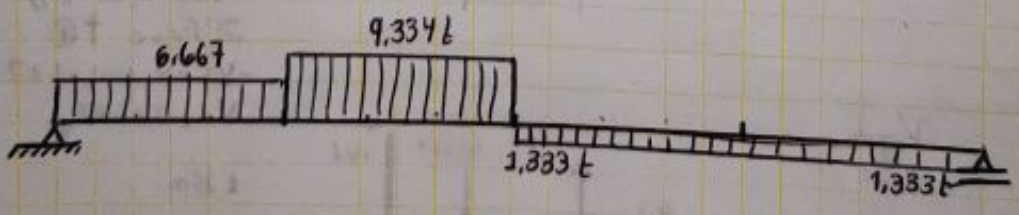
Cálculo de Momento

CI $M_1=0$
CI $M_{2a}=6,667(1,5)=10\,tm$
CI $M_{2d}=M_{2a}+8=18\,tm$
CI $M_3=6,667(3)+2,667(1,5)+8=32\,tm$
CD $M_{4a}=28+1,333(1,5)=30\,tm$
CD $M_{4d}=1,333(1,5)=2\,tm$
CD $M_5=0$



Cálculo de Cortante

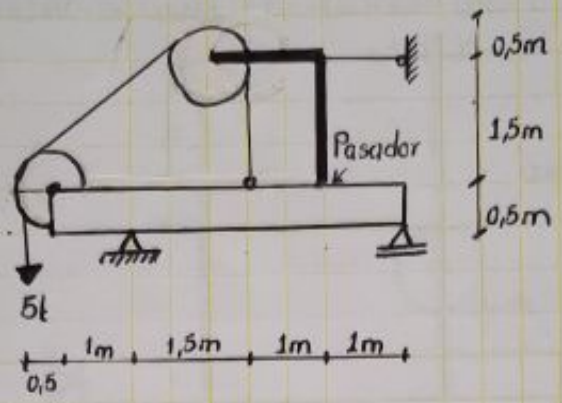
CI $Q_{1d}=6,667\,t$
CI $Q_{2a}=6,667\,t$
CI $Q_{2d}=6,667+2,667=9,334\,t$
CI $Q_{3a}=6,667+2,667=9,334\,t$
CD $Q_{3d}=-1,333\,t$
CD $Q_{5a}=-1,333\,t$



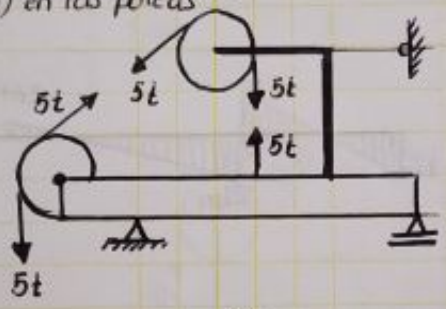
Cálculo de Normales

CI $N_1=0$
CI $N_{2a}=0$
CI $N_{2d}=-16\,t$
CI $N_{4a}=-16\,t$
CD $N_{4d}=0$
CD $N_5=0$

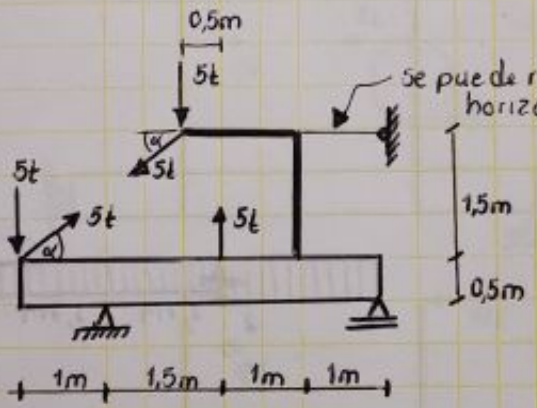
5.- Analizar la Viga determinando sus diagramas de esfuerzos internos



1.- Análisis (en el aparejo) en las poleas



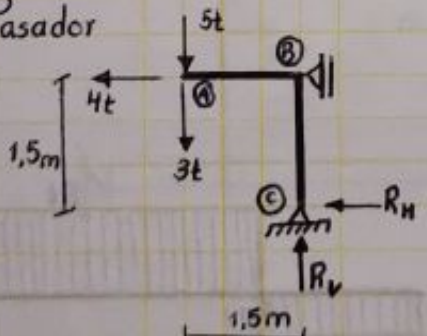
2.- Análisis del pasador



se puede reemplazar por un apoyo movil horizontal solo para calcular las reacc

$\alpha = arctg\left(\frac{1,5}{2}\right) = 36,87°$

Pasador



$\Sigma F_{V=0} \uparrow \oplus$

$-5-3+R_V=0$

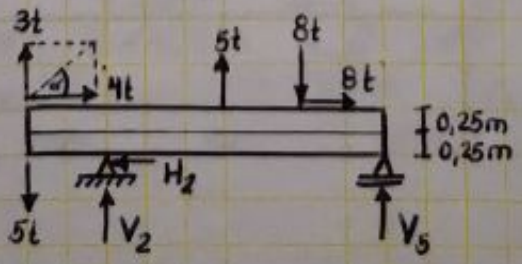
$R_V = 8t$ //

$\Sigma M_{B=0} \curvearrowright \oplus$

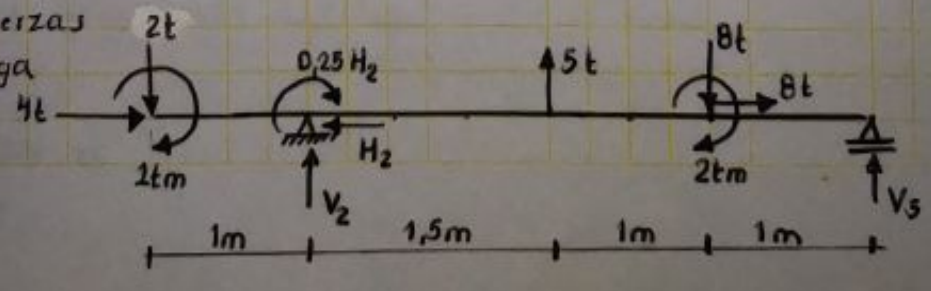
$-(5+3)(1,5)+R_H(1,5)=0$

$R_H = 8t$ //

Viga



Trasladamos las fuerzas al baricentro de la viga



Reacciones

$\Sigma F_{H=0} \rightarrow \oplus$

$+4 - H_2 + 8 = 0$

$H_2 = 12t$ //

$\Sigma M_{②} = 0 \curvearrowright \oplus$

$1 - 2(1) + 0{,}25(12) - 5(1{,}5) + 8(2{,}5) + 2 - V_5(3{,}5) = 0$

$V_5 = 4{,}714t$ //

$\Sigma F_{V=0} \uparrow \oplus$

$-2 + V_2 + 5 - 8 + 4{,}714 = 0$

$V_2 = 0{,}286t$ //

Cálculo de Momentos

C1 $M_{1d} = 1tm$

C1 $M_{2a} = 1 - 2(1) = -1tm$

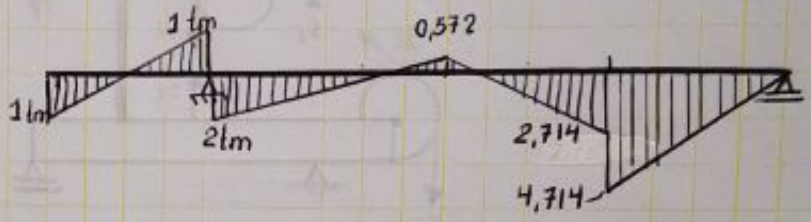
C1 $M_{2d} = M_{2a} + 0{,}25(12) = 2tm$

CD $M_3 = -2 - 8(1) + 4{,}714(2) = -0{,}572tm$

CD $M_{4a} = -2 + 4{,}714(1) = 2{,}714tm$

CD $M_{4d} = 4{,}714(1) = 4{,}714tm$

CD $M_5 = 0$ //



Cálculo de Cortante

C1 $Q_{1d} = -2t$

C1 $Q_{2a} = -2t$

C1 $Q_{2d} = -2 + 0{,}286 = -1{,}714t$ //

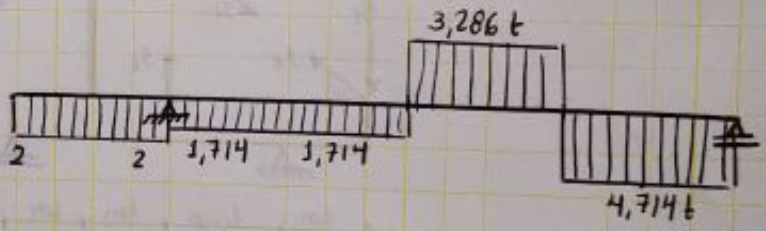
C1 $Q_{3a} = -2 + 0{,}286 = -1{,}714t$

C1 $Q_{3d} = -2 + 0{,}286 + 5 = 3{,}286t$

CD $Q_{4a} = 8 - 4{,}714 = 3{,}286t$

CD $Q_{4d} = -4{,}714t$ //

CD $Q_{5a} = -4{,}714t$ //



Cálculo de Normales

C1 $N_{1d} = -4t$

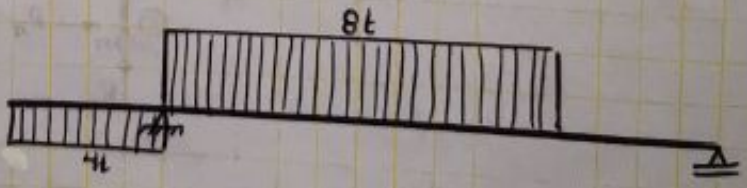
C1 $N_{2a} = -4t$

C1 $N_{2d} = -4 + 12 = 8t$

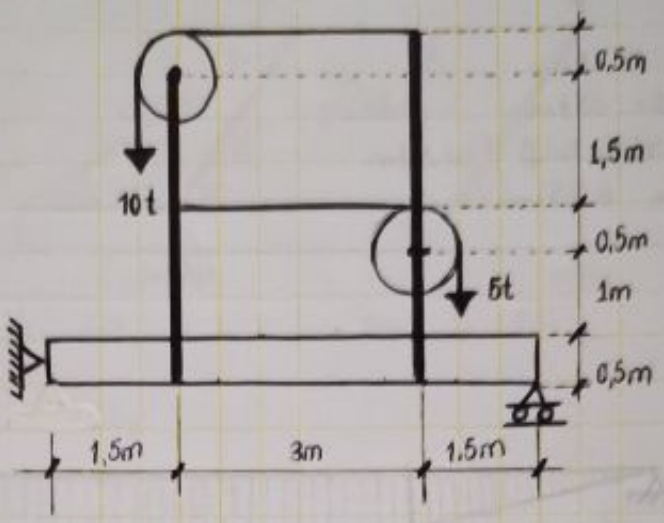
CD $N_{4a} = 8t$

CD $N_{4d} = 0$

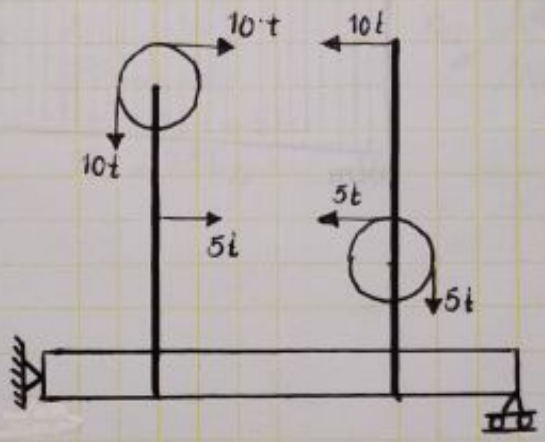
CD $N_5 = 0$

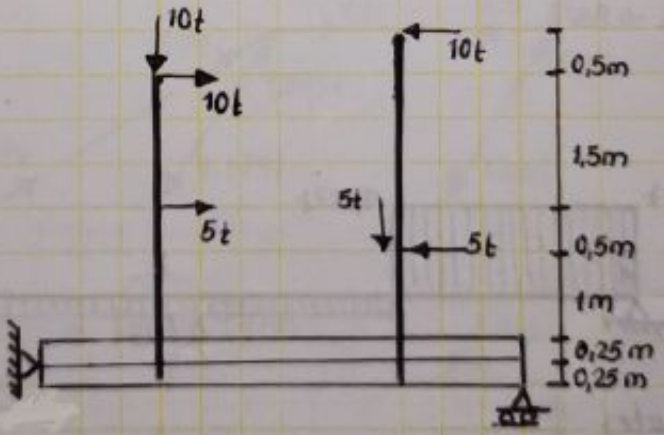


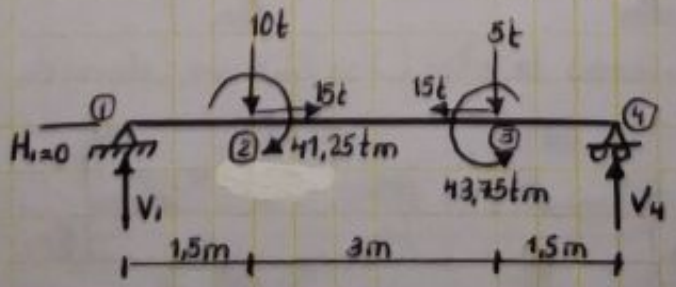
6 Calcular reacciones, diagramar momento



1. Análisis en las poleas







Cálculo de Reacciones

$\sum M_{①} = 0$ ↻ (+)

$10(1,5) + 41,25 + 5(4,5) - 43,75 - V_4(6) = 0$

$V_4 = 5,833\,t$ //

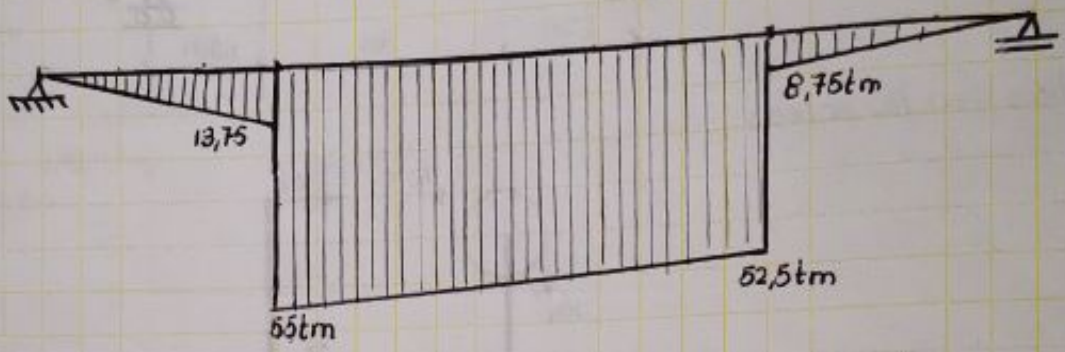
$\sum F_V = 0$ ↑ (+)

$V_1 - 10 - 5 + 5,833 = 0$

$V_1 = 9,167\,t$ //

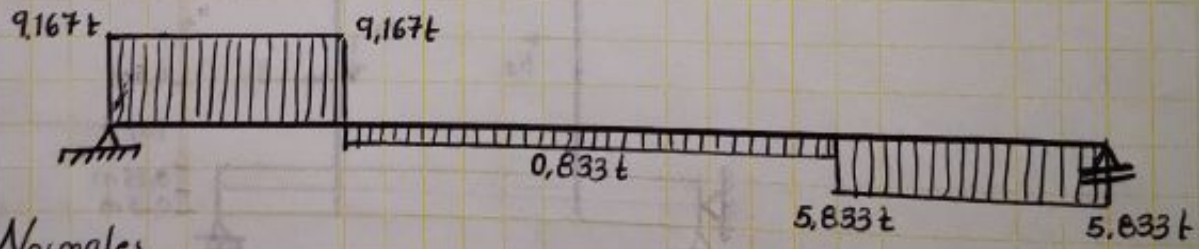
Cálculo de Momento

CI $M_1 = 0$
CI $M_{2a} = 9,167(1,5) = 13,75\ tm$
CI $M_{2d} = M_{2a} + 41,25 = 55\ tm$
CI $M_{3a} = 9,167(4,5) + 41,25 - 10(3) = 52,5\ tm$
CI $M_{3d} = M_{3a} - 43,75 = 8,75\ tm$
CD $M_4 = 0$



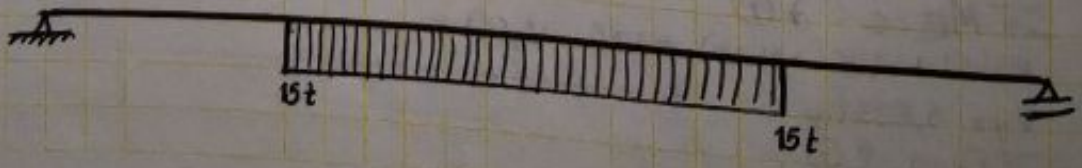
Cálculo de Cortante

CI $Q_{1d} = 9,167\ t$
CI $Q_{2a} = 9,167\ t$
CI $Q_{2d} = 9,167 - 10 = -0,833\ t$
CI $Q_{3a} = 9,167 - 10 = -0,833\ t$
CD $Q_{3d} = -5,833\ t$
CD $Q_{4a} = -5,833\ t$

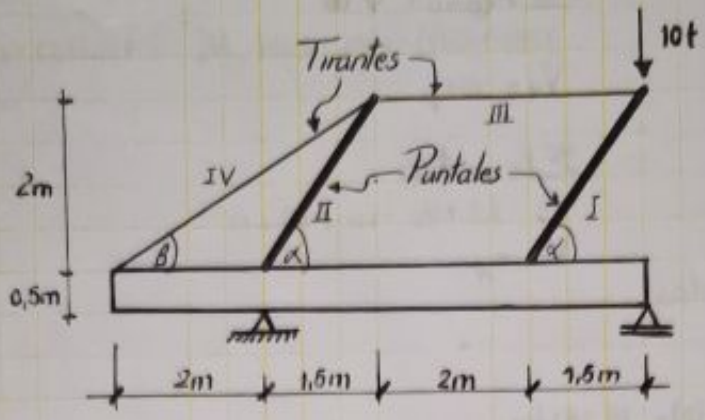


Cálculo de Normales

CI $N_1 = 0$
CI $N_{2a} = 0$
CI $N_{2d} = -15\ t$
CI $N_{3a} = -15\ t$
CD $N_{3d} = 0$
CD $N_4 = 0$

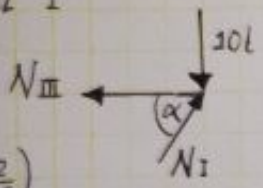


7.- Diagramar esfuerzos en el elemento horizontal



1 Análisis de puntales y tirantes

~Puntal I



$\alpha = \text{arctg}\left(\frac{2}{1,5}\right)$

$\alpha = 53,13°$

$\sum F_V = 0 \uparrow \oplus$

$N_I \operatorname{Sen}\alpha - 10 = 0$

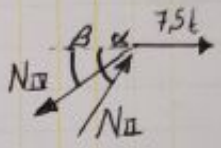
$N_I = 12,5\,t$ //

$\sum F_H = 0 \rightarrow \oplus$

$-N_{III} + 12,5 \operatorname{Cos}\alpha = 0$

$N_{III} = 7,5\,t$ //

~Puntal II



$\beta = \text{arctg}\left(\frac{2}{3,5}\right) = 29,745°$

$\sum F_H = 0 \rightarrow \oplus$

$-N_{IV} \operatorname{Cos}\beta + N_{II} \operatorname{Cos}\alpha + 7,5 = 0$

$-0,8682\,N_{IV} + 0,600\,N_{II} = -7,5$

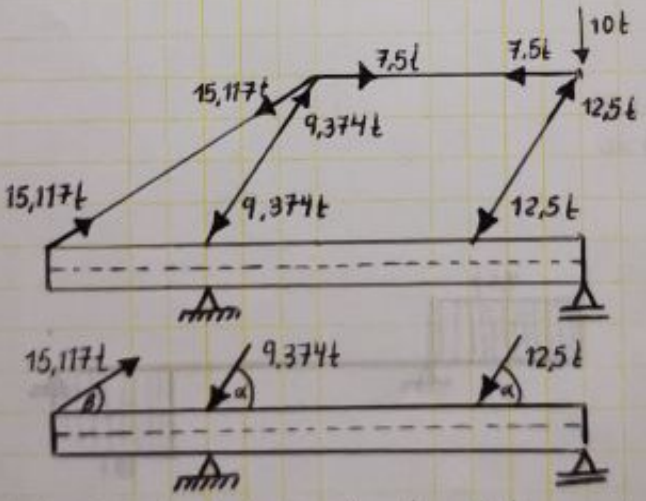
$\sum F_V = 0 \uparrow \oplus$

$-N_{IV} \operatorname{Sen}\beta + N_{II} \operatorname{Sen}\alpha = 0$

$-0,4961\,N_{IV} + 0,8\,N_{II} = 0$ //

$N_{IV} = 15,117\,t$ //

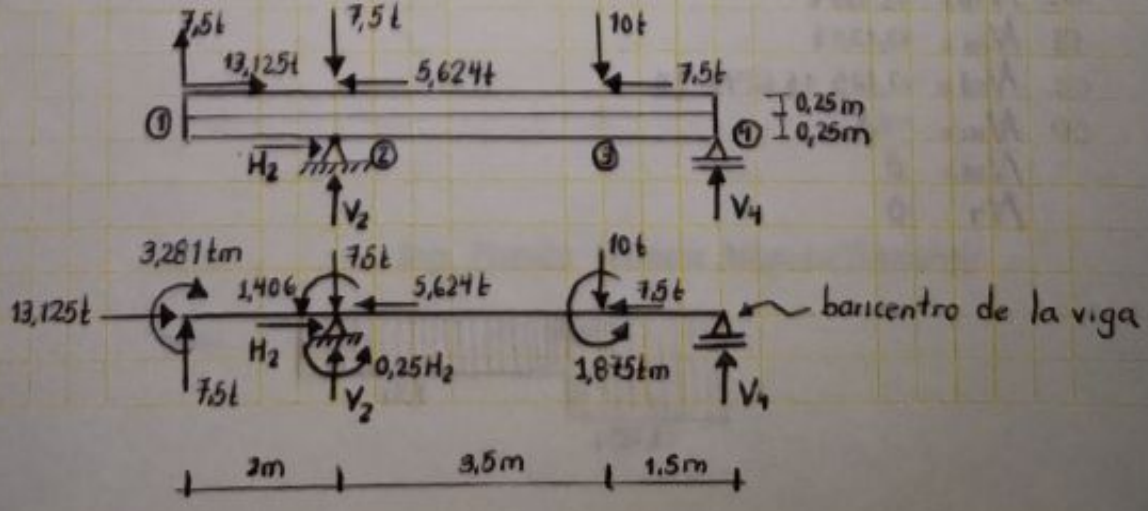
$N_{II} = 9,374\,t$ //



$\alpha = 53,13$

$\beta = 29,745°$

Las cargas y los apoyos deberán trasladarse hasta el baricentro de la viga.



Cálculo de Reacciones

$\Sigma F_H = 0 \rightarrow \oplus$

$13,125 + H_2 - 5,625 - 7,5 = 0$

$H_2 = 0$ //

$\Sigma M_{(2)} = 0 \curvearrowright \oplus$

$3,281 + 7,5(2) - 1,406 - 0,25(\cancel{H_2}^{\,0}) + 10(3,5) - 1,875 - V_4(5) = 0$

$V_4 = 10t$ //

$\Sigma F_V = 0 \uparrow \oplus$

$7,5 - 7,5 + V_2 - 10 + 10 = 0$

$V_2 = 0$ //

Calculo de Momentos

CI $M_{1d} = 3,281 tm$

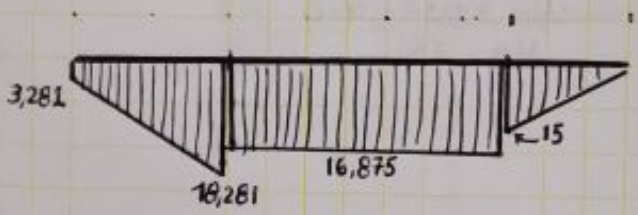
CI $M_{2a} = 3,281 + 7,5(2) = 18,281 tm$

CI $M_{2d} = M_{2a} - 1,406 = 16,875 tm$

CD M_{3a} $10(1,5) + 1,875 = 16,875 tm$

CD $M_{3d} = 10(1,5) = 15 tm$

$M_4 = 0$



Cálculo de Cortantes

CI $Q_{1d} = 7,5$

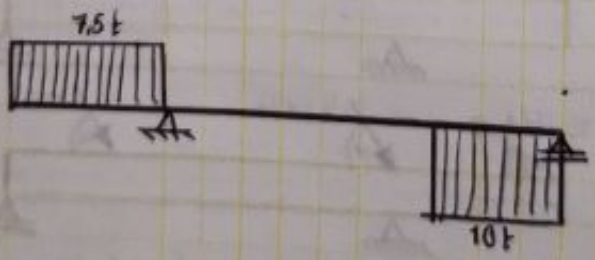
CI $Q_{2a} = 7,5$

CI $Q_{2d} = 7,5 - 7,5 = 0$

CI $Q_{3a} = +7,5 - 7,5 = 0$

CD $Q_{3d} = -10t$

CD $Q_4 = -10t$



Cálculo de Normal

CI $N_{1d} = -13,125 t$

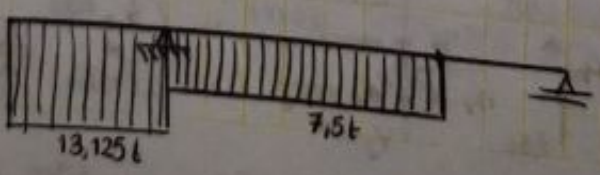
CI $N_{2a} = -13,125 t$

CI $N_{2d} = -13,125 + 5,624 = -7,5$

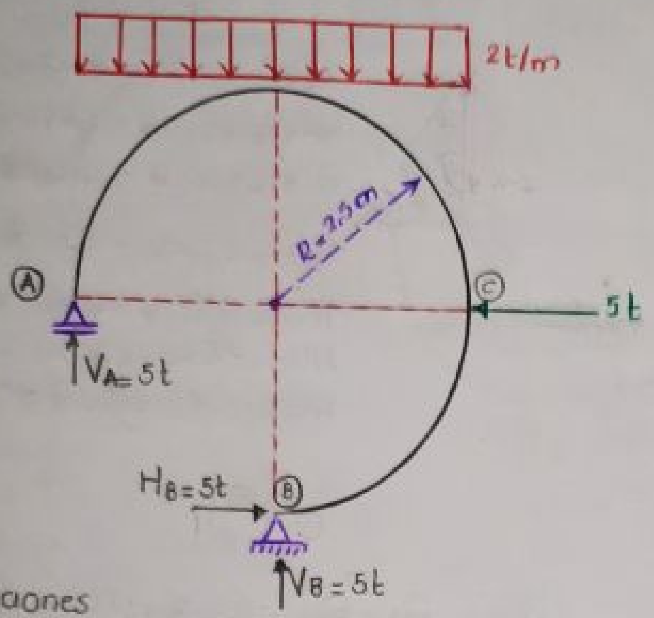
CD $N_{3a} = -7,5$

$N_{3d} = 0$

N_4 0



Diagramar Momento, Normal y Cortante para la siguiente estructura



1.- Reacciones

$\sum M_B = 0$ ↻ ⊕

$-5(2{,}5) + V_A(2{,}5) = 0 \quad \therefore V_A = 5t$

$\sum F_V = 0$ ↑ ⊕

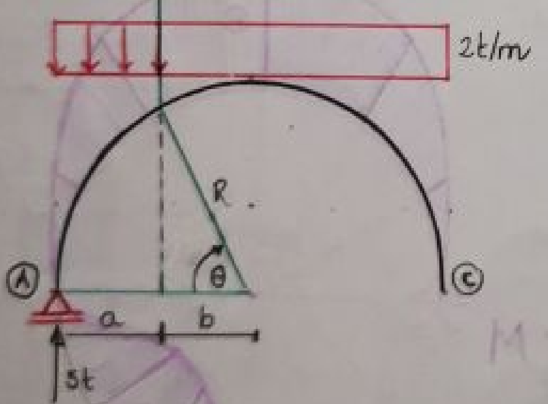
$5 - 2(5) + V_B = 0 \quad \therefore V_B = 5t$

$\sum F_H = 0$ → ⊕

$H_B = 5t$

2- Momentos

* Tramo $\overline{AC}$ $\quad 0 \leq \theta \leq 180$



$a = R - b$

$b = R\cos\theta$

$a = R - R\cos\theta = R(1-\cos\theta)$

$M(\theta) = 5 \cdot a - \frac{2(a)^2}{2} = 5R(1-\cos\theta) - R^2(1-\cos\theta)^2$

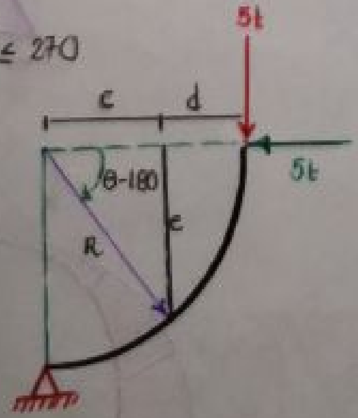
$M(\theta) = 5R - 5R\cos\theta - R^2(1 - 2\cos\theta + \cos^2\theta)$

$M(\theta) = 5R - 5R\cos\theta - R^2 + 2R^2\cos\theta - R^2\cos^2\theta$

$M(\theta) = 5(2{,}5) - 5(2{,}5)\cos\theta - (2{,}5)^2 + 2(2{,}5)^2\cos\theta - (2{,}5)^2\cos^2\theta$

$M(\theta) = -6{,}25\cos^2\theta + 6{,}25$

* Tramo $\overline{CB}$ $\quad 180 \leq \theta \leq 270$



$c = R\cos\alpha \quad ; e = R \cdot \mathrm{Sen}\,\alpha$

$d = R - c = R - R\cos\alpha = R(1-\cos\alpha)$

$M_2(\theta) = 5R(1-\cos\alpha) - 5(R \cdot \mathrm{Sen}\,\alpha)$

$M_2(\theta) = 5(2{,}5)(1-\cos\alpha) - 5(2{,}5 \cdot \mathrm{Sen}\,\alpha)$

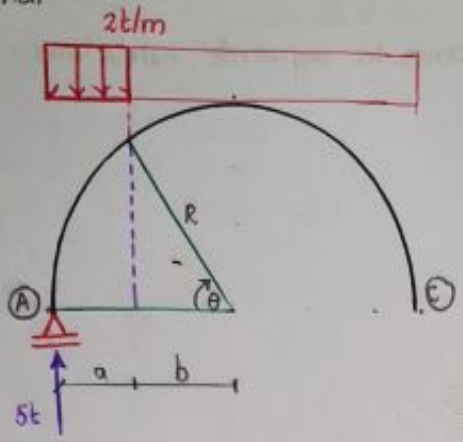
$M_2(\theta) = -12{,}5\cos\alpha - 12{,}5\,\mathrm{Sen}\,\alpha + 12{,}5$

$\alpha = \theta - 180$

$M_2(\theta) = -12{,}5\cos(\theta - 180) - 12{,}5\,\mathrm{Sen}(\theta - 180) + 12{,}5$

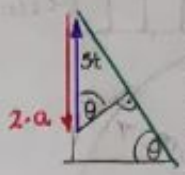
3.-Corte- Normal

* Tramo AC



$a = R - b \quad ; b = R\cos\theta$

$a = R(1-\cos\theta)$



$Q(\theta) = 5\cdot\operatorname{Sen}\theta - 2a\operatorname{Sen}\theta$

$Q(\theta) = 5\operatorname{Sen}\theta - 2R(1-\cos\theta)\operatorname{Sen}\theta$

$Q(\theta) = 5\operatorname{Sen}\theta - 2R\operatorname{Sen}\theta + 2R\operatorname{Sen}\theta\cos\theta$

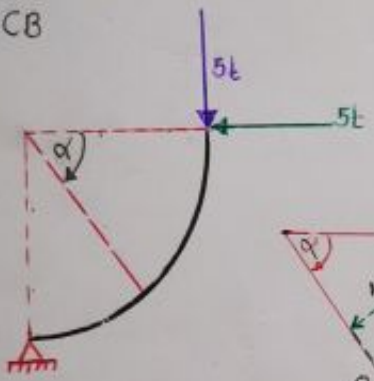
$Q(\theta) = 5\operatorname{Sen}\theta - 5\operatorname{Sen}\theta + 5\operatorname{Sen}\theta\cos\theta$

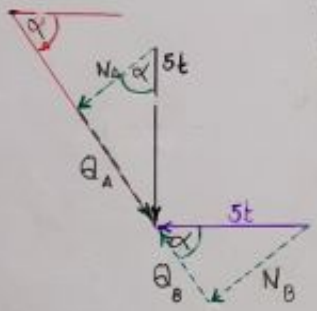
$N(\theta) = -5\cos\theta + 2a\cos\theta$

$N(\theta) = -5\cos\theta + 2(2.5)(1-\cos\theta)\cos\theta$

$N(\theta) = -5\cos\theta + 5\cos\theta - 5\cos^2\theta$

* Tramo CB





$Q(\theta) = -Q_A + Q_B = -5\operatorname{Sen}\alpha + 5\cos\alpha$

$Q(\theta) = -5\operatorname{Sen}\alpha + 5\cos\alpha$

$N(\theta) = -N_A - N_B$

$N(\theta) = -5\cos\alpha - 5\operatorname{Sen}\alpha$

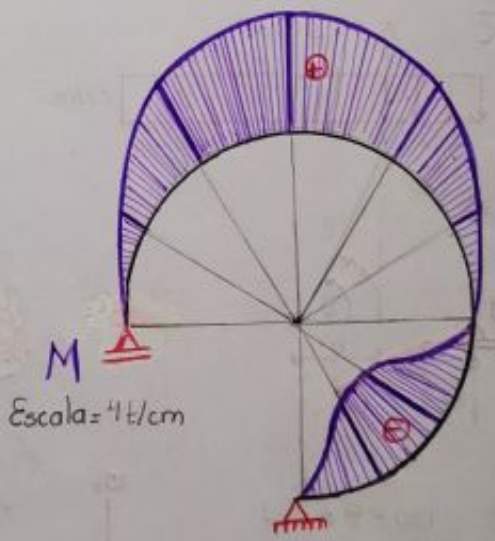
con $\alpha = \theta - 180$

$Q(\theta) = -5\operatorname{Sen}(\theta-180) + 5\cos(\theta-180)$ //

$N(\theta) = -5\cos(\theta-180) - 5\operatorname{Sen}(\theta-180)$ //

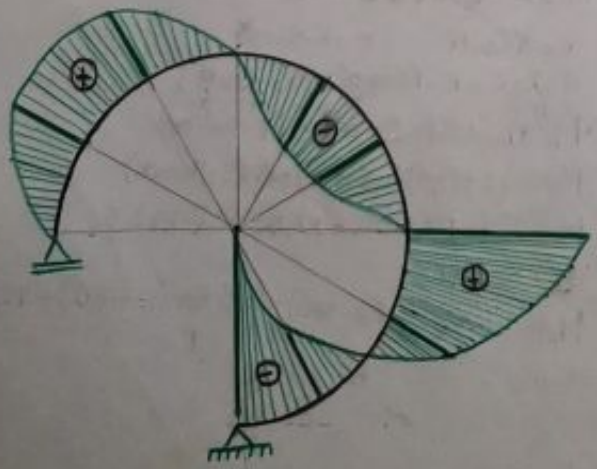
4.-Diagramas

θ	Mto	Cortante	Normal
0	0	0	-5
30	1,563	2,165	-3,75
60	4,688	2,165	-1,25
90	6,25	0	0
120	4,688	-2,165	-1,25
150	1,563	-2,165	-3,75
180	0	0/ 5	-5
210	-4,575	1,830	-6,830
240	-4,575	-1,830	-6,830
270	0	-5	-5

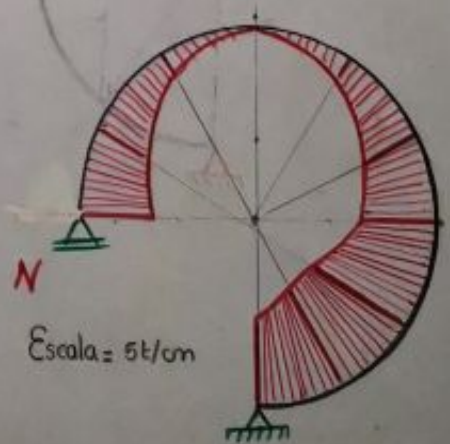


M

Escala = 4t/cm



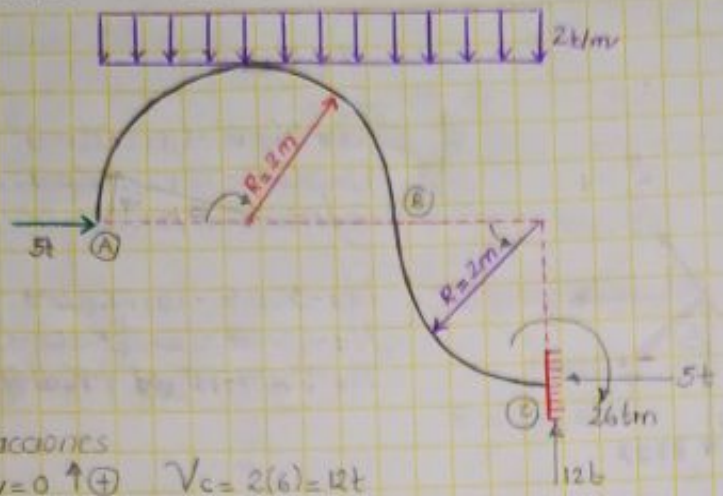
Q Escala = 2t/cm



N

Escala = 5t/cm

Diagramar los esfuerzos internos



1.- Reacciones

$\Sigma F_V = 0 \uparrow \oplus \quad V_c = 2(6) = 12t$

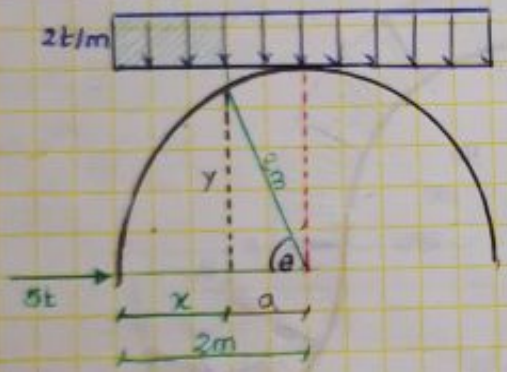
$\Sigma F_H = 0 \rightarrow \oplus \quad H_c = 5t$

$\Sigma M_c = 0 \circlearrowleft \oplus$

$5(2) - 2(6)(3) + M_c = 0 \quad \therefore M_c = 26tm$

2 - Ecuaciones de Momento

* Tramo AB



$x = 2 - a \quad ; a = 2 \cdot \cos\theta$

$x = 2 - 2\cos\theta$

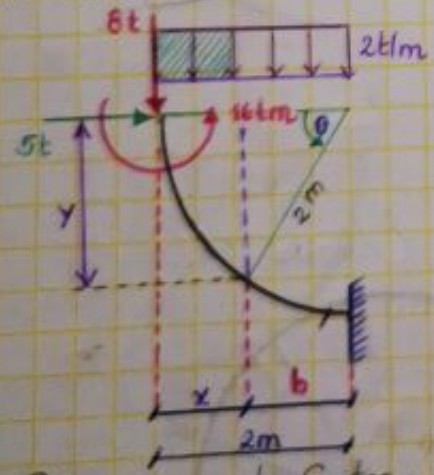
$x = 2(1 - \cos\theta)$

$Y = 2\,\mathrm{Sen}\theta$

$M = -5 \cdot Y - 2\frac{x^2}{2} = -5(2\,\mathrm{Sen}\theta) - [2(1-\cos\theta)]^2$

$M = -10\,\mathrm{Sen}\,x - 4\cos^2 x + 8\cos x - 4$ //

* Tramo BC



$M = -16 - 8 \cdot x + 5Y - \frac{2x^2}{2}$

$x = 2 - b \quad ; b = 2 \cdot \cos\theta$

$x = 2 - 2\cos\theta$

$x = 2(1 - \cos\theta)$

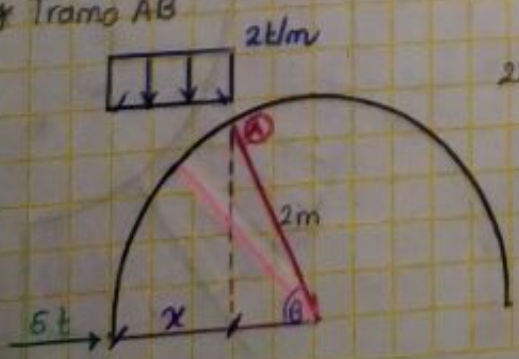
$Y = 2\,\mathrm{Sen}\theta$

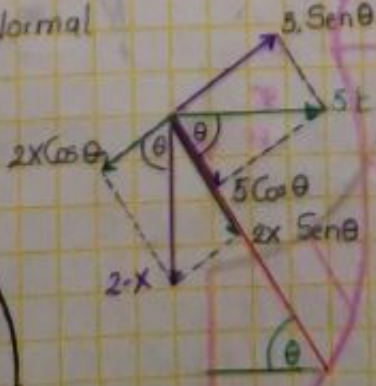
$M = -16 - 8[2(1-\cos\theta)] + 5(2\,\mathrm{Sen}\theta) - [2(1-\cos\theta)]^2$

$M = 10\,\mathrm{Sen}(x) - 4\cos^2(x) + 24\cos(x) - 36$ //

3.- Ecuaciones de Cortes y Normal

* Tramo AB





$x = 2(1 - \cos\theta)$

$Q = -5\cos\theta - 2x\,\mathrm{Sen}\theta$

$Q = -5\cos\theta - 2[2(1-\cos\theta)]\,\mathrm{Sen}\theta$

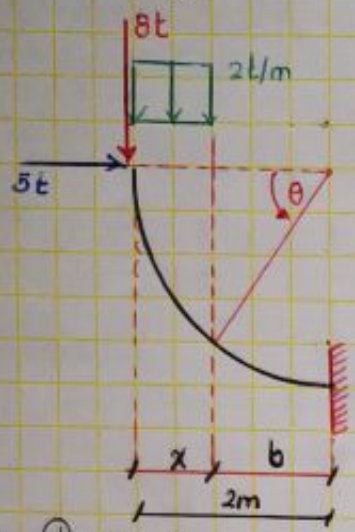
$Q = (4\cos(x) - 4)\,\mathrm{Sen}(x) - 5\cos(x)$ //

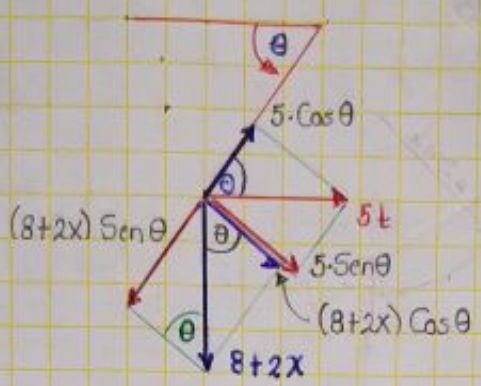
$N = 2x \cdot \cos\theta - 5\,\mathrm{Sen}\theta$

$N = 2[2(1-\cos\theta)]\cos\theta - 5\,\mathrm{Sen}\theta$

$N = -5\,\mathrm{Sen}(x) - 4\cos^2(x) + 4\cos(x)$

* Tramo BC





$x = 2(1-\cos\theta)$

$Q = 5\cos\theta - (8+2x)\sin\theta$

$Q = 5\cos\theta - (8+2[2-2\cos\theta])\sin\theta$

$Q = (4\cos x - 12)\sin(x) + 5\cos(x)$ //

$N = -5\sin\theta - (8+2x)\cos\theta$

$N = -5\sin\theta - (8+2[2-2\cos\theta])\cos\theta$

$N = -5\sin\theta - 12\cos\theta + 4\cos^2\theta$ //

4.- Diagramas

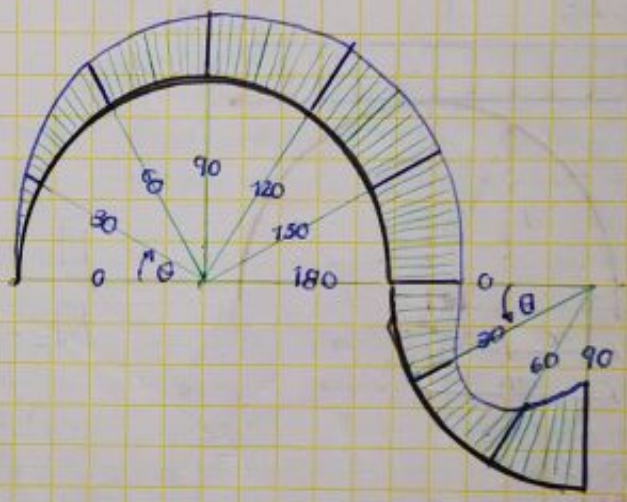
Tramo AB

x	Momento	Cortante	Normal
0	0	-5	0
30	-5,072	-4,598	-2,036
60	-9,66	-4,232	-3,33
90	-14	-4	-5
120	-17,66	-2,696	-7,33
150	-18,93	0,598	-8,964
180	-16	5	-8

Tramo BC

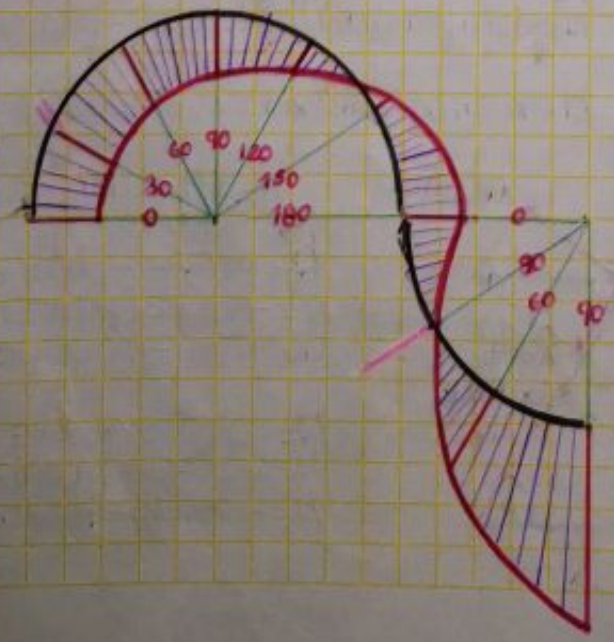
x	Momento	Cortante	Normal
0	-16	5	-8
30	-13,215	0,062	-9,89
60	-16,34	-6,160	-9,333
90	-26	-12,000	-5

Momento Escala = 20tm/cm

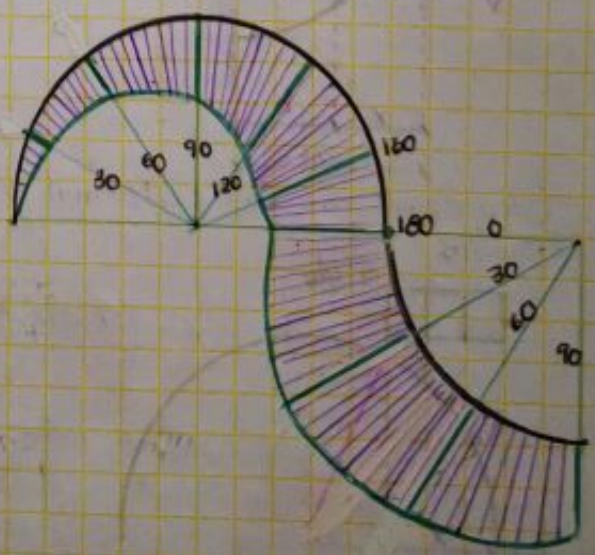


E = 20 tm/cm

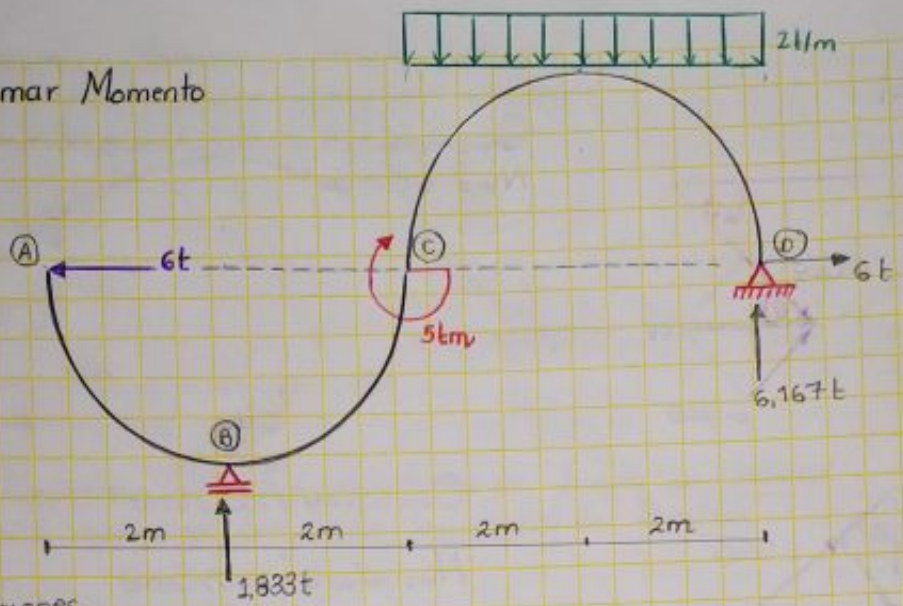
Cortante Escala = 5t/cm



Normal Escala = 5t/cm



Diagramar Momento



1.- Reacciones

$\sum M_{(D)} = 0$ ↺ (+)

$V_B(6) + 5 - 2(4)(2) = 0$ ∴ $V_B = 1,833\ t$ //

$\sum F_V = 0$ ↑ (+)

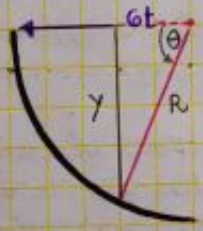
$1,833 - 2(4) + V_D = 0$ ∴ $V_D = 6,167\ t$ //

$\sum F_H = 0$ → (+)

$H_D = 6t$ //

2.- Momentas

* Tramo AB

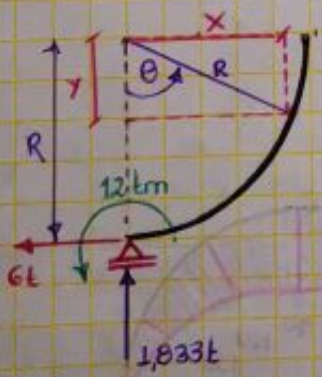


$M_\theta = -6 \cdot Y$ donde $Y = R \cdot Sen\theta$

$M_\theta = -6R\,Sen\theta$

$M_\theta = -12\,Sen\theta$

* Tramo BC



$M_\theta = -12 + 1,833 \cdot X + 6(R - Y)$

$R = 2m$

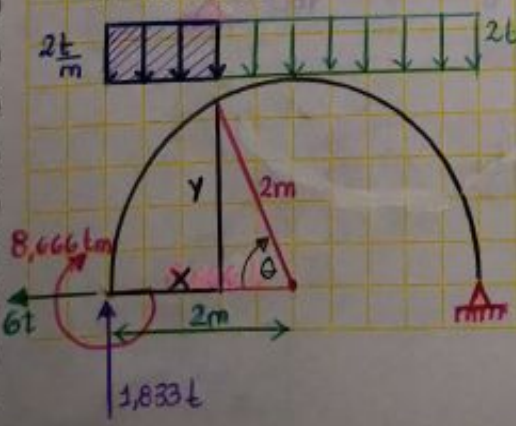
$Y = R\,Cos\theta = 2\,Cos\theta$

$X = R\,Sen\theta = 2\,Sen\theta$

$M_\theta = -12 + 1,833(2\,Sen\theta) + 6(2 - 2\,Cos\theta)$

$M_\theta = 3,666\,Sen(\theta) - 12 \cdot Cos(\theta)$

* Tramo CD



$Y = 2 \cdot Sen\theta$

$X = 2 - 2\,Cos\theta$

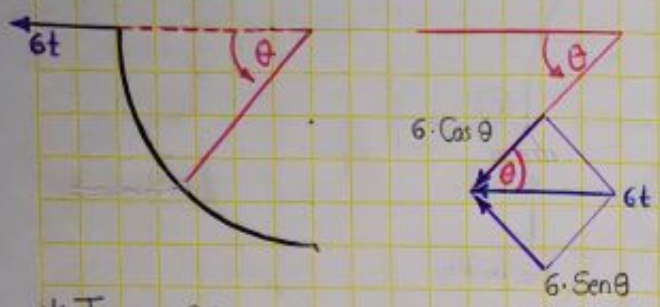
$M_\theta = 8,666 + 1,883X + 6 \cdot Y - 2\frac{X^2}{2}$

$M_\theta = 8,666 + 1,883 \cdot (2 - 2\,Cos\theta) + 6(2\,Sen\theta) - (2 - 2\,Cos\theta)^2$

$M_\theta = 12\,Sen\theta - 4\,Cos^2\theta + 4,334\,Cos\theta + 8,332$ //

3.- Cortante y Normal

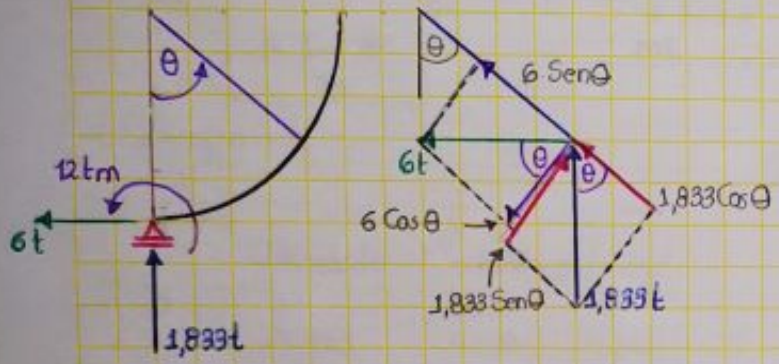
* Tramo AB



$Q_\theta = -6\cos\theta$

$N_\theta = 6\sin\theta$

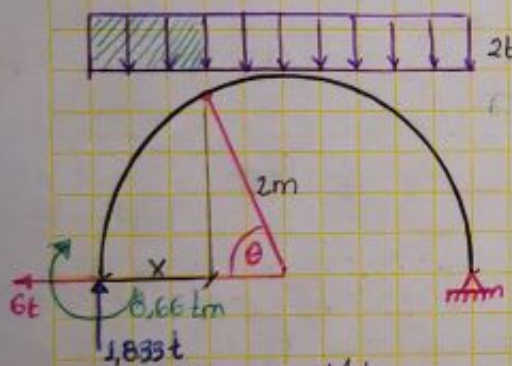
* Tramo BC

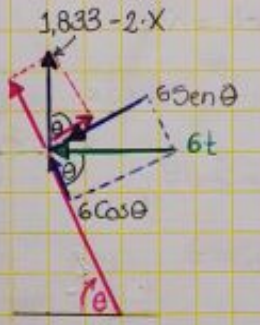


$Q_\theta = 6\sin\theta + 1{,}833\cos\theta$

$N_\theta = 6\cos\theta - 1{,}833\sin\theta$

* Tramo CD





$X = 2 - 2\cos\theta$

$Q_\theta = (1{,}833 - 2X)\sin\theta + 6\cos\theta$

$Q_\theta = (1{,}833 - 2[2 - 2\cos\theta])\sin\theta + 6\cos\theta$

$Q_\theta = (4\cos\theta - 2{,}167)\sin\theta + 6\cos\theta$ //

$N_\theta = 6\sin\theta - (1{,}833 - 2X)\cos\theta$

$N_\theta = 6\sin\theta - (1{,}833 - 2[2 - 2\cos\theta])\cos\theta$

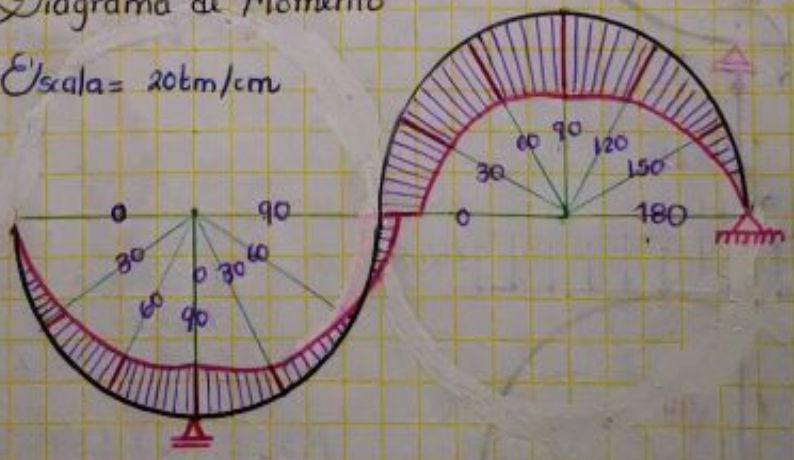
$N_\theta = 6\sin\theta - 4\cos^2\theta + 2{,}167\cos\theta$ //

4.- Diagrama de Mto

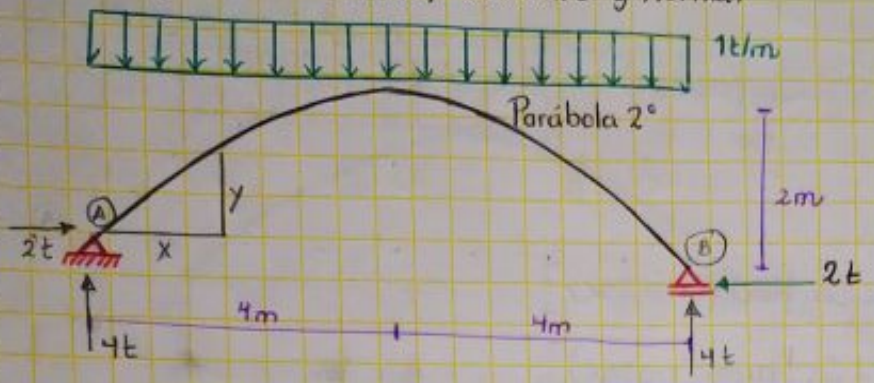
X	Momento
0°	0
30°	-6
60°	-10,892
90°	-12
0°	-12
30°	-8,56
60°	-2,825
90°	3,666
0°	8,666
30°	15,085
60°	19,891
90°	20,332
120°	15,557
150°	7,58
180°	0

Diagrama de Momento

Escala = 20tm/cm



Diagramar Momento, Cortante y Normal



Los diagramas de esfuerzos internos en barras parabólicas las realiza con respecto a una recta, que es la proyección horizontal de la curva

1- Ecuación de la curva

Datos

Origen = Pto (A)

Pto Conocido $X=0$, $Y=0$

Vertice $h=4$, $K=2$

$(X-h)^2 = -4a(Y-K)$

$(0-4)^2 = -4a(0-2)$

$16 = +8a$

$a = 2$ //

Ecuacion → $(X-4)^2 = -4(2)(Y-2)$

despejamos Y

$X^2 - 8X + 16 = -8(Y-2)$

$X^2 - 8X + 16 = -8Y + 16$

$Y = \frac{8X - X^2}{8}$ ∴ $\boxed{Y = X - \frac{X^2}{8}}$

2. Reacciones

$\sum M_A = 0$ ↻ (+)

$1(8)(4) - V_B(8) = 0$ ∴ $V_B = 4t$ //

$\sum F_V = 0$ ↑ (+)

$V_A = 8 - 4 = 4t$ //

$\sum F_H = 0$ → (+)

$H_A = 2t$

3- Ecuación de Momento

$M_X = 4 \cdot X - 2 \cdot Y - 1 \cdot \frac{X^2}{2} = 4X - 2\left(X - \frac{X^2}{8}\right) - \frac{X^2}{2}$

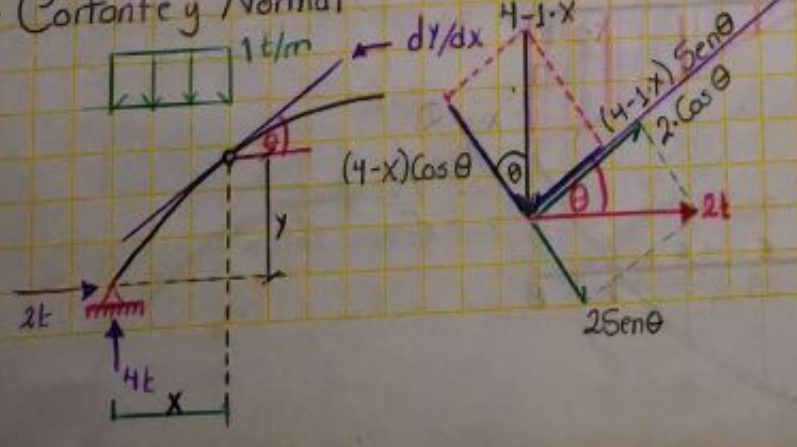
$M_X = 4X - 2X + \frac{X^2}{4} - \frac{X^2}{2}$

$M_X = 2X - \frac{X^2}{4}$

$\frac{dY}{dx} = tg\theta = 1 - \frac{2X}{8} = 1 - \frac{X}{4}$ //

$\theta = arctg(1 - 0{,}25X)$

4.- Cortante y Normal



$Q_x = (4-x)\cos\theta - 2\operatorname{Sen}\theta$ $\quad \theta = \operatorname{arctg}(1-0{,}25x)$

$N_x = (4-x)\operatorname{Sen}\theta - 2\cos\theta$

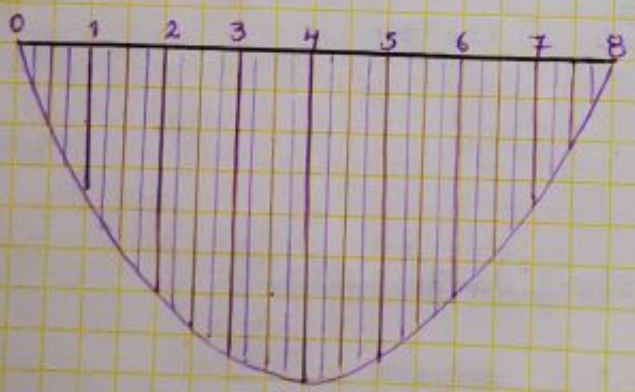
Reemplazamos θ en Q_x y N_x

$$Q_x = (4-x)\cos[\operatorname{arctg}(1-0{,}25x)] - 2\operatorname{Sen}[\operatorname{arctg}(1-0{,}25x)]$$

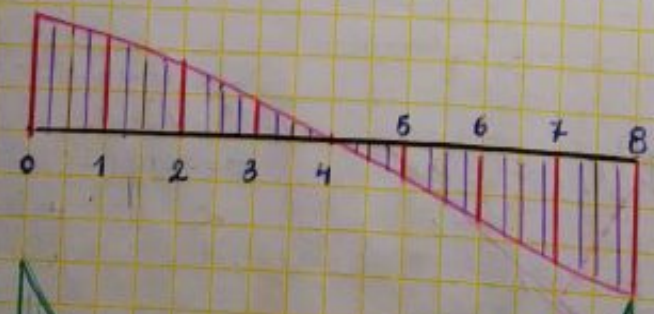
$$N_x = (4-x)\operatorname{Sen}[\operatorname{arctg}(1-0{,}25x)] - 2\cos[\operatorname{arctg}(1-0{,}25x)]$$

x	Momento	Cortante	Normal
0	0	1,4142	1,4142
1	1,75	1,2	0,2
2	3	0,894	-0,894
3	3,75	0,485	-1,698
4	4	0	-2
5	3,75	-0,485	-1,698
6	3	-0,894	-0,894
7	1,75	-1,2	0,2
8	0	-1,414	1,4142

Diagramas



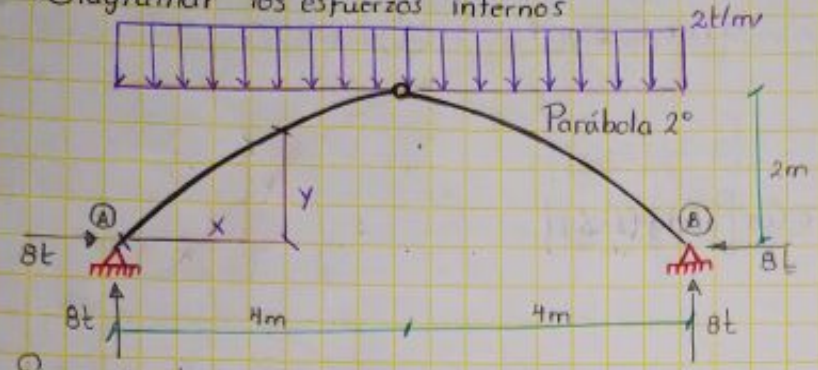
Momento
Esc = 1tn/cm



Cortante
Esc = 1t/cm

Normal
Esc = 1t/cm

Diagramar los esfuerzos internos



1.- Ecuación de la barra parabólica

Datos

Origen del eje cartesiano = pto A = (0,0)

Vertice = (4,2)

$$(x-h)^2 = -4a(y-k)$$
$$(0-4)^2 = -4a(0-2)$$
$$16 = -4a(-2)$$
$$16 = 8a$$
$$a = 2 //$$

$$(x-4)^2 = -4(2)(y-2)$$
$$x^2 - 8x + 16 = -8(y-2)$$
$$x^2 - 8x + 16 = -8y + 16$$
$$y = \frac{8x - x^2}{8}$$
$$y = x - \frac{x^2}{8} //$$

2.- Reacciones

$\Sigma M_A = 0 \curvearrowright \oplus$

$2(8)(4) - V_B(8) = 0 \quad \therefore V_B = 8t //$

$\Sigma F_V = 0 \uparrow \oplus$

$V_A - 2(8) + 8 = 0 \quad \therefore V_A = 8t //$

$\Sigma M_C = 0 \curvearrowright \oplus$ (Izq)

$8(4) - 2(4)(2) - H_A(2) = 0 \quad \therefore H_A = 8t$

$\Sigma F_H = 0 \rightarrow \oplus$

$H_B = 8t$

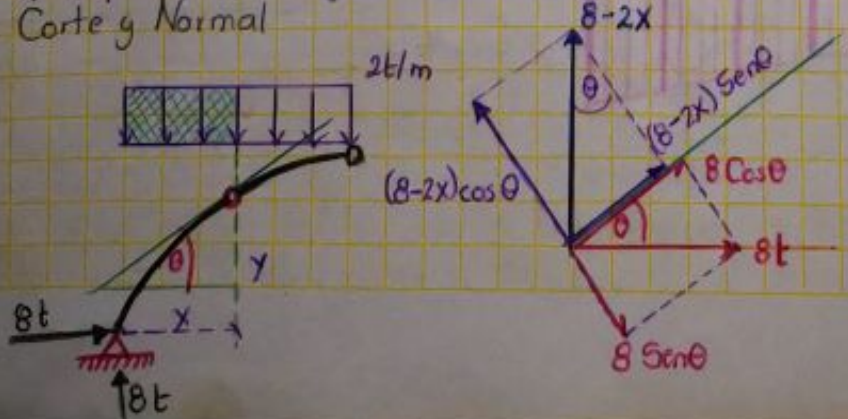
4.- Ecuación de Momento

$$M = 8 \cdot x - 8y - 2\frac{x^2}{2} = 8x - 8y - x^2$$

$$M = 8x - 8\left(\frac{8x - x^2}{8}\right) - x^2 = 8x - 8x + x^2 - x^2 = 0$$

No presenta diagrama de Mto flector

5. Corte y Normal



$$\frac{dy}{dx} = tg\theta = 1 - \frac{x}{4} //$$

$$\theta = arctg\left(1 - \frac{x}{4}\right) //$$

$Q = (8-2x)\cos\theta - 8\,\mathrm{Sen}\,\theta$ donde $\theta = \mathrm{arctg}\left(1-\frac{x}{4}\right)$

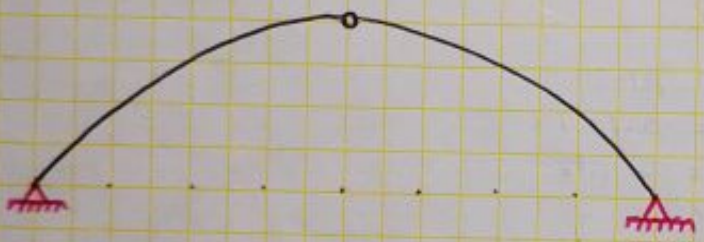
$N = -(8-2x)\,\mathrm{Sen}\,\theta - 8\cos\theta$

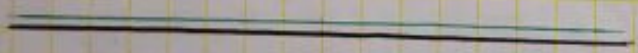
$$Q = (8-2x)\cos\left[\mathrm{arctg}\left(1-\tfrac{x}{4}\right)\right] - 8\cdot\mathrm{Sen}\left[\mathrm{arctg}\left(1-\tfrac{x}{4}\right)\right]$$

$$N = -(8-2x)\,\mathrm{Sen}\left[\mathrm{arctg}\left(1-\tfrac{x}{4}\right)\right] - 8\cos\left[\mathrm{arctg}\left(1-\tfrac{x}{4}\right)\right]$$

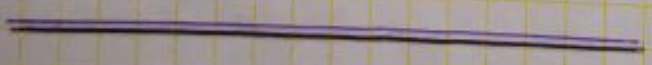
x	Cortante	Normal
0	0	-11,31
1	0	-10
2	0	-8,944
3	0	-8,246
4	0	-8

6 - Diagramas

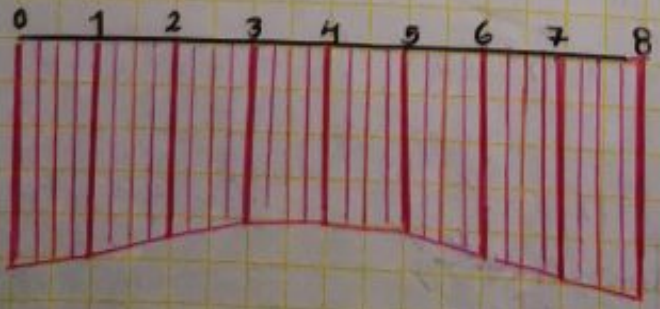




Momento



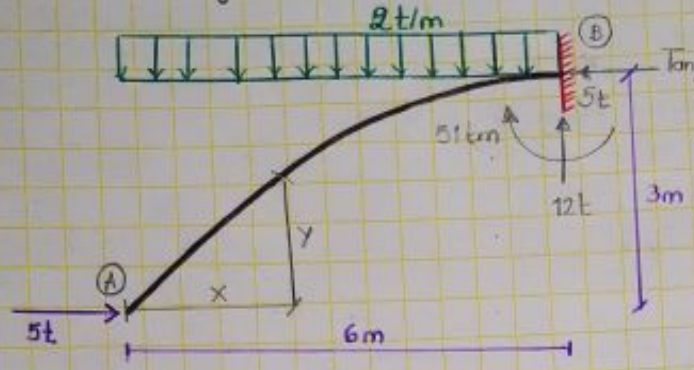
Cortante



Normal

Esc = 4t/cm

Obtener los diagramas caracteristicos



1.- Ecuación de la barra parabólica

Datos

Eje cartesiano de referencia = Pto Ⓐ = (0,0)

Vertice = pto B = (6,3)

$$(x-h)^2 = -4a(y-k)$$

$$(0-6)^2 = -4a(0-3)$$

$$36 = 12a$$

$$a = 3$$

$$(x-6)^2 = -4(3)(y-3)$$

$$Y = x - \frac{x^2}{12} //$$

2.- Reacciones

$\sum F_V = 0 \quad \uparrow \oplus$

$-2(6) + V_B = 0 \quad \therefore V_B = 12t$

$\sum F_H = 0 \rightarrow \oplus \quad \therefore H_B = 5t$

$\sum M_B = 0 \quad \circlearrowleft \oplus$

$-2(6)(3) - 5(3) + M_B = 0 \quad \therefore M_B = 51tm$

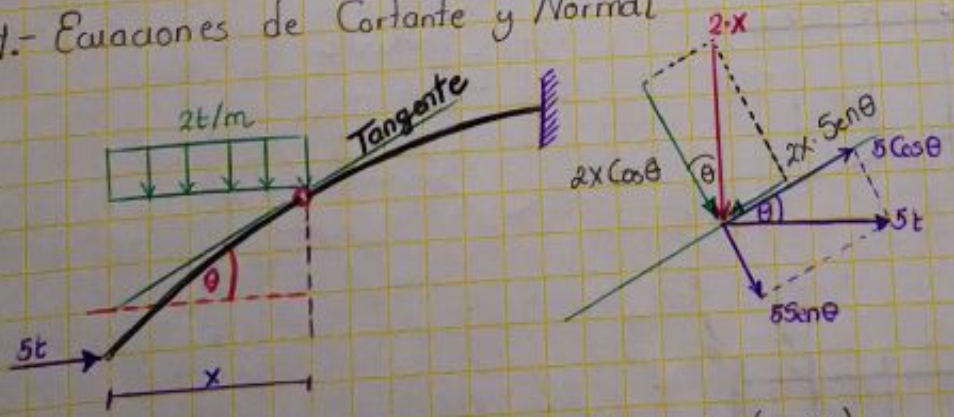
3.- Ecuación de Momento

$$M_x = -5Y - \frac{2x^2}{2} = -5Y - x^2$$

$$M_x = -5\left(x - \frac{x^2}{12}\right) - x^2 = -5x + \frac{5x^2}{12} - x^2$$

$$M_x = -5x - \frac{7x^2}{12} //$$

4.- Ecuaciones de Cortante y Normal



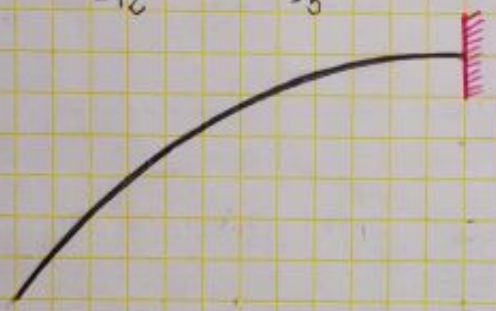
$$\frac{dy}{dx} = Tag\theta = 1 - \frac{x}{6} \Rightarrow \theta = arctg\left(1 - \frac{x}{6}\right)$$

$$Q_x = -2x\cos\theta - 5\sin\theta$$
$$Q_x = -2x\cos\left[\operatorname{arctg}\left(1-\frac{x}{6}\right)\right] - 5\,\mathrm{Sen}\left[\operatorname{arctg}\left(1-\frac{x}{6}\right)\right]//$$

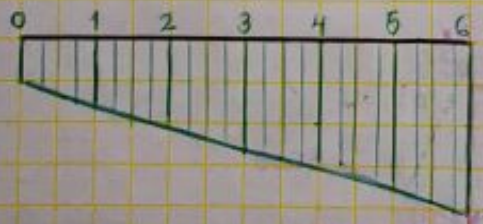
$$N_x = 2x\,\mathrm{Sen}\,\theta - 5\cos\theta$$
$$N_x = 2x\,\mathrm{Sen}\left[\operatorname{arctg}\left(1-\frac{x}{6}\right)\right] - 5\cos\left[\operatorname{arctg}\left(1-\frac{x}{6}\right)\right]$$

5.- Diagramas

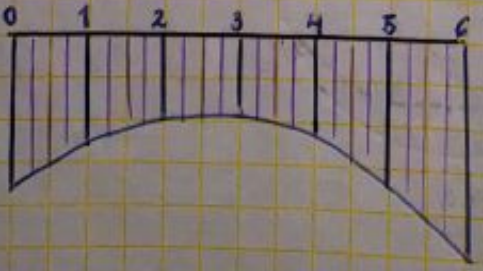
x	Momento	Cortante	Normal
0	0	-3,535	-3,535
1	-5,5833	-4,787	-2,561
2	-12,333	-6,102	-1,941
3	-20,25	-7,603	-1,789
4	-29,33	-9,171	-2,21
5	-39,583	-10,686	-3,288
6	-51	-12	-5



Momento
Esc= 20tm/cm

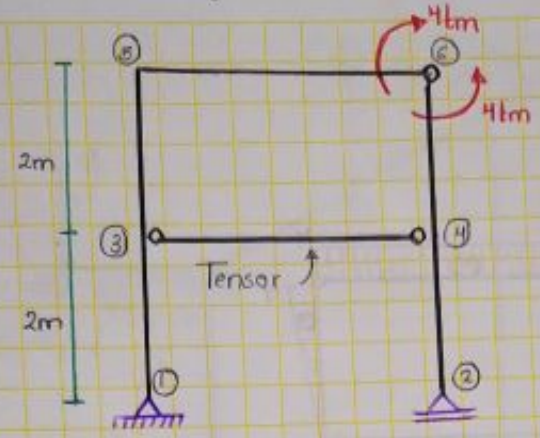


Cortante
Esc= 6t/cm

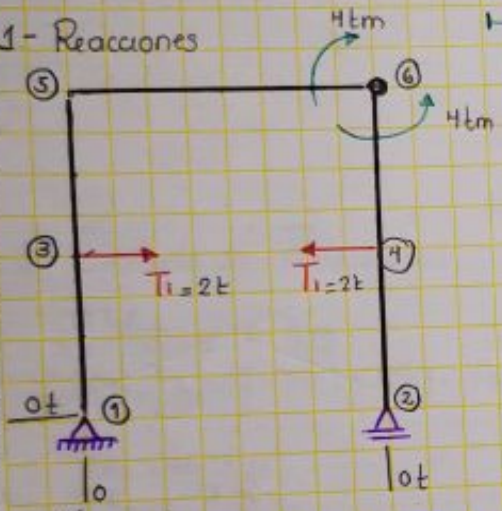


Normal
Esc= 2t/cm

Diagramar Momento, Cortante y Normal



1- Reacciones



$\sum M_{⑥}=0$ ↻ (+)

$-4 + T_1(2)=0$ ∴ $T_1=2t$

$\sum M_{①}=0$ ↻ (+)

$V_2=0t$

$\sum F_V=0$ ↑ (+)

$V_1=0t$

$\sum F_H=0$ → (+)

$H_1=0t$

2 Momentos

Barra 1-3 $M_1=0$

$M_3=0$

Barra 3-5 $M_3=0$

$M_5=-2(2)=-4tm$

Barra 5-6 $M_5=-4tm$

$M_6=-4tm$

Barra 2-4 $M_2=0$

$M_4=0$

Barra 4-6 $M_4=0$

$M_4=-2(2)=-4tm$

3 Cortantes

Barra 1-3 $Q_1=0$

$Q_3=0$

Barra 3-5 $Q_{3-5}=-2t$

Barra 5-6 $Q_{5-6}=0t$

Barra 2-4 $Q_{24}=0t$

Barra 4-6 $Q_{46}=2t$

4- Normales

Barra 1-3 $N_{13}=0t$

Barra 3-5 $N_{3-5}=0t$

Barra 5-6 $N_{5-6}=-2t$

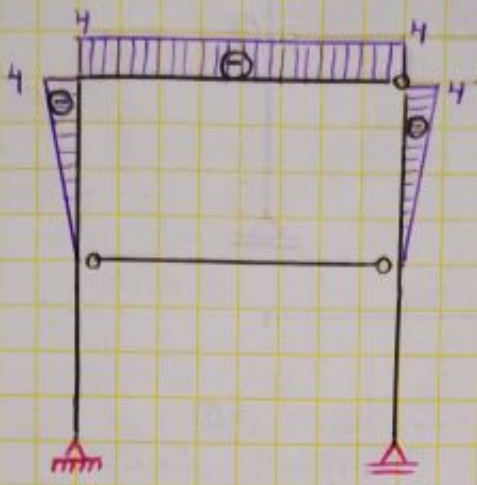
Barra 2-4 $N_{2-4}=0t$

Barra 4-6 $N_{4-6}=0t$

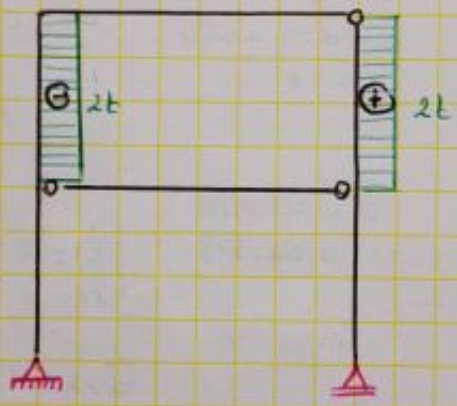
Barra 3-4 $N_{3-4}=2t$

5- Diagramas

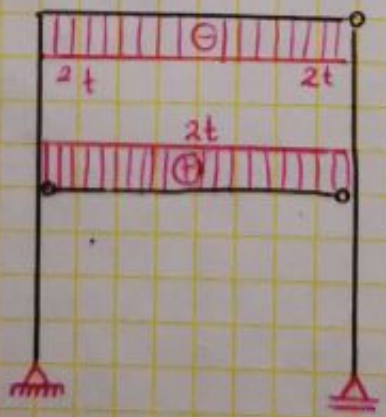
Momentos



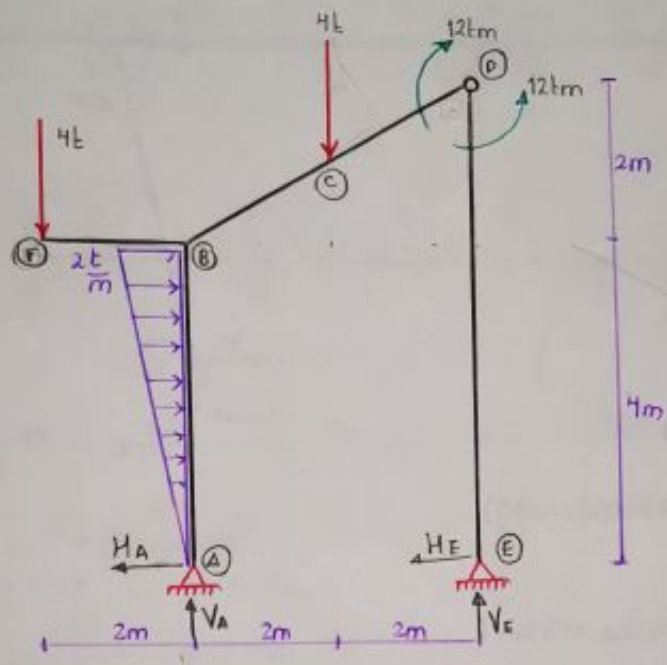
Cortante



Normal



Obtener los diagramas de esfuerzos internos para la siguiente estructura



1.- Calculo de Reacaones

$\sum M_{(A)}=0$ ↷ ⊕

$\frac{2(4)}{2}.\left[\frac{2}{3}(4)\right]-4(2)+4(2)-V_E(4)=0$

$V_E=2,667t$ //

$\sum F_V=0$ ↑⊕

$V_A-4-4+2,667=0$

$V_A=5,333t$ //

$\sum M_D=0$ ↷ ⊕ (barra D-E)

$-12+H_E(6)=0$

$H_E=2t$ //

$\sum F_H=0$ → ⊕

$-H_A+\frac{2(4)}{2}-2=0$

$H_A=2t$ //

2.- Momentos ↷ ⊕

$M_A=0\,tm$

$M_{BA}=2(4)-\frac{2(4)}{2}.\left[\frac{1}{3}.4\right]$

$M_{BA}=2,667\,tm$

$M_F=0tm$

$M_{BF}=-4(2)=-8tm$

$M_{BC}=2(4)-\frac{2(4)}{2}.\left[\frac{1}{3}.4\right]-4(2)=-5,333\,tm$

$M_C=5,333(2)+2(5)-\frac{2(4)}{2}.\left[\frac{1}{3}.4+1\right]-4(4)=-4,6673\,tm$

$M_D=0tm$

$M_{DC}=-12tm$

$M_{DE}=-12tm$

$M_E=0tm$

3.- Cortantes

$Q_A=2t$

$Q_{BA}=2-\frac{2(4)}{2}=-2t$

$Q_F=-4t$

$Q_{BF}=-4t$

Cortantes

$Q_{BC}=1,333\cos(26,565)-2\,\mathrm{Sen}(26,565)$

$Q_{BC}=0,2981t$

$Q_{CD}=0,2981-4\cos(26,565)$

$Q_{CD}=-3,2796$

$Q_{DC}=-3,2796$

$Q_{DE}=2t$

$Q_E=2t$

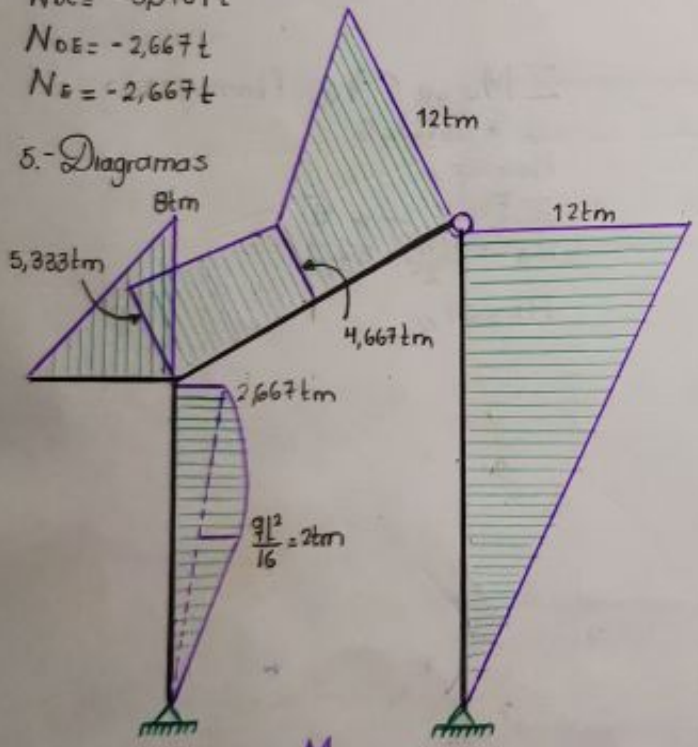
4 - Normales

$N_A = -5{,}333\,t$
$N_{BA} = -5{,}333\,t$
$N_F = 0$
$N_{BF} = 0$
$N_{BC} = ?$

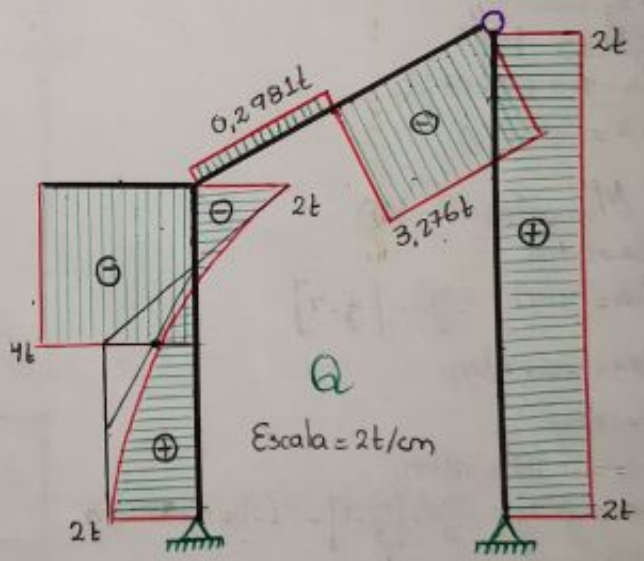
$N_{BC} = -2\,\text{Cos}(26{,}565) - 1{,}333\,\text{Sen}(26{,}565)$
$N_{BC} = -2{,}385\,t$

$N_{CD} = -2{,}385 + 4 \cdot \text{Sen}(26{,}565) = -0{,}5961\,t$
$N_{DC} = -0{,}5961\,t$
$N_{DE} = -2{,}667\,t$
$N_E = -2{,}667\,t$

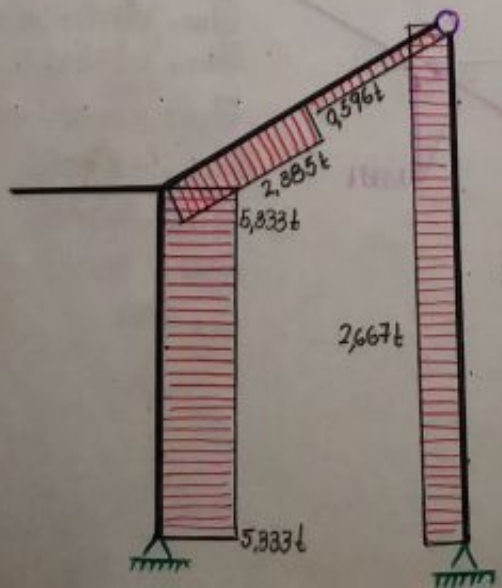
5.- Diagramas



M
Escala = 4tm/cm

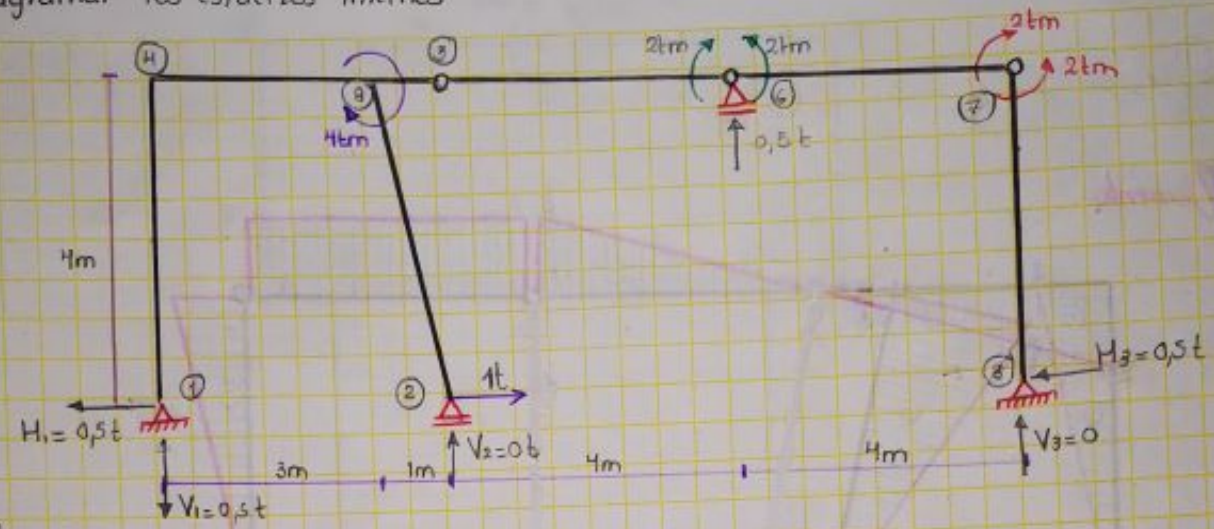


Q
Escala = 2t/cm



N
Escala = 5t/cm

Diagramar los esfuerzos internos



1.- Reacciones

$\sum M_7 = 0$ ↻ ⊕ (abajo)

$-2 + H_3(4) = 0 \quad \therefore H_3 = 0,5t$ //

$\sum F_H = 0 \rightarrow ⊕$

$-H_1 + 1 - 0,5 = 0 \quad \therefore H_1 = 0,5t$ //

$\sum M_6 = 0$ ↻ ⊕ (Derecha)

$-2 + 0,5(4) - V_3(4) = 0 \quad \therefore V_3 = 0$

$\sum M_5 = 0$ ↻ ⊕ (Derecha)

$0,5(4) - V_6(4) = 0 \quad \therefore V_6 = 0,5t$ //

$\sum M_8 = 0$ ↻ ⊕ (Izquierda)

$4 + 0,5(4) - 1(4) + V_1(4) = 0 \quad \therefore V_1 = 0,5t$

$\sum F_V = 0$ ↑ ⊕

$-0,5 + V_2 + 0,5 = 0 \quad \therefore V_2 = 0$

2.- Momentos

* Barra 1-4 $M_1 = 0$
  $M_4 = 0,5(4) = 2tm$
* Barra 4-8 $M_4 = 2tm$
  $M_8 = 0,5(4) - 0,5(3) = 0,5tm$
* Barra 2-8 $M_2 = 0$
  $M_8 = -1(4) = -4tm$
* Barra 3-7 $M_3 = 0tm$
  $M_7 = -0,5(4) = -2tm$ //
* Barra 6-7 $M_6 = -0,5(4) = -2tm$
  $M_7 = -0,5(4) = -2tm$ //
* Barra 5-6 $M_5 = +0,5(4) - 0,5(4) = 0tm$ //
  $M_6 = 2tm$ //
* Barra 5-8 $M_5 = 0tm$ //
  $M_8 = +0,5(5) - 0,5(4) = +0,5tm$ //

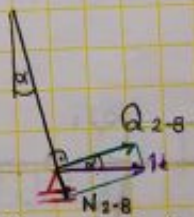
3- Cortantes

Barra 1-4 $Q_{1-4} = 0,5t$

Barra 4-8 $Q_{4-8} = -0,5t$

Barra 2-8

$\alpha = \text{arctg}\left(\frac{1}{4}\right) = 14,036$



$Q_{2-8} = -1\ Cos(14,036) = -0,97t$ //

Barra 3-7 $Q_{3-7} = 0,5t$

Barra 6-7 $Q_{6-7} = 0t$

Barra 5-6 $Q_{5-6} = -0,5t$

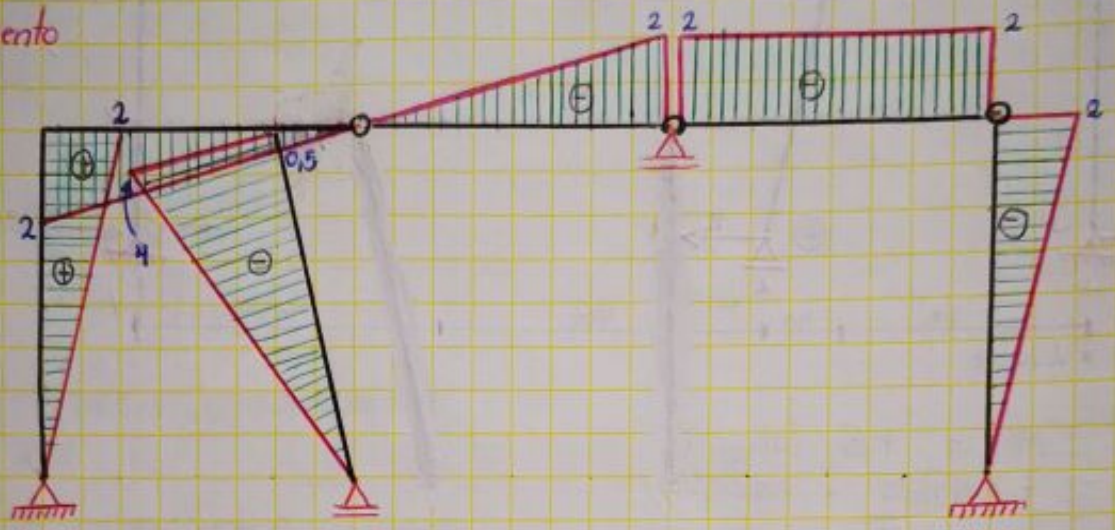
Barra 8-5 $Q_{8-5} = -0,5t$

4.- Normales

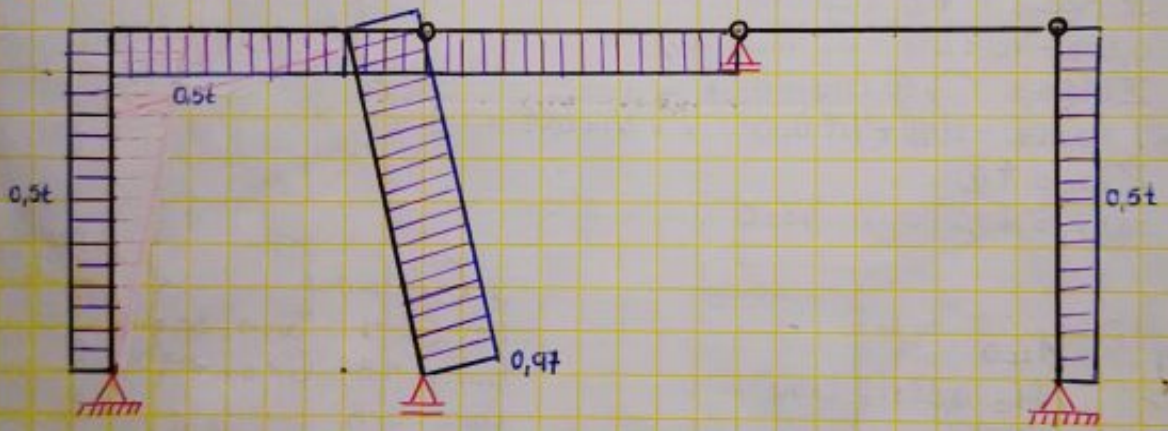
* Barra 1-4 $N_{1-4} = 0,5t$
* Barra 4-8 $N_{4-8} = 0,5t$
* Barra 2-8 $N_{2-8} = -1 \cdot Sen(14,036) = -0,243t$ //
* Barra 3-7 $N_{3-7} = 0t$
* Barra 6-7 $N_{6-7} = -0,5t$
* Barra 5-6 $N_{5-6} = -0,5t$
* Barra 5-8 $N_{5-8} = -0,5t$

5 - Diagramas

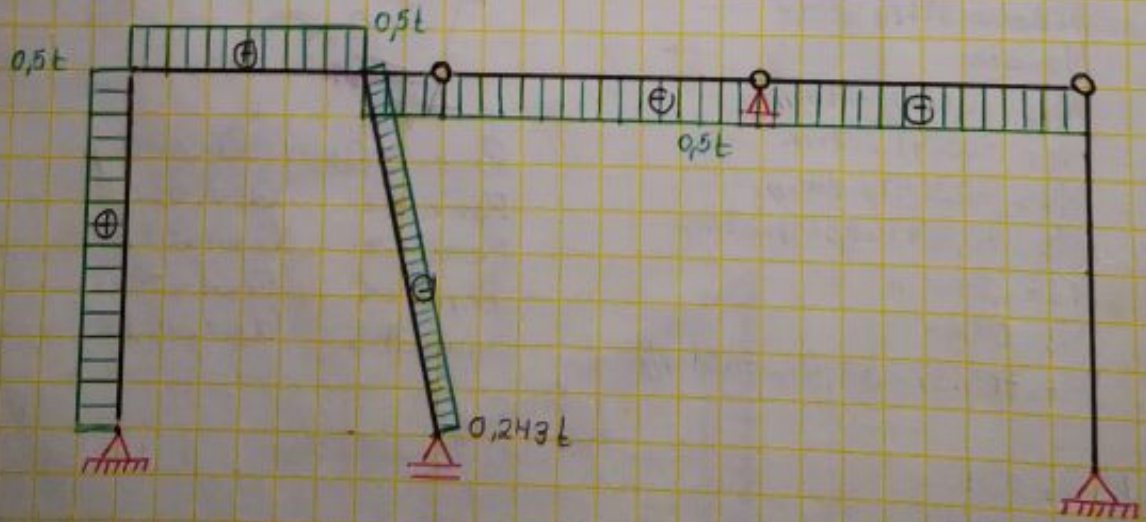
Momento



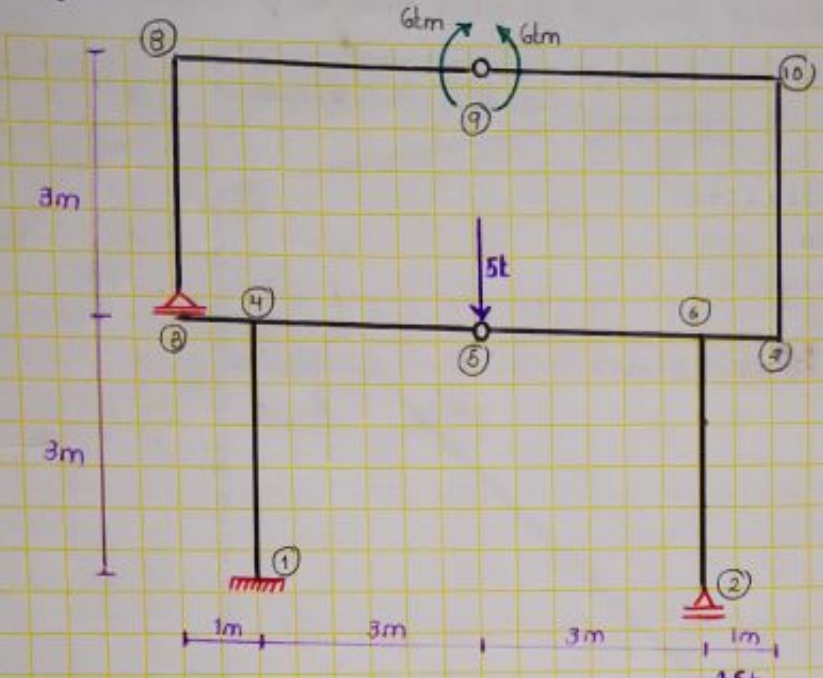
Cortante



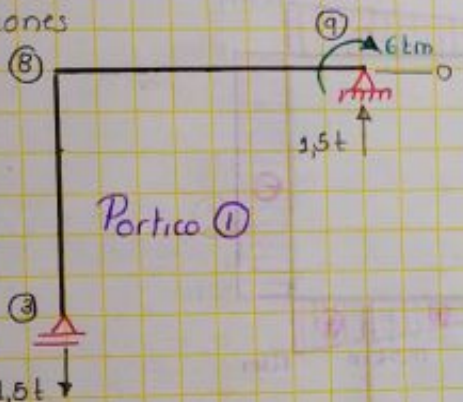
Normal



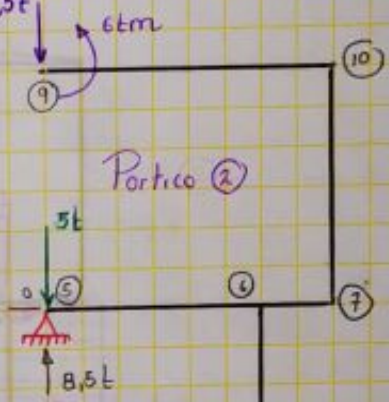
Obtener el diagrama de Momento



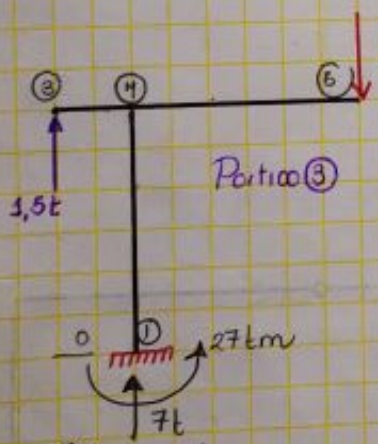
1- Reacciones



$\sum M_9 = 0 \curvearrowright +$
$6 - V_3(4) = 0$
$V_3 = 1,5t$
$\sum F_V = 0 \uparrow +$
$V_9 = 1,5t$



$\sum M_8 = 0 \curvearrowright +$
$-6 + V_2(3) = 0$
$V_2 = 2t$
$\sum F_V = 0 \uparrow +$
$V_5 - 5 - 1,5 - 2 = 0$
$V_5 = 8,5t$



$\sum F_V = 0 \curvearrowright +$
$V_1 + 1,5 - 8,5 = 0 \quad \therefore V_1 = 7t //$
$\sum F_H = 0 \rightarrow +$
$H_1 = 0t$
$\sum M_{①} = 0 \curvearrowright +$
$-M_1 + 8,5(3) + 1,5(1) = 0$
$M_1 = 27tm //$

2 Momentos

- Portico ①
  - * Barra 3-8: $M_3 = 0$, $M_8 = 0$
  - * Barra 8-9: $M_8 = 0$, $M_9 = -1,5(4) = -6tm$
- Portico 2
  - * Barra 9-10: $M_9 = -6tm$, $M_{10} = -6 - 1,5(4) = -12tm$
  - * Barra 10-7: $M_{10} = -12tm$, $M_7 = -12tm$
  - * Barra 5-6: $M_5 = 0tm$, $M_6 = (8,5-5)(3) = 10,5tm$
  - * Barra 2-6: $M_2 = M_6 = 0$
  - * Barra 6-7: $M_6 = (8,5-5)(3) = 10,5tm$, $M_7 = (8,5-5)(4) - 2(1) = 12tm$

- Pórtico ③

* Barra 3-4 $M_3 = 0$

$M_4 = 1,5(1) = 1,5\,tm$

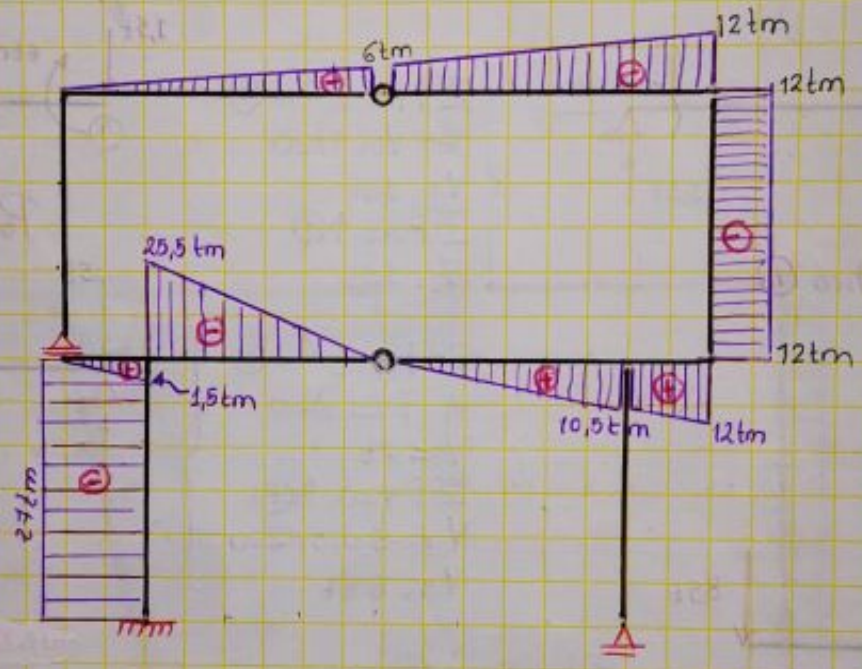
* Barra 1-4 $M_1 = -27\,tm$

$M_4 = -27\,tm$

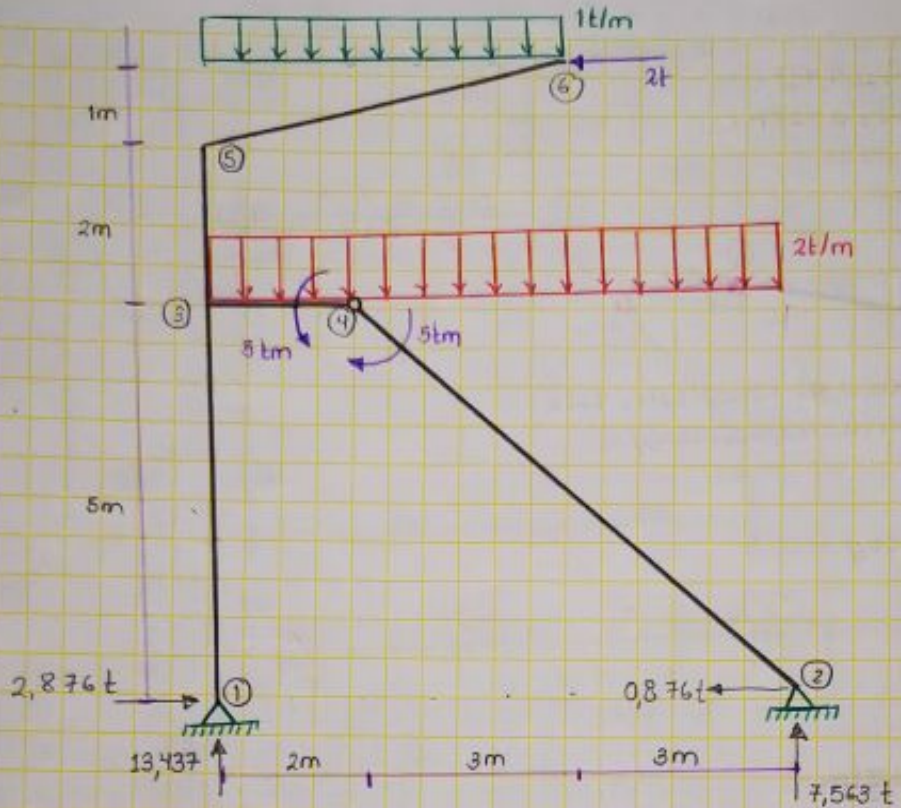
* Barra 4-5 $M_5 = 0\,tm$

$M_4 = -8,5(3) = -25,5\,tm$

3. Diagrama



Diagramar Momento y cortante para la siguiente estructura



1-Cálculo de Reacciones

$\sum M_{①} = 0 \curvearrowright +$

$2(8)(4) + 1(5)(2,5) - 2\cdot(8) - V_2(8) = 0 \quad \therefore V_2 = 7,563\ t$

$\sum F_V = 0 \uparrow +$

$V_1 - 1(5) - 2(8) + 7,563 = 0 \quad \therefore V_1 = 13,437\ t$

$\sum M_{④} = 0 \curvearrowright + \text{ (Derecho)}$

$5 + 2(6)(3) - 7,563(6) + H_2(5) = 0 \quad \therefore H_2 = 0,876\ t$

$\sum F_H = 0 \rightarrow +$

$H_1 - 2 - 0,876 = 0 \quad \therefore H_1 = 2,876\ t$

2 · Momento

Barra 1-3 $M_1 = 0$

$M_3 = -2,876(5) = -14,38\ tm$

Barra 2-4 $M_2 = 0$

$M_4 = -0,876(5) + 7,563(6) - 2(6)(3) = 5\ tm //$

Barra 3-4

$M_3 = -2,876(5) - 2(3) + 1(5)(2,5) = -7,88$

$M_4 = 5\ tm$

Barra 5-6 $M_6 = 0$

$M_5 = 2(1) - 1(5)(2,5) = -10,5\ tm$

Barra 3-5 $M_5 = -10,5\ tm$

$M_3 = 2(3) - 1(5)(2,5) = -6,5\ tm$

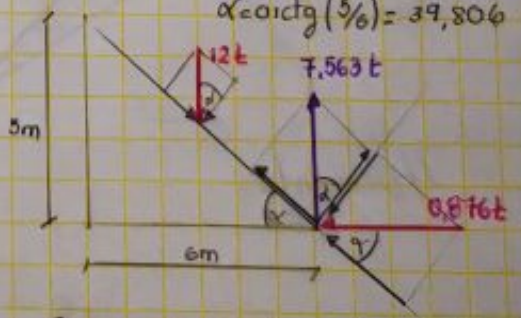
3.-Cortantes

Barra 1-3

$Q_{1-3} = -2,876\ t$

Barra 2-4

$\alpha = \text{arctg}(5/6) = 39,806$



$Q_2 = 0,876\ \text{Sen}(39,806) - 7,563\ \text{Cos}(39,806)$

$Q_2 = -5,249\ t //$

$Q_4 = -5,249 + 12\cdot\text{Cos}(39,806)$

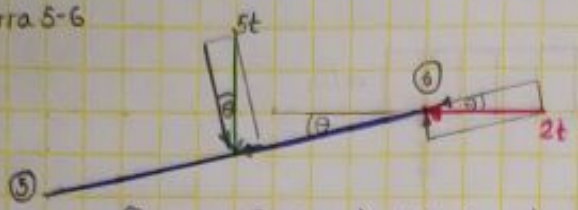
$Q_4 = 3,9694\ t //$

Barra 3-4

$Q_4 = -7{,}563 + 2(6) = 4{,}437\ t$

$Q_3 = 4{,}437 + 2(2) = 8{,}437\ t$

Barra 5-6



$\theta = \text{arctg}\left(\frac{1}{5}\right) = 11{,}31°$

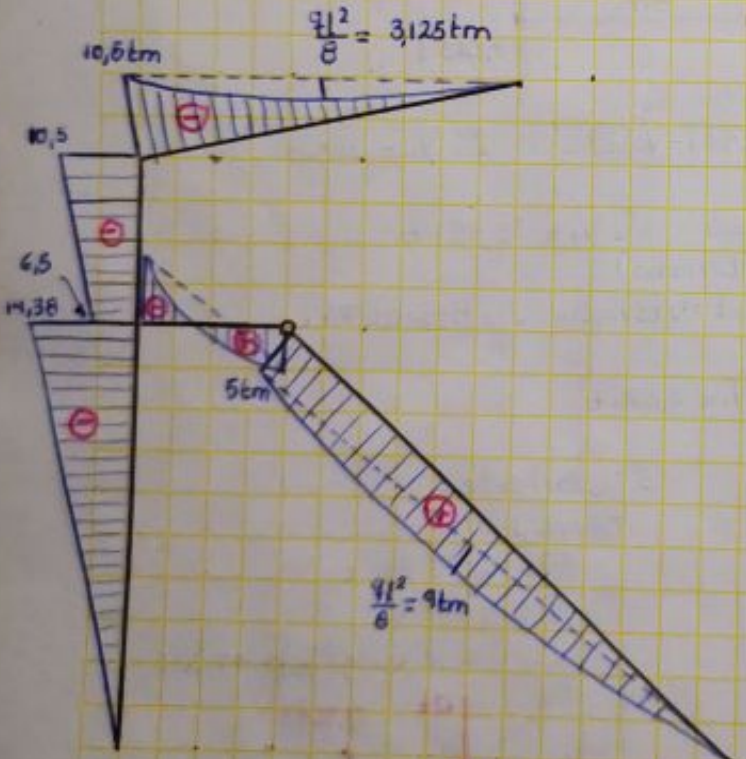
$Q_5 = -2\,\text{Sen}(11{,}31) + 5\,\text{Cos}(11{,}31) = 4{,}51\ t$

$Q_6 = -2\,\text{Sen}(11{,}31) = -0{,}392\ t$

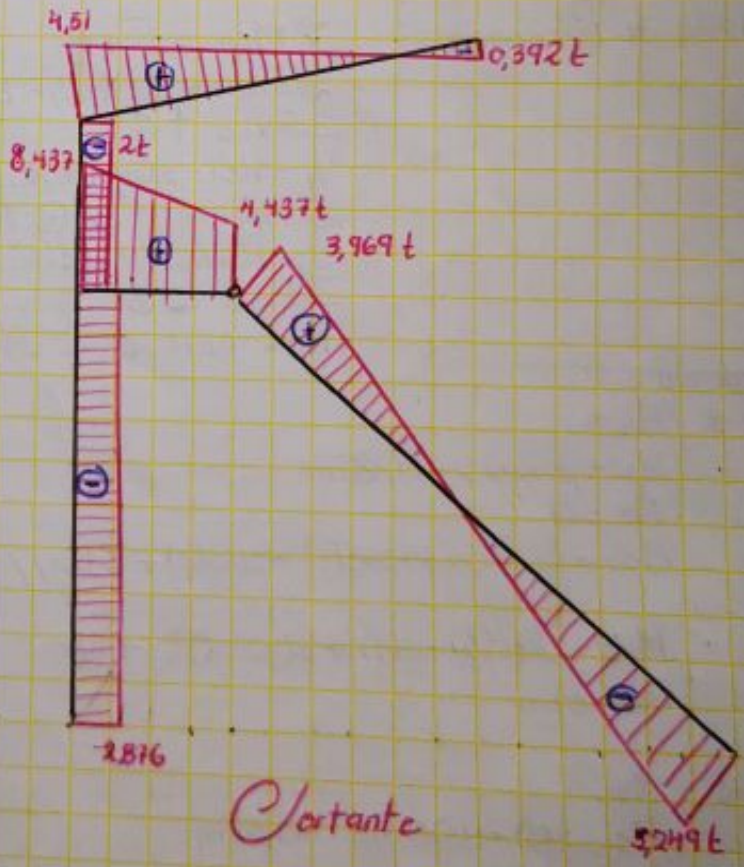
Barra 8-5

$Q_{35} = -2\ t$

4-Diagramas

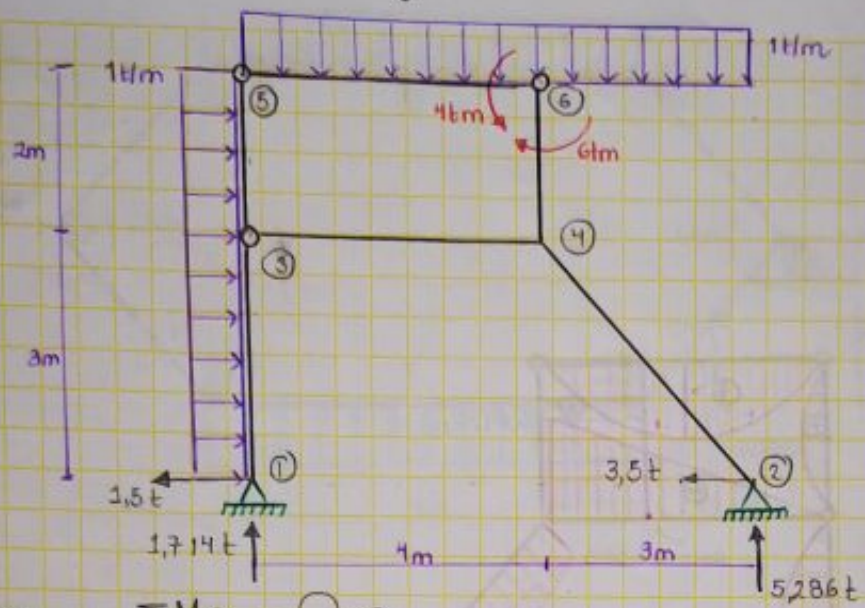


Momento



Cortante

Diagramar Momento en la siguiente estructura



1- Reacciones

$\Sigma M_{①} = 0$ ↻ ⊕

$1(5)(2,5) + 1(7)(3,5) - V_2(7) = 0$ ∴ $V_2 = 5,286t$ //

$\Sigma F_V = 0$ ↑ ⊕

$V_1 - 1(7) + 5,286t = 0$ ∴ $V_1 = 1,714t$ //

$\Sigma M_{③} = 0$ ↻ ⊕

$H_1(3) - 1(3)(1,5) = 0$ ∴ $H_1 = 1,5t$ //

$\Sigma F_H = 0$ → ⊕

$1(5) - 1,5 - H_2 = 0$ ∴ H_2 $3,5t$ //

2- Momentos

Barra 1-3

$M_1 = 0$

$M_3 = 1,5(3) - 1(3)(1,5) = 0$

$\bar{M}_{13} = \frac{ql^2}{8} = \frac{1(3)^2}{8} = 1,125\,tm$

Barra 2-4

$M_2 = 0$

$M_4 = 5,286(3) - 3,5(3) - 1(3)(1,5) = 0,858\,tm$

Barra 4-6

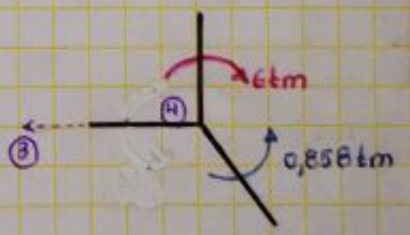
$M_6 = 6\,tm$

$M_4 = 6\,tm$

Barra 3-4

$M_3 = 0$

$M_4 = -6 - 3,5(3) + 5,286(3) - 1(3)(1,5) = -5,142$



$M_4 = -6 + 0,858 = -5,142\,tm$

Barra 3-5

$\bar{M}_{35} = \frac{ql^2}{8} = \frac{1(2)^2}{8}$

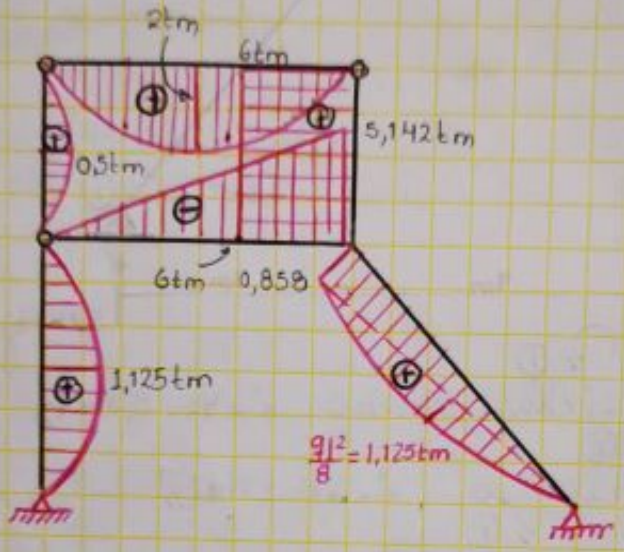
$\bar{M}_{35} = 0,5\,tm$

Barra 5-6

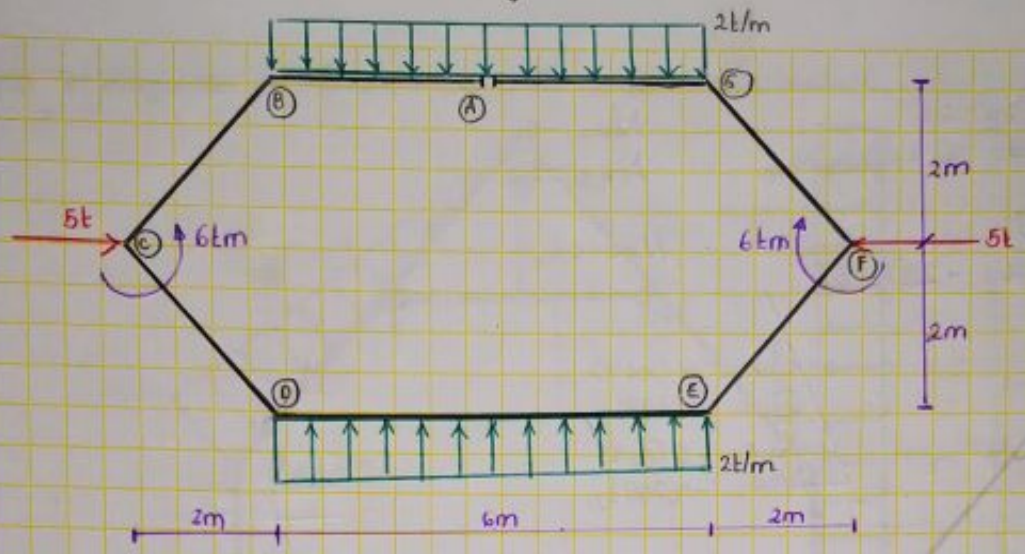
$\bar{M}_{56} = \frac{1(4)^2}{8}$

$\bar{M}_{56} = 2\,tm$

3. Diagrama



Para la estructura autosustentada diagramar los esfuerzos internos



1.- Momento

Barra AB $M_A = 0$ tm

$M_B = -2(3)(1,5) = -9$ tm

Barra BC $M_B = -2(3)(1,5) = -9$ tm

$M_C = -21$ tm

Barra CD $M_C = -6 + 2(3)(3,5) = 15$ tm

$M_D = 2(3)(1,5) + 5(2) - 6 = 13$ tm

Barra DE $M_D = 13$ tm

$M_E = 2(3)(1,5) + 5(2) - 6 = 13$ tm

Barra AG

$M_A = 0$

$M_G = -2(3)(1,5) = -9$ tm

Barra GF $M_G = -9$ tm

$M_F = -2(3)(3,5) = -21$ tm

Barra FE $M_F = 2(3)(3,5) - 6 = 15$ tm

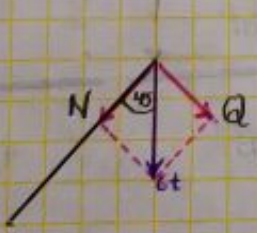
$M_E = 2(3)(1,5) + 5(2) - 6 = 13$ tm

2.- Cortante y Normal

Barra AB $Q_A = 0$ $N_A = 0$

$Q_B = 2(3) = 6$ tm $N_B = 0$

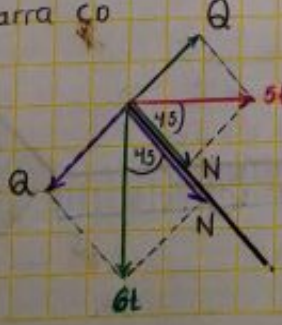
Barra BC



$Q_{BC} = 6 \cdot \text{Sen}45 = 4,243$ t

$N_{BC} = -6 \cdot \text{Cos}45 = -4,243$ t

Barra CD



$Q_{CD} = 5\,\text{Sen}45 - 6\,\text{Sen}45 =$

$Q_{CD} = -0,707$ t //

$N_{CD} = -5\,\text{Cos}45 - 6\,\text{Cos}45$

$N_{CD} = -7,778$ t //

Barra DE

$Q_D = -6\,t$

$Q_E = -6 + 2(6) = 6\,t$ //

$N_D = -5\,t$

$N_E = -5\,t$

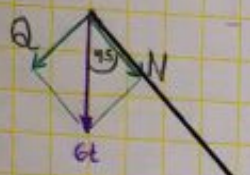
Barra AG

$Q_A = 0\,t$

$Q_G = -2(3) = -6\,t$

$N_{AG} = 0\,t$

Barra GF



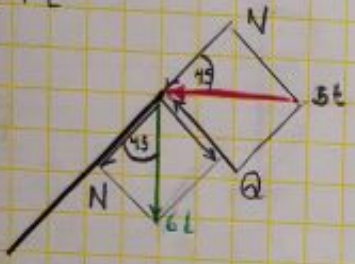
$Q_{GF} = -6 \cdot \mathrm{Sen}45$

$Q_{GF} = -4{,}243\,t$ //

$N_{GF} = -6\,\mathrm{Cos}45$

$N_{GF} = -4{,}243\,t$ //

Barra FE



$Q_{FE} = -5\,\mathrm{Sen}45 + 6 \cdot \mathrm{Sen}45$

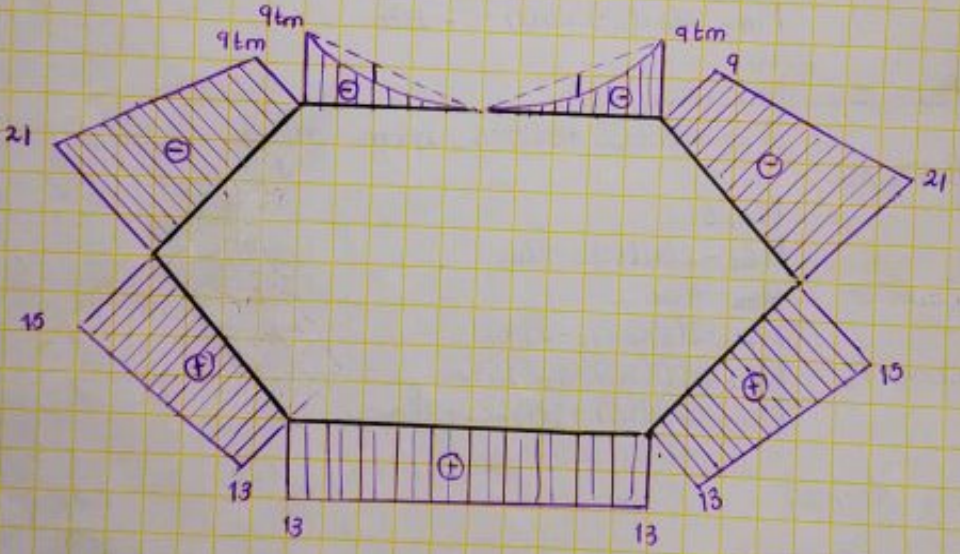
$Q_{FE} = 0{,}707\,t$ //

$N_{FE} = -5\,\mathrm{Cos}45 - 6\,\mathrm{Cos}45$

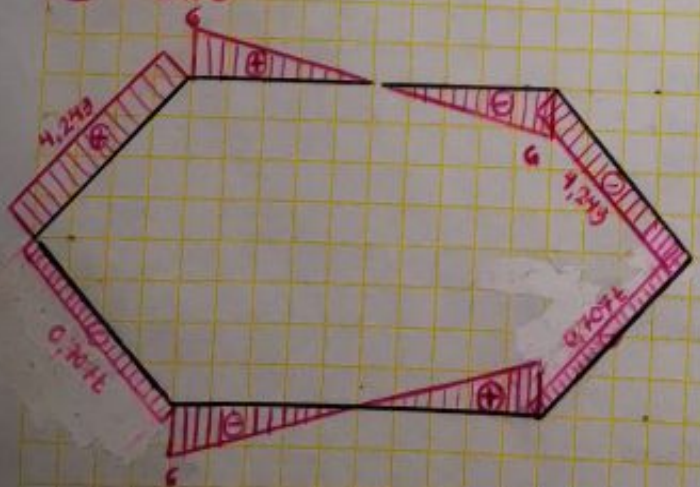
$N_{FE} = -7{,}778\,t$ //

3. Diagramas

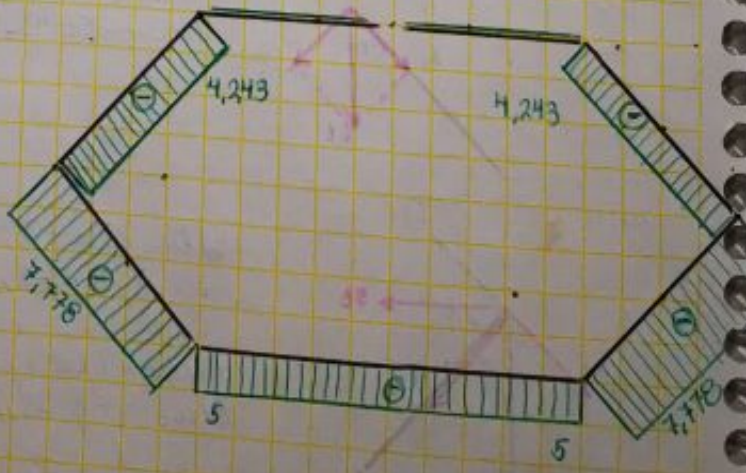
Momento



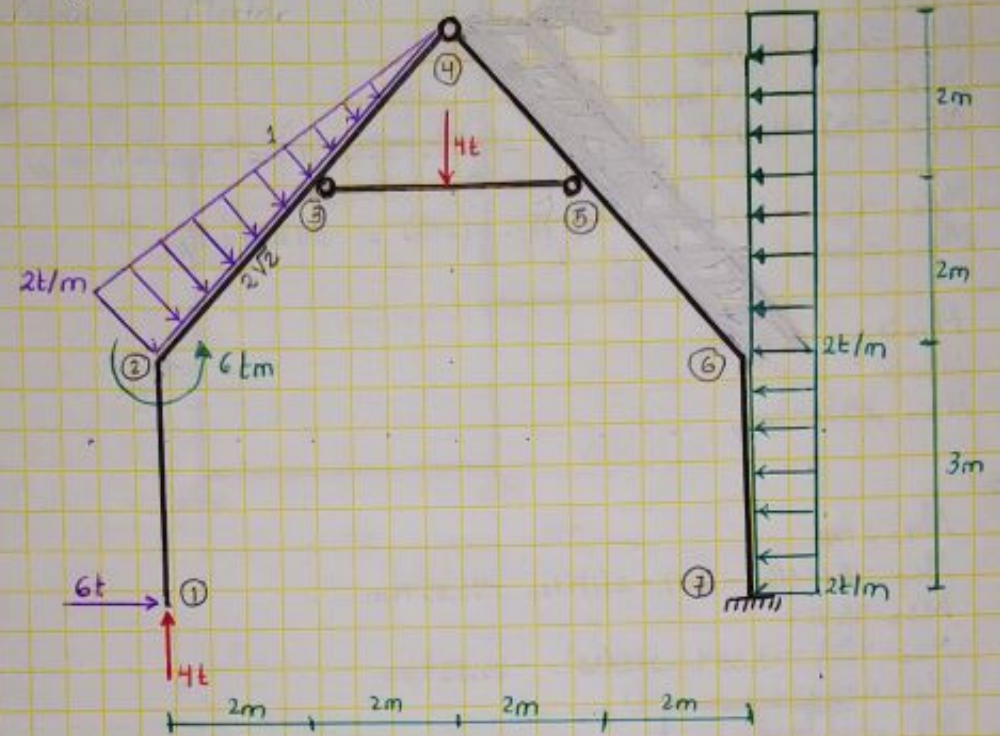
Cortante



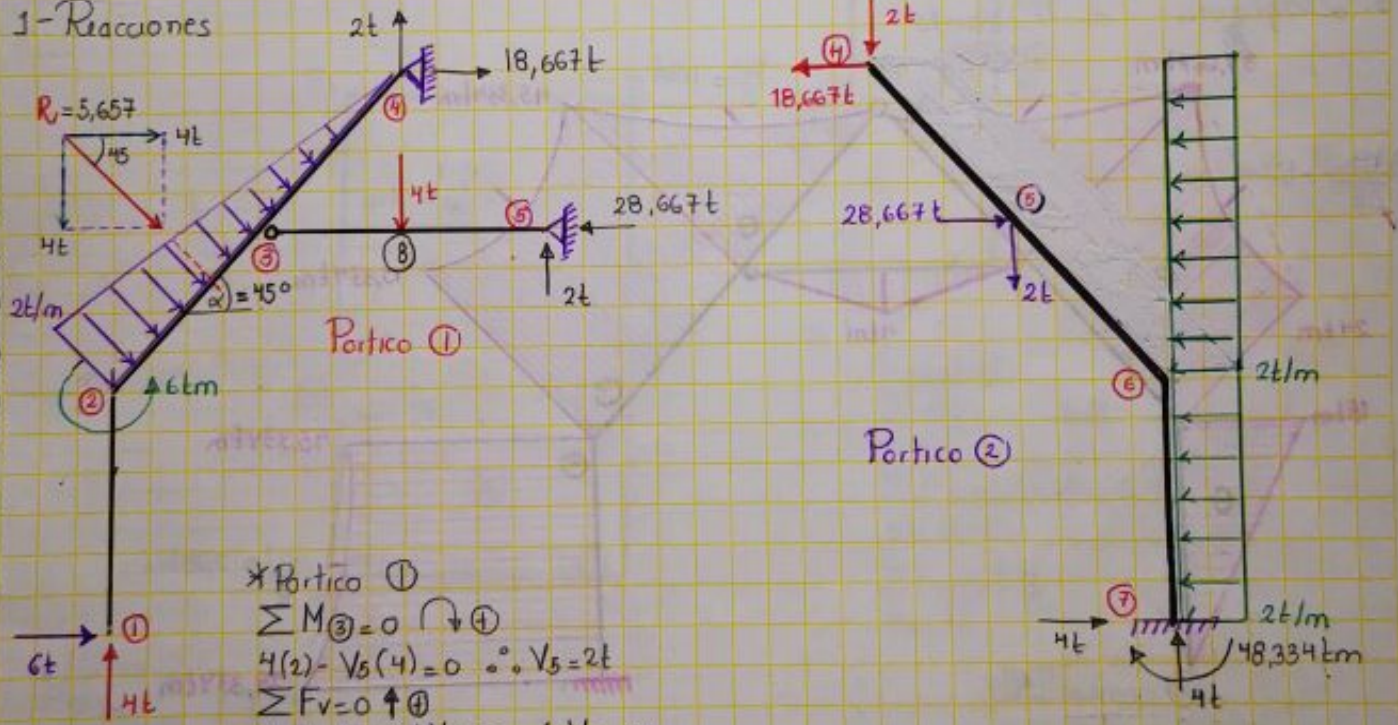
Normal



Obtener el diagrama de momento para la siguiente estructura



1- Reacciones



*Portico ①

$\sum M_{③}=0 \curvearrowright +$

$4(2) - V_5(4) = 0 \quad \therefore V_5 = 2t$

$\sum F_V = 0 \uparrow +$

$4 - 4 - 4 + 2 + V_4 = 0 \quad \therefore V_4 = 2t$

$\sum M_{④}=0 \curvearrowright +$

$-2(2) - 5{,}657 \cdot \left[\frac{2}{3}(4\sqrt{2})\right] - 6 - 6(7) + 4(4) + H_5(2) = 0$

$H_5 = 28{,}667t$

$\sum F_H = 0 \rightarrow +$

$+6 + 4 - 28{,}667 + H_4 = 0$

$H_4 = 18{,}667t$ //

*Portico ②

$\sum F_V = 0 \uparrow +$

$-2 - 2 + V_7 = 0 \quad \therefore V_7 = 4t$

$\sum F_H = 0 \rightarrow +$

$-18{,}667 + 28{,}667 - 2(7) + H_7 = 0 \quad \therefore H_7 = 4t$

$\sum M_{⑦}=0 \curvearrowright +$

$-18{,}667(7) - 2(4) + 28{,}667(5) - 2(2) - 2(7)(3{,}5) + M_7 =$

$M_7 = 48{,}334tm$ //

2.- Momentos

Barra 1-2 $M_1 = 0$

$M_2 = -6(3) = -18\,tm$ //

Barra 2-3 $M_2 = -6(3) - 6 = -24\,tm$ //

$M_3 = -6(5) + 4(2) - 6 - \frac{1(2\sqrt{2})}{2}^2 - \frac{1(2\sqrt{2})}{2} \cdot \left[\frac{2}{3}(2\sqrt{2})\right] = -34{,}667\,tm$ //

Barra 3-4 $M_3 = -18{,}667(2) + 2(2) - \frac{1(2\sqrt{2})}{2} \cdot \frac{1}{3}(2\sqrt{2}) = -34{,}667\,tm$ //

$M_4 = 0\,tm$

Barra 3-5 $M_3 = 0$

$M_5 = 0$

$M_8 = 2(2) = 4\,tm$ //

Pórtico 2

Barra 4-5

$M_4 = 0\,tm$

$M_5 = -18{,}667(2) - 2(2) - 2(2)(1) = -45{,}334\,tm$

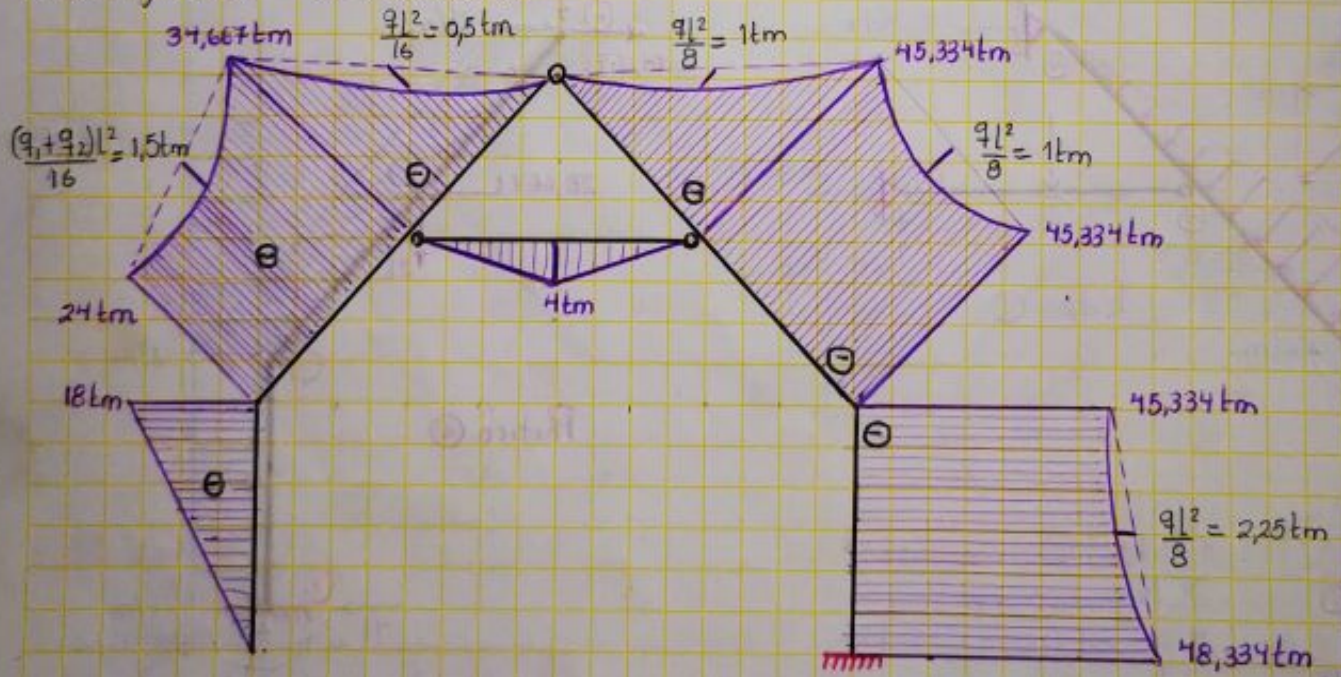
Barra 5-6 $M_5 = -45{,}334$

$M_6 = 4(3) - 48{,}334 - 2(3)(1{,}5) = -45{,}334\,tm$

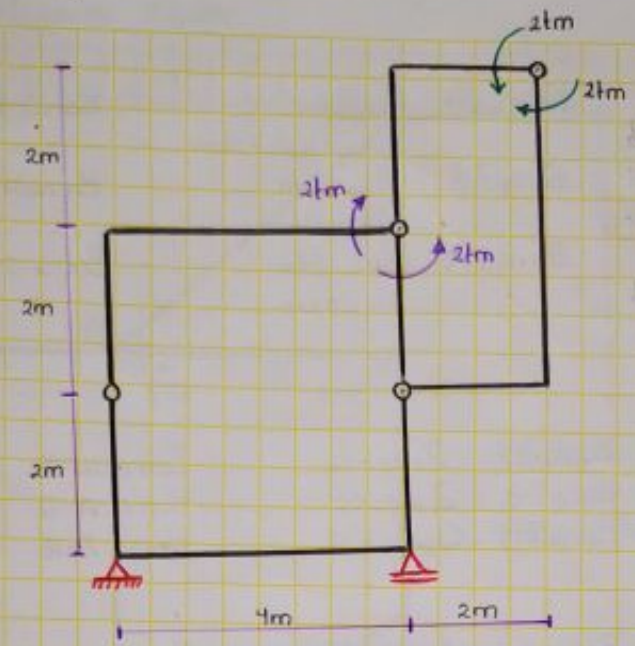
Barra 6-7 $M_6 = -45{,}334\,tm$

$M_7 = -48{,}334\,tm$

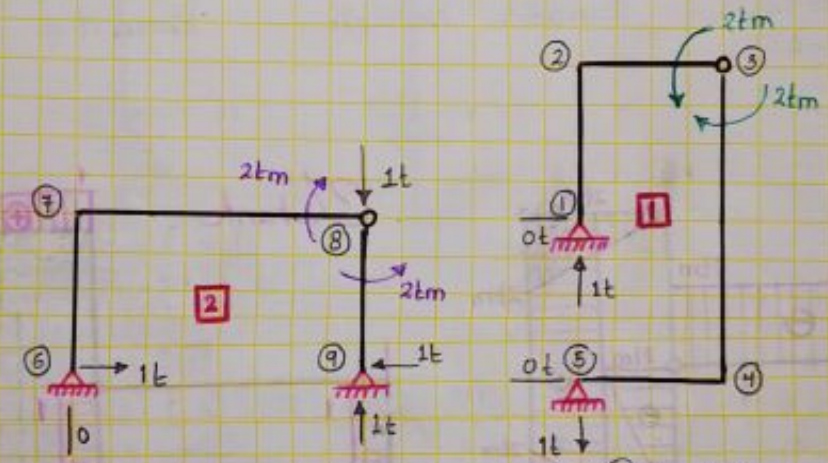
3.- Diagrama de Momento

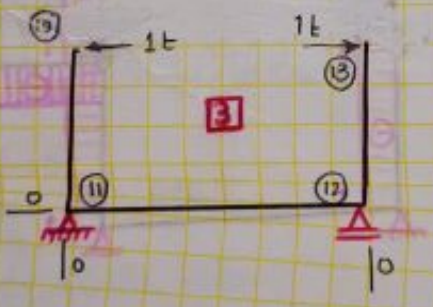


Diagramar los esfuerzos internos



1. Reacaones





Portico ①

$\sum M_{⑤}=0$ ↷ ⊕ $\Rightarrow H_1=0$

$\sum F_H=0$ → ⊕ $\Rightarrow H_5=0t$

$\sum M_{③}=0$ (Izq) ↷ ⊕

$V_1(2)-2=0 \therefore V_1=1t$ //

$\sum F_V=0$ ↑ ⊕

$V_5=1t$

* Portico 2

$\sum M_{⑥}=0$ ↷ ⊕

$1(4)-V_9(4)=0 \therefore V_9=1t$

$\sum F_V=0$ ↑ ⊕

$V_6=0t$

$\sum M_{⑧}=0$ ↷ ⊕ (abajo)

$-2+H_9(2)=0 \therefore H_9=1t$

$\sum F_H=0$ → ⊕

$H_6=1t$ //

2.- Momentos

Barra 1-2	$M_1 = 0$ tm	Barra 6-7	$M_6 = 0$ tm	Barra 10-11	$M_{10} = 0$ tm
	$M_2 = 0$ tm		$M_7 = -1(2) = -2$ tm		$M_{11} = -1(2) = -2$ tm
Barra 2-3	$M_2 = 0$ tm	Barra 7-8	$M_7 = -2$ tm	Barra 11-12	$M_{11} = -2$ tm
	$M_3 = 1(2) = 2$ tm		$M_8 = -2$ tm		$M_{12} = -2$ tm
Barra 3-4	$M_3 = 2$ tm	Barra 8-9	$M_8 = -2$ tm	Barra 12-13	$M_{12} = -2$ tm
	$M_4 = 1(2) = 2$ tm		$M_9 = 0$ tm		$M_{13} = 0$ tm
Barra 4-5	$M_4 = -1(2) = -2$ tm				
	$M_5 = 0$ tm				

3.- Cortantes

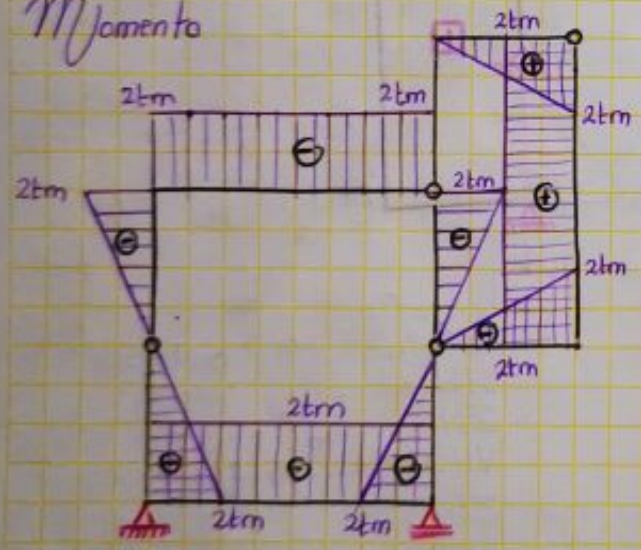
Barra 1-2	$Q_{1-2} = 0$ t	Barra 6-7	$Q_{6-7} = -1$ t	Barra 10-11	$Q_{10-11} = -1$ t
Barra 2-3	$Q_{2-3} = 1$ t	Barra 7-8	$Q_{7-8} = 0$ t	Barra 11-12	$Q_{11-12} = 0$ t
Barra 3-4	$Q_{3-4} = 0$ t	Barra 8-9	$Q_{8-9} = +1$ t	Barra 12-13	$Q_{12-13} = 1$ t //
Barra 5-4	$Q_{5-4} = -1$ t				

4.- Normales

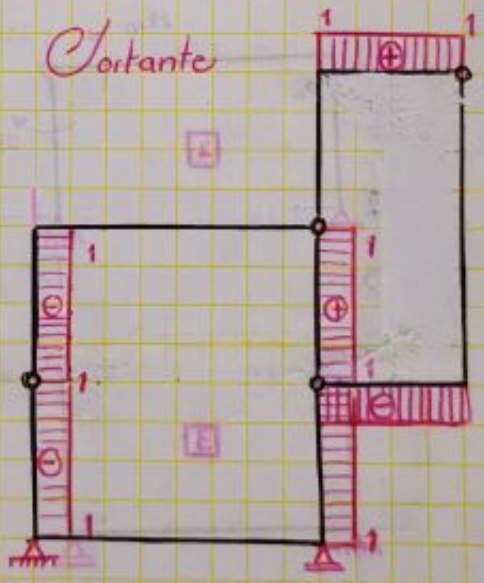
Barra 1-2	$N_{1-2} = -1$ t	Barra 6-7	$N_{6-7} = 0$ t	Barra 10-11	$N_{10-11} = 0$ t
Barra 2-3	$N_{2-3} = 0$ t	Barra 7-8	$N_{7-8} = -1$ t	Barra 11-12	$N_{11-12} = 1$ t
Barra 3-4	$N_{3-4} = 1$ t	Barra 8-9	$N_{8-9} = -1$ t	Barra 12-13	$N_{12-13} = 0$ t
Barra 4-5	$N_{4-5} = 0$ t				

6.- Diagramas

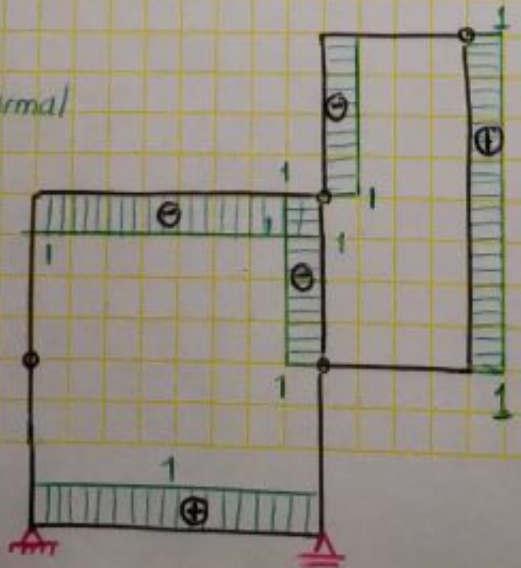
Momento



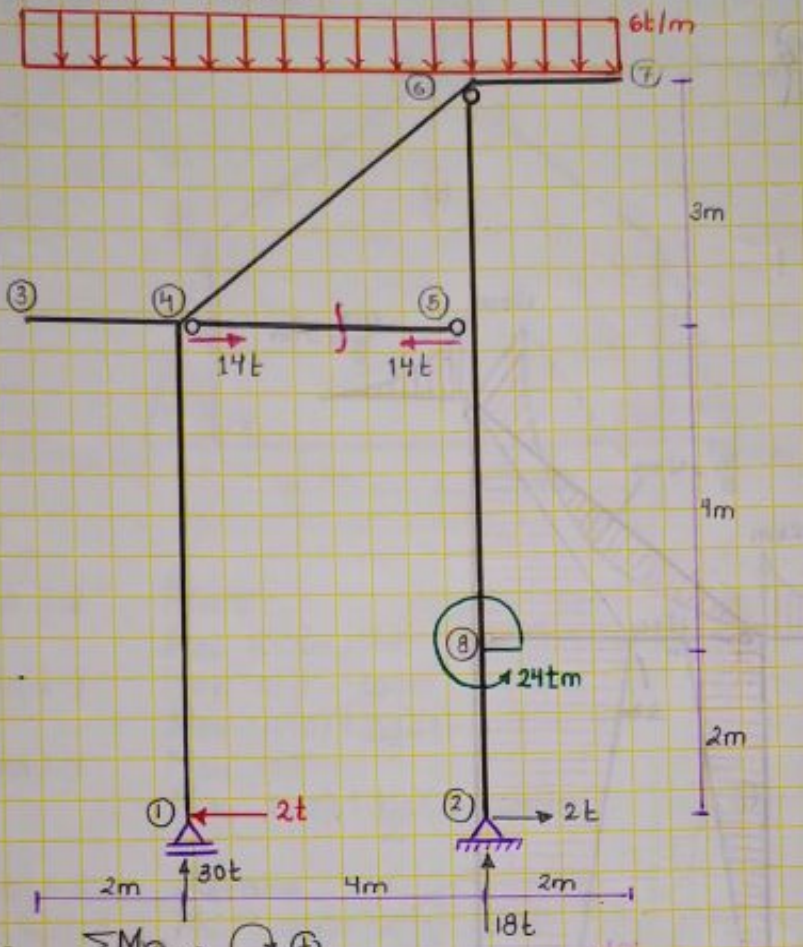
Cortante



Normal



Diagramar Momento Flector



1-Reacciones $\sum M_{①}=0$ ↻ ⊕

$6(8)(2) - 24 - V_2(4)=0$

$V_2 = 18t$ //

$\sum F_V=0$ ↑ ⊕

$V_1 - 6(8) + 18 = 0 \quad \therefore V_1 = 30t$ //

$\sum F_H=0$ → ⊕

$H_2 = 2t$ //

$\sum M_{⑥}=0$ ↻ ⊕ (abajo)

$N_{6-6}(3) - 24 - 2(9)=0 \quad \therefore N_{6-6} = 14t$

2-Momento

Barra 1-4 $M_1=0$

$M_4 = 2(6) = 12tm$

Barra 3-4 $M_3=0$

$M_4 = -6(2)(1) = -12tm$

Barra 4-6 $M_4 = 2(6) - 6(2)(1) = 0tm$

$M_6 = 30(4) + 2(9) - 14(3) - 6(6)(3) = -12tm$

Barra 6-7 $M_6 = -6(2)(1) = -12tm$

$M_7 = 0tm$

Barra 2-8 $M_2 = 0tm$

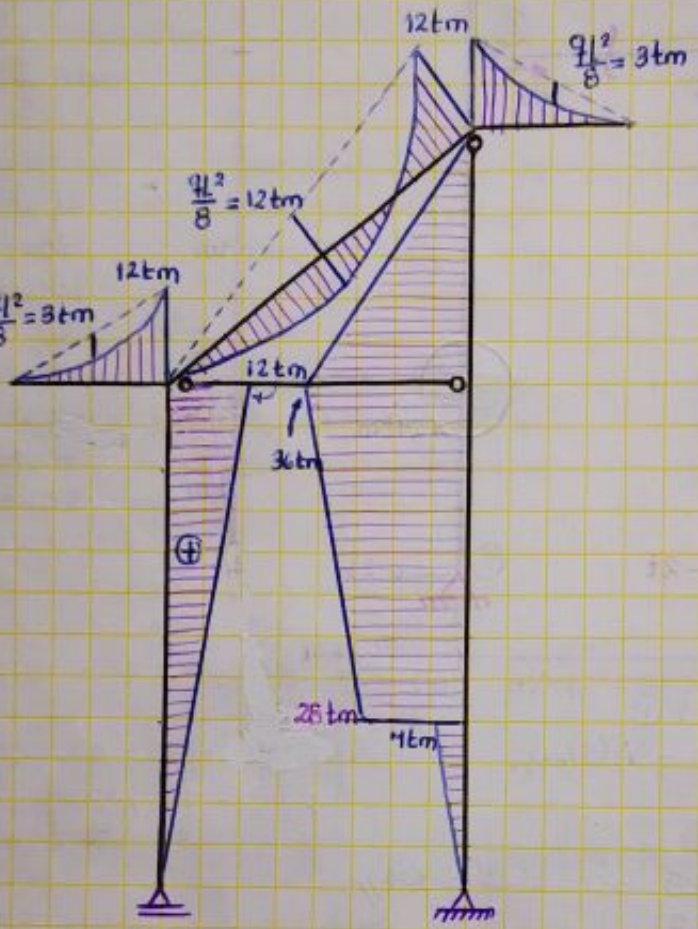
$M_8 = 2(2) = 4tm$

Barra 8-5 $M_8 = 2(2) + 24 = 28tm$

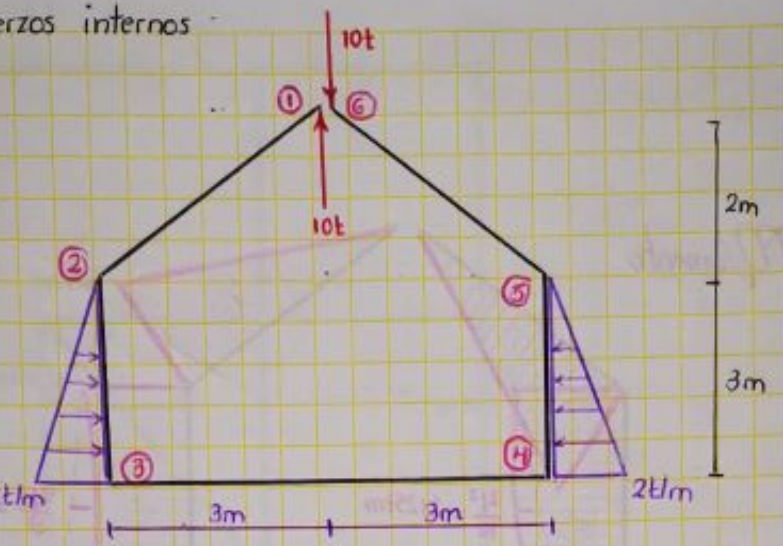
$M_5 = 2(6) + 24 = 36tm$

Barra 5-6 $M_5 = 2(9) + 24 - 14(3) = 0tm$

3 - Diagrama de Momento



Diagramar los esfuerzos internos



-Momentos

Barra 1-2 $M_1 = 0$

$M_2 = 10(3) = 30tm$

Barra 2-3 $M_2 = -10(3) = -30tm$

$M_3 = -10(3) + \frac{2(3)}{2}(1) = -27tm$

Barra 3-4 $M_3 = -27tm$

$M_4 = 10(3) + \frac{2(3)}{2}(1) = 33tm$

Barra 5-6 $M_6 = 0tm$

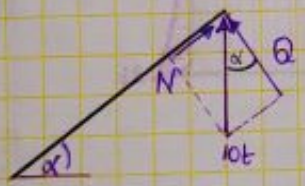
$M_5 = -10(3) = -30tm$

Barra 5-4 $M_5 = 10(3) = 30tm$

$M_4 = 10(3) + \frac{2(3)}{2}(1) = 33tm$

-Cortante y Normal

Barra 1-2



$\alpha = \text{arctg}\left(\frac{2}{3}\right) = 33,69°$

$Q_{1-2} = -10 \cdot \text{Cos}\,33,69 = -8,321t$ //

$N_{1-2} = 10 \cdot \text{Sen}(33,69) = 5,547t$

Barra 2-3

$Q_2 = 0t$

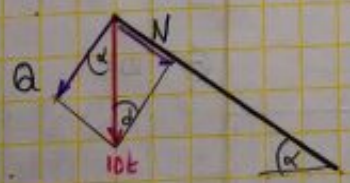
$Q_3 = \frac{2(3)}{2} = 3t$ //

$N_{2-3} = 10t$ //

Barra 3-4 $Q_{3-4} = 10t$ //

$N_{3-4} = -\frac{2(3)}{2} = -3t$ //

Barra 6-5



$Q_{6-5} = -10 \cdot \text{Cos}(33,69) = -8,32t$ //

$N_{6-5} = -10\,\text{Sen}(33,69) = -5,55t$ //

Barra 5-4

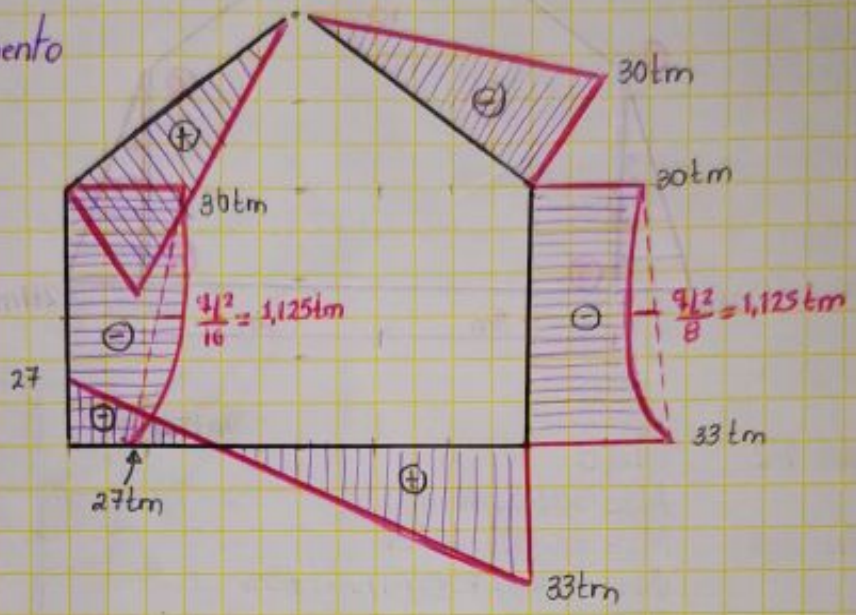
$Q_5 = 0t$

$Q_4 = -\frac{2(3)}{2} = -3t$ //

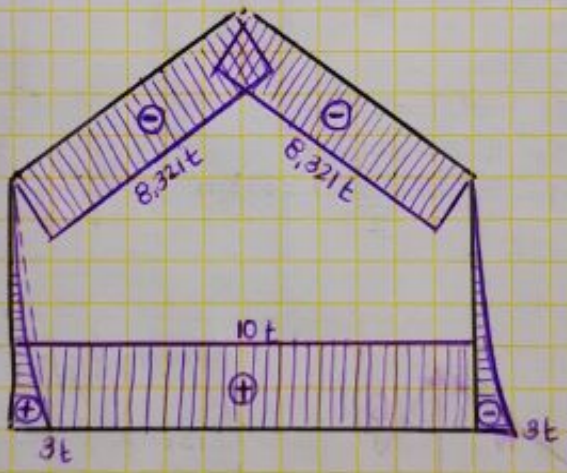
$N_{5-4} = -10t$

3. Diagramas

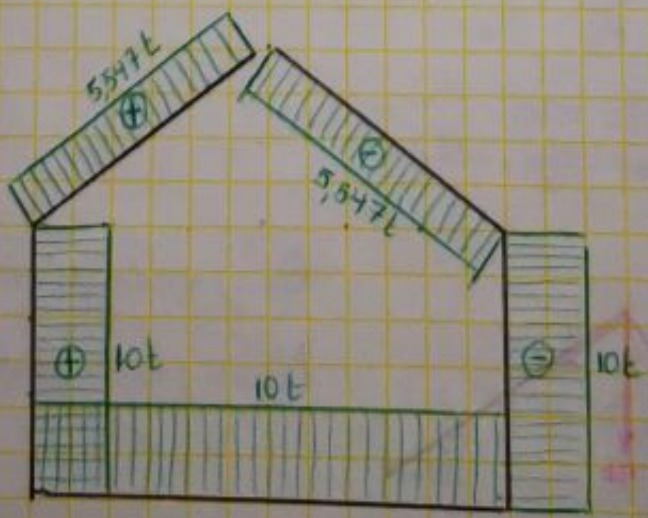
Momento



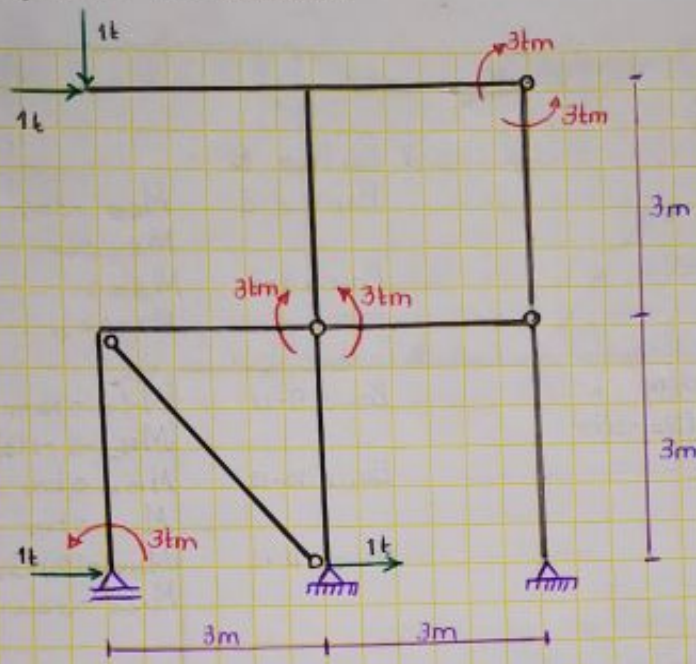
Cortante



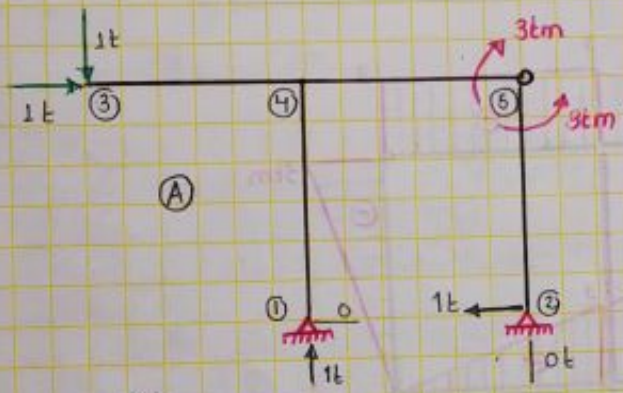
Normal

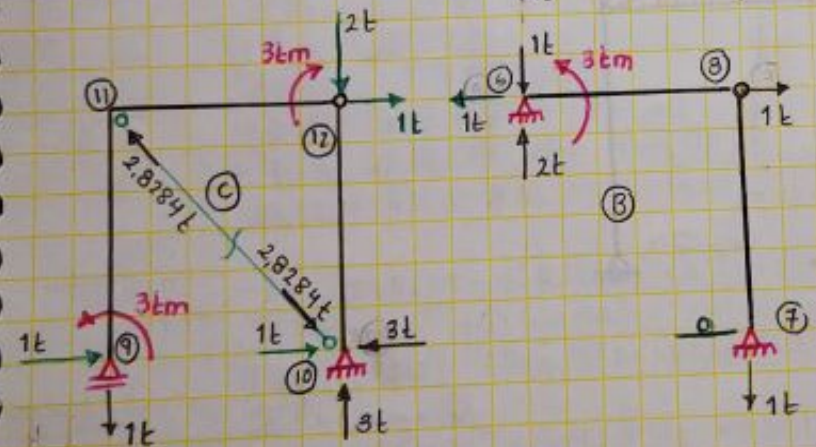


Obtener el diagrama de Momento



1. Reacciones





* Portico A

$\Sigma M_{①}=0$ ↷ (+)

$1(3)-1(3)+V_2(3)=0 \quad \therefore V_2=0t$

$\Sigma F_V=0$ ↑ (+)

$V_1=1t$

$\Sigma M_{②}=0$ ↷ (+) (abajo)

$-3+H_2(3)=0 \quad \therefore H_2=1t$

$\Sigma F_H=0$ → (+) $\quad \therefore H_1=0$

* Portico B

$\Sigma M_{⑧}=0$ ↷ (+) (abajo)

$H_7=0t$

$\Sigma F_H=0$ → (+)

$H_6=1t$

$\Sigma M_{⑥}=0$ ↷ (+)

$-3+V_7(3)=0 \quad \therefore V_7=1t$

$\Sigma F_V=0$ ↑ (+)

$V_6=2t$

* Portico C

$\Sigma M_9=0$ ↷ (+)

$-3+3+2(3)+1(3)-V_{10}(3)=0$

$V_{10}=3t$ //

$\Sigma F_V=0$ ↑ (+)

$-V_9-2+3=0 \quad \therefore V_9=1t$

$\Sigma F_H=0$ → (+)

$H_{10}=3t$

$\Sigma M_{⑫}=0$ ↷ (+) (abajo)

$2(3)-N_{10-11}\cdot(1,5\sqrt{2})=0$

$N_{10-11}=2,8284\,t$ //

3.- Momentos

* Portico Ⓐ

Barra 1-4 $M_{1-4} = 0\,tm$

Barra 2-5 $M_2 = 0\,tm$

$M_5 = -1(3) = -3\,tm$

Barra 3-4 $M_3 = 0\,tm$

$M_4 = -1(3) = -3\,tm$

Barra 4-5 $M_4 = -1(3) = -3\,tm$

$M_5 = -1(6) + 1(3) = -3\,tm$

* Portico B

Barra 6-8 $M_6 = -3\,tm$

$M_8 = 0\,tm$

Barra 8-7 $M_8 = 0$

$M_7 = 0\,tm$

* Portico C

Barra 9-11 $M_9 = -3\,tm$

$M_{11} = -3 - 1(3) = -6\,tm$

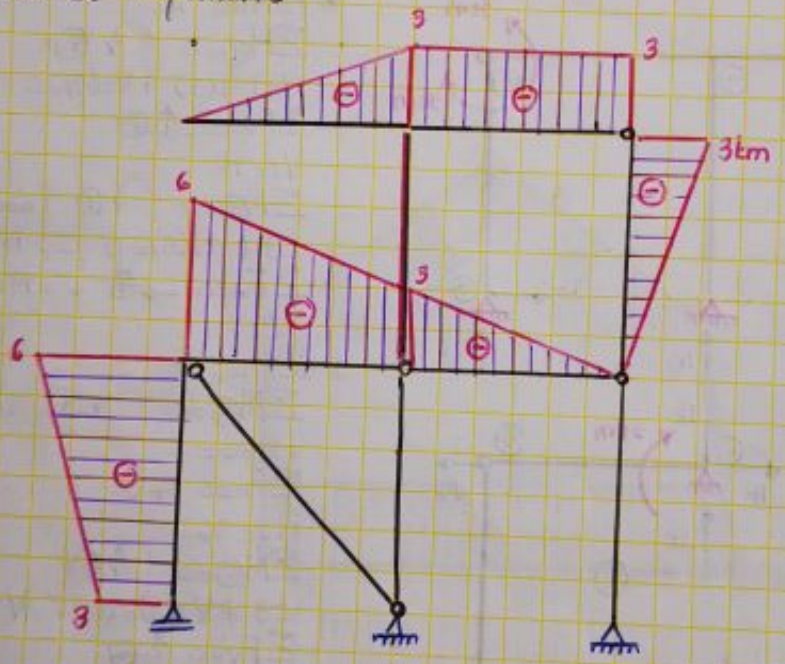
Barra 10-12 $M_{10} = 0\,tm$

$M_{12} = 0\,tm$

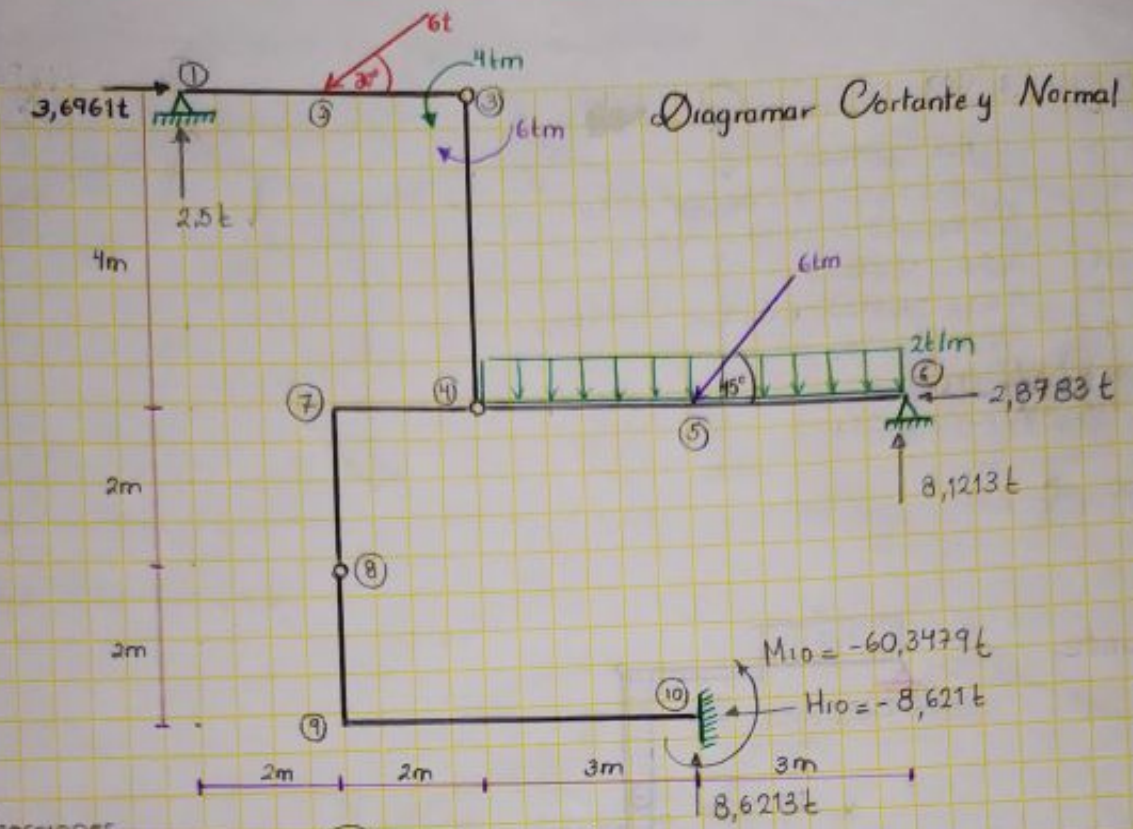
Barra 11-12 $M_{11} = -1(3) - 3 = -6\,tm$

$M_{12} = -3\,tm$

4- Diagrama de Momento



Diagramar Cortante y Normal



1 Reacciones

$\sum M_{③} = 0$ ↷ ⊕ (Barra 1-3)

$+V_1(4) - 4 - 6 \cdot Sen30 \cdot 2 = 0 \quad \therefore V_1 = 2,5t$ //

$\sum M_{④} = 0$ ↷ ⊕ (Barra 4-6)

$6\,Sen45(3) + 2(6)(3) - V_6(6) = 0 \quad \therefore V_6 = 8,1213t$ //

$\sum F_V = 0$ ↑⊕

$2,5 - 6\,Sen30 - 2(6) - 6\,Sen45 + 8,1213 + V_{10} = 0 \quad \therefore V_{10} = 8,6213t$ //

$\sum M_8 = 0$ ↷ ⊕ (abajo)

$-M_{10} - 8,6213(5) + H_{10}(2) = 0$ ①

$\sum M_1 = 0$ ↷ ⊕ (izq)

$-8,6213(3) + H_{10}(4) - M_{10} = 0$

$-M_{10} = 25,8639 - 4H_{10}$ (en ①)

$25,8639 - 4H_{10} - 8,6213(5) + 2H_{10} = 0 \quad \therefore H_{10} = -8,621t$ //

$-M_{10} = 25,8639 - 4(-8,621) \quad \therefore M_{10} = -60,3479\,tm$

$\sum M_{④} = 0$ ↷ ⊕ (arriba)

$H_1(4) + 2,5(4) - 6\,Sen30(2) - 6\,Cos30(4) - 4 + 6 = 0 \quad \therefore H_1 = 3,6961t$

$\sum F_H = 0$ →⊕

$3,6961 - 6\,Cos30 - 6\,Cos45 - H_6 + 8,621 = 0 \quad \therefore H_6 = 2,8783t$

2 - Cortante

Barra 1-2

$Q_1 = 2,5$ //

$Q_2 = 2,5$ //

Barra 2-3

$Q_2 = 2,5 - 6\,Sen30 = -0,5t$

$Q_3 = -0,5t$ //

Barra 3-4

$Q_3 = 3,6961 - 6\,Cos30 = -1,5t$ //

$Q_4 = -1,5t$ //

Barra 5-6

$Q_6 = -8,1213t$

$Q_5 = -8,1213 + 2(3) = -2,1213t$ //

Barra 5-4

$Q_5 = -8,1213 + 2(3) + 6\,Sen45 = 2,1213t$ //

$Q_4 = 2,1213 + 2(3) = 8,1213t$

Barra 9-10

$Q_{10} = -8,6213t$ //

$Q_9 = -8,6213t$ //

Barra 9-7 $Q_9 = -8,621t$ //

$Q_7 = -8,621t$ //

Barra 7-4 $Q_7 = 8,6213t$ //

$Q_4 = 8,6213t$ //

3-Normales

Barra 1-2 $N_{1-2} = -3{,}6961\,t$ //

Barra 2-3 $N_{2-3} = -3{,}6961 + 6\,\text{Cos}30 = 1{,}5\,t$ //

Barra 3-4 $N_{3-4} = 2{,}5 - 6\,\text{Sen}30 = -0{,}5\,t$ //

Barra 5-6 $N_{5-6} = -2{,}8783\,t$

Barra 4-5 $N_{4-5} = -2{,}8783 - 6\,\text{Cos}45 = -7{,}121\,t$ //

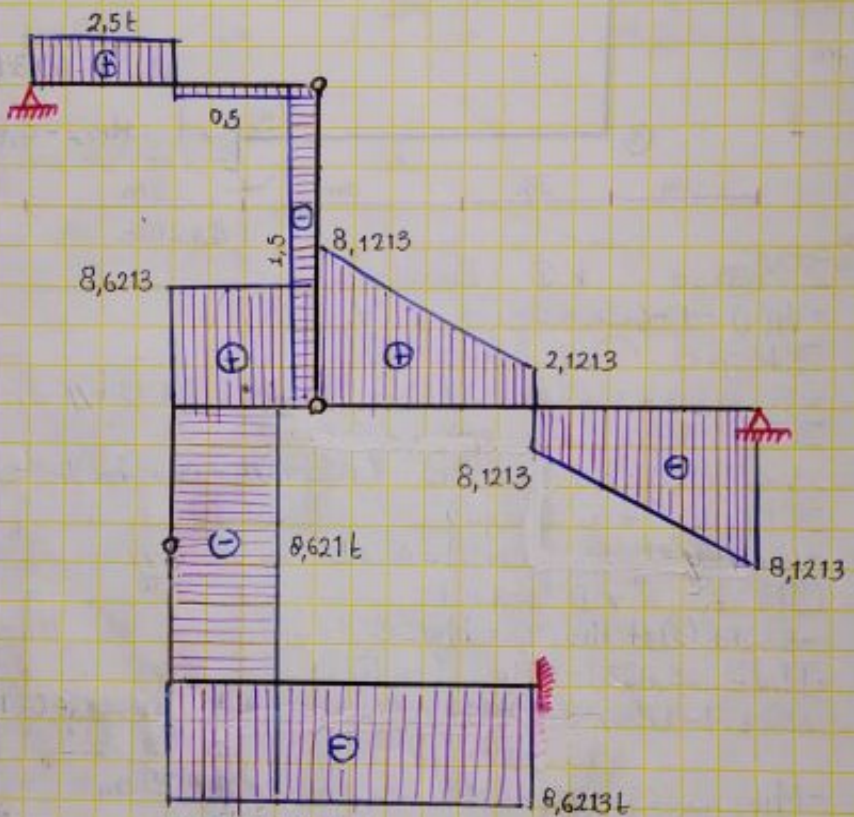
Barra 9-10 $N_{9-10} = 8{,}621\,t$

Barra 7-9 $N_{7-9} = -8{,}6213\,t$

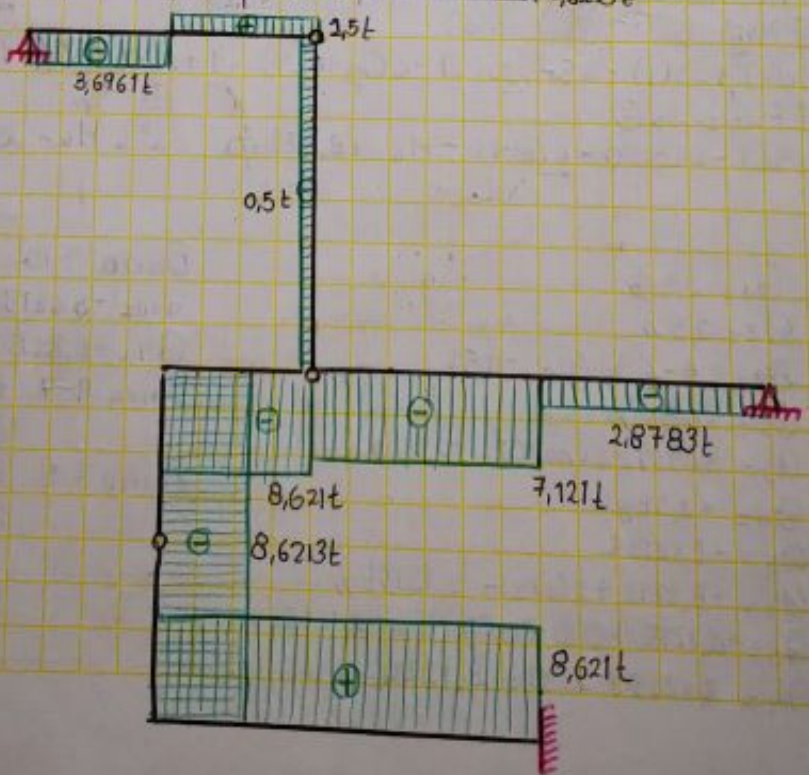
Barra 7-4 $N_{7-4} = -8{,}621\,t$

4-Diagramas

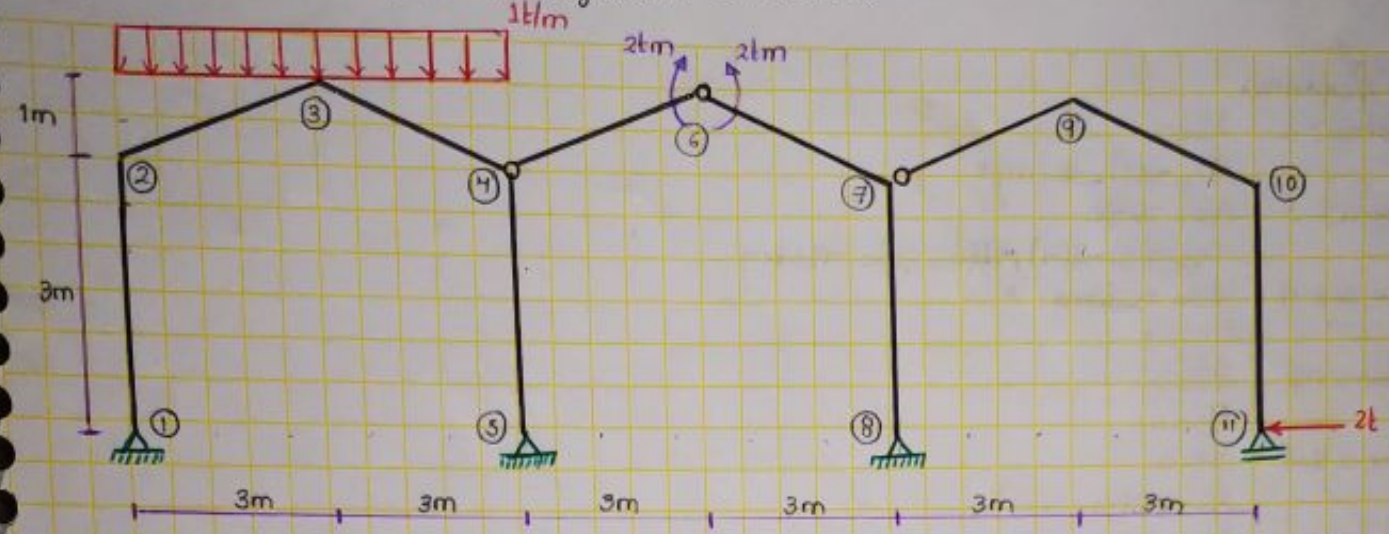
Cortante



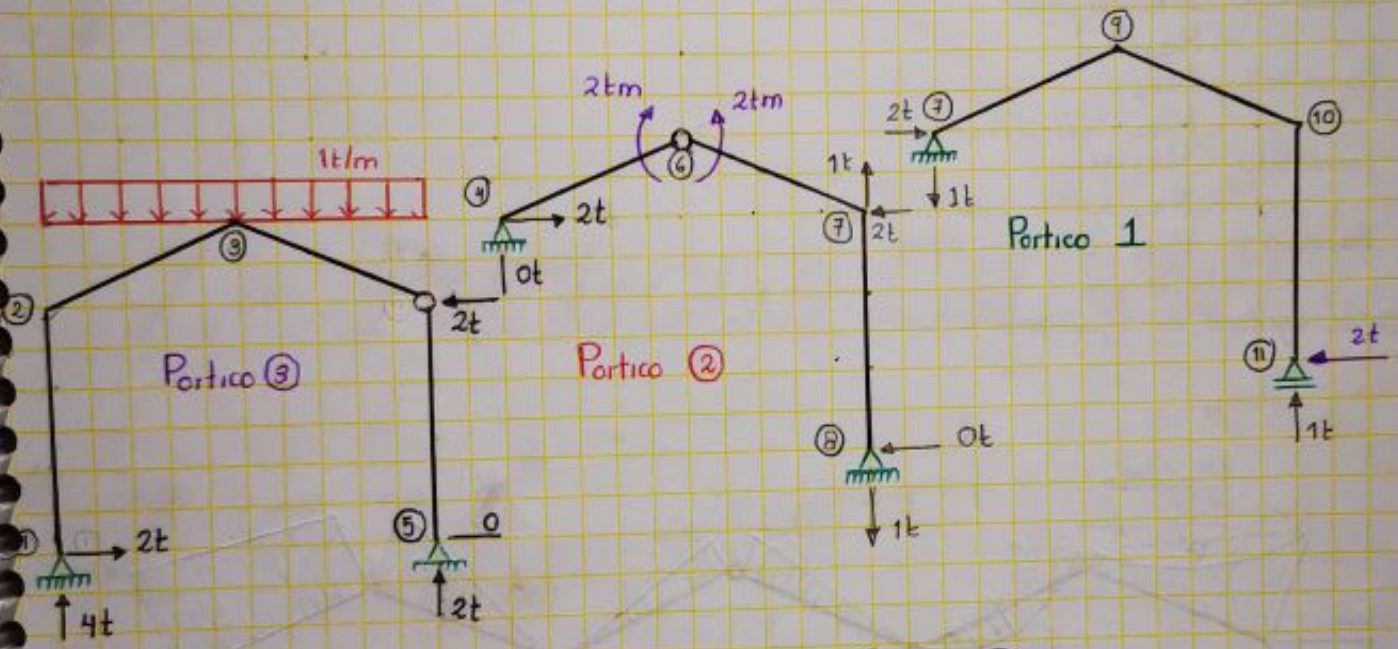
Normal



Diagramar Momento para la siguiente estructura



1.- Separar la estructura



* Portico ①
$\sum M_7 = 0$ ↷ ⊕
$2(3) - V_{11}(6) = 0$ ∴ $V_{11} = 1t$ //
$\sum F_V = 0$ ↑ ⊕
$V_7 = 1t$
$\sum F_H = 0$ → ⊕
$H_7 = 2t$

* Portico ③
$\sum M_1 = 0$ ↷ ⊕
$1(6)(3) - 2(3) - V_5(6) = 0$ ∴ $V_5 = 2t$ //
$\sum F_V = 0$ ↑ ⊕
$V_1 - 1(6) + 2 = 0$ ∴ $V_1 = 4t$
$\sum M_4 = 0$ ↷ ⊕ (abajo)
$H_5 = 0t$
$\sum F_H = 0t$
$H_1 = 2t$

* Portico ②
$\sum M_6 = 0$ ↷ ⊕ (izq)
$-H_4(1) - V_4(3) + 2 = 0$
$H_4 = 2 - 3V_4$ ①
$\sum M_8 = 0$ ↷ ⊕
$H_4(3) - V_4(6) - 2(3) = 0$
$3H_4 - 6V_4 - 6 = 0$ ②
Reemplazamos ① en ②
$3(2-3V_4) - 6V_4 - 6 = 0$
$6 - 9V_4 - 6V_4 - 6 = 0$
$V_4 = 0t$ //
$H_4 = 2 - 3(0) = 2t$ //
$\sum F_H = 0$ → ⊕
$H_8 - 2 + 2 = 0$ ∴ $H_4 = 0t$
$\sum F_V = 0$ ↑ ⊕
$V_8 = 1t$

2.- Momentos

Barra 1-2 $M_1 = 0$

$M_2 = -2(3) = -6\,tm$

Barra 2-3 $M_2 = -6\,tm$

$M_3 = 4(3) - 2(4) - 1(3)(1,5) = -0,5\,tm$

Barra 3-4 $M_3 = -0,5\,tm$

$M_4 = 0\,t$

Barra 4-5 $M_4 = 0$

$M_5 = 0$

Barra 4-6 $M_4 = 0$

$M_6 = -2\,tm$

Barra 6-7 $M_6 = -2\,tm$

$M_7 = 0\,tm$

Barra 7-8 $M_7 = 0\,tm$

$M_8 = 0\,tm$

Barra 7-9 $M_7 = 0$

$M_9 = -1(3) - 2(1) = -5\,tm$

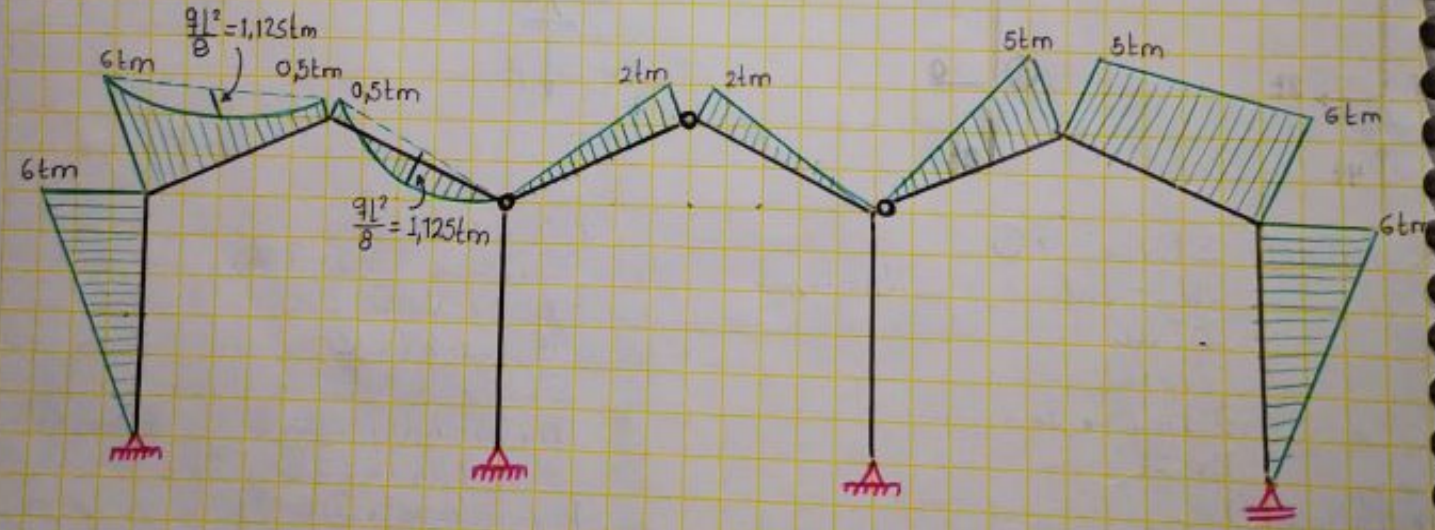
Barra 9-10 $M_9 = -5\,tm$

$M_{10} = -2(3) = -6\,tm$

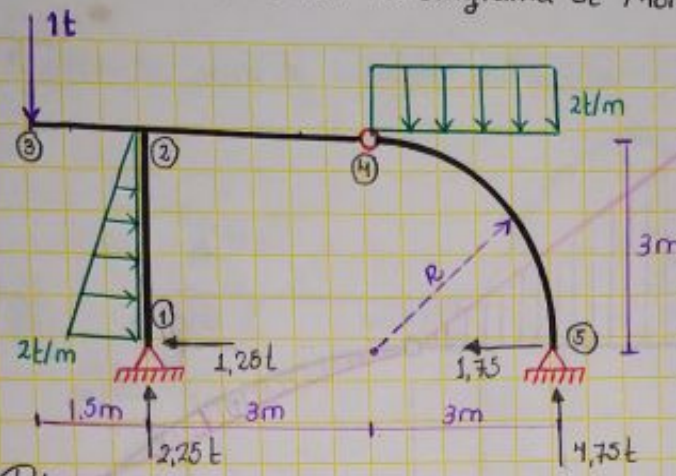
Barra 10-11 $M_{10} = -6\,tm$

$M_{11} = 0\,tm$

3- Diagrama de Momento



Para la siguiente estructura obtener el diagrama de Momento



Diagramar Momento

1- Cálculo de Reacciones

$\sum M_{①}=0$ ↻ ⊕

$\frac{2(3)}{2}(1) - 1(1,5) + 2(3)(4,5) - V_5(6) = 0$

$V_5 = 4,75t$ //

$\sum F_V = 0$ ↑ ⊕

$-1 + V_1 - 2(3) + 4,75 = 0$

$V_1 = 2,25t$ //

$\sum M_4$ (DER) ↻ ⊕

$2(3)(1,5) - 4,75(3) + H_5(3) = 0$

$H_5 = 1,75t$ //

$\sum F_H = 0$ → ⊕

$\sum F_H = 0$ → ⊕

$\frac{2(3)}{2} - H_1 - 1,75 = 0$

$H_1 = 1,25t$

2- Momentos

Barra 1-2

$M_1 = 0$

$M_2 = 1,25(3) - \frac{2(3)}{2}.2 = -2,25tm$

Barra 3-2

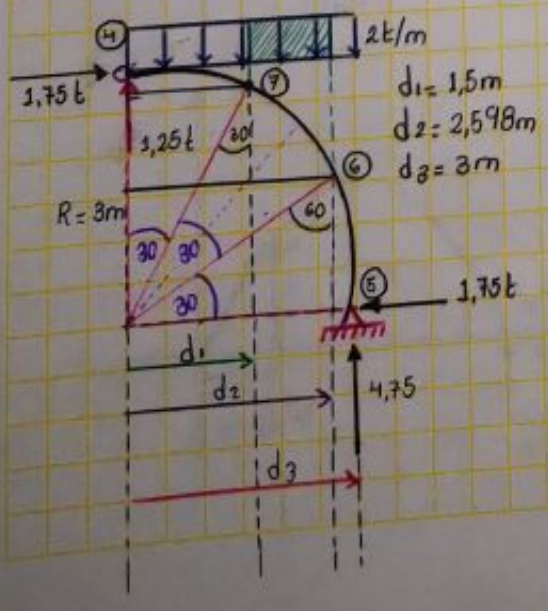
$M_3 = 0$

$M_2 = -1(1,5) = -1,5tm$

Barra 2-4

$M_2 = 1,25(3) - \frac{2(3)}{2}.2 - 1(1,5) = -3,75tm$

$M_4 = 0tm$



$d_1 = 1,5m$

$d_2 = 2,598m$

$d_3 = 3m$

$M_{\theta=30} = 1,25 \cdot d_1 - 2\,d_1 \cdot \frac{d_1}{2} + 1,75(R - R\cos\alpha)$

$M_{(\theta=30)} = 1,25(1,5) - (1,5)^2 + 1,75(3 - 3\cos 30)$

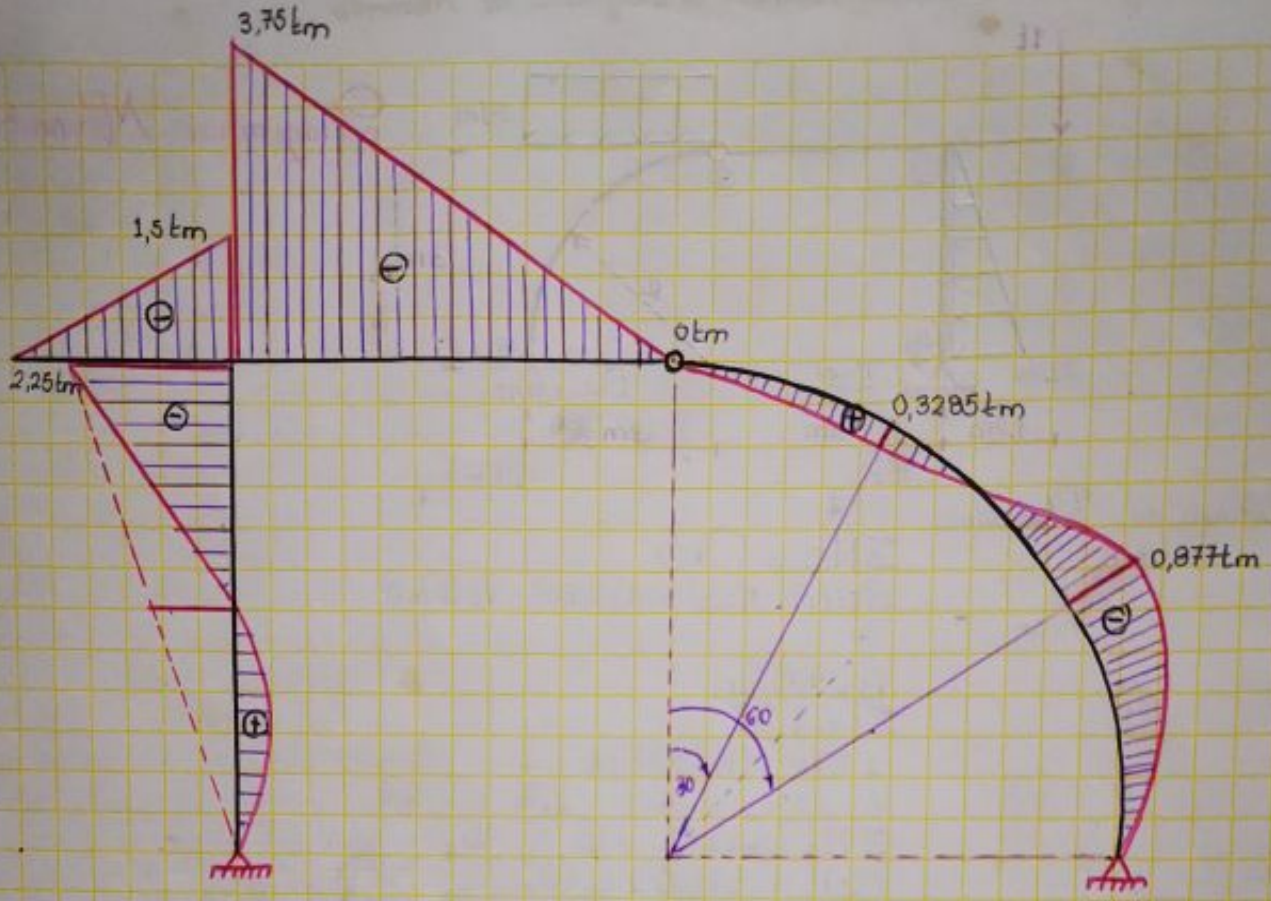
$M_{(\theta=30)} = 0,3285tm$

$M_{(\theta=60)} = 1,25 \cdot d_2 - 2\,d_2 \cdot \frac{d_2}{2} + 1,75(R - R\cos\alpha)$

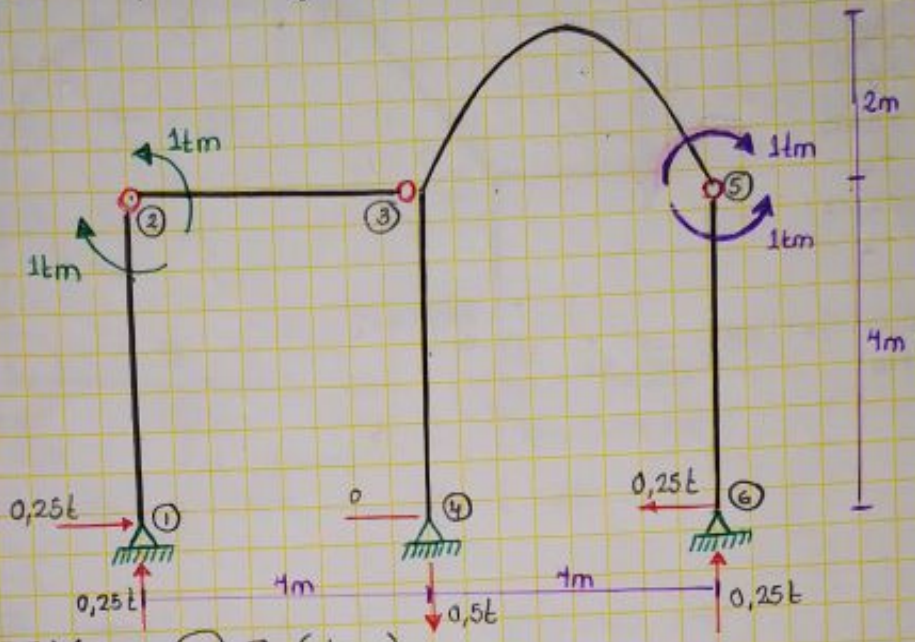
$M_{(\theta=60)} = 1,25(2598) - (2,598)^2 + 1,75(3 - 3\cos 60)$

$M_{(\theta=60)} = -0,877tm$

$M_{\theta=90°} = 0$ //


3,75 tm
1,5 tm
2,25 tm
0 tm
0,3285 tm
0,877 tm
60
30

Diagramar Momento para la siguiente estructura



1- Reacciones

$\sum M_{②}=0$ ↻ ⊕ (abajo)

$1 - H_1(4)=0 \quad \therefore H_1 = 0,25\ t$

$\sum M_{⑤}=0$ ↻ ⊕ (abajo)

$-1 + H_6(4)=0 \quad \therefore H_6 = 0,25t$

$\sum F_H=0 \rightarrow ⊕$

$H_4 = 0\ t$

$\sum M_{③}=0$ ↻ ⊕ (Izquierda)

$V_1(4) - 0,25(4)=0 \quad \therefore V_1 = 0,25t$

$\sum M_{③}=0$ ↻ ⊕ (Derecho)

$0,25(4) - V_6(4)=0 \quad \therefore V_6 = 0,25t$

$\sum F_V=0$ ↑ ⊕

$0,25 - V_4 + 0,25=0 \quad \therefore V_4 = 0,5t$

2- Momentos

Borra 1-2 $M_1 = 0$

$M_2 = -0,25(4) = -1tm$

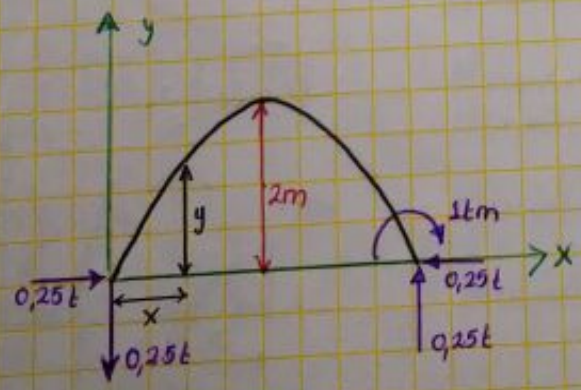
Borra 2-3 $M_2 = -1tm$

$M_3 = 0\ tm$

Borra 3-4 $M_3 = 0$

$M_4 = 0$

Borra 3-5



$y = f(x)$ Parabola

$(X-h)^2 = -4a(Y-K)$

$P(0,0)$; $V(h,K) = (2,2)$

$(0-2)^2 = -4a(0-2)$

$4 = +8a$

$a = 0,5$

$(x-2)^2 = -4(0,5)(Y-2)$

$Y = -0,5X^2 + 2X$

$M = -0,25X - 0,25Y = -0,25X - 0,25(-0,5X^2 + 2X)$

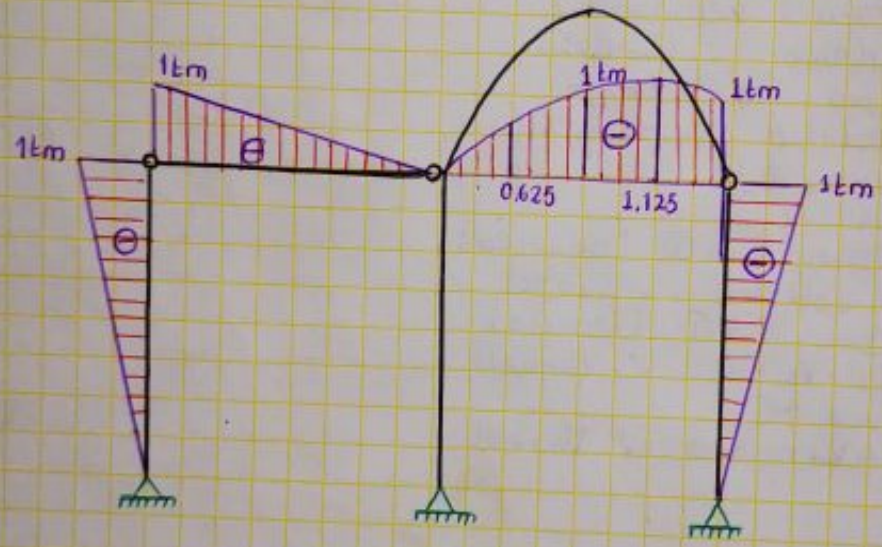
$M_x = 0{,}125x^2 - 0{,}75x$

x	M
0	0
1	-0,625
2	-1
3	-1,125
4	-1

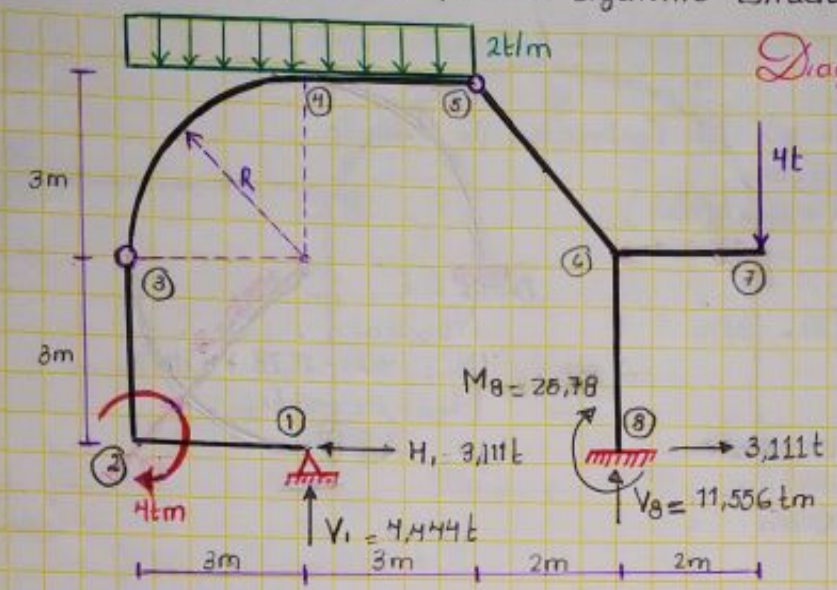
Barra 5-6

$M_5 = -0{,}25(4) = -1\,tm$

$M_6 = 0\,tm$



Obtener el diagrama de Momento para la siguiente estructura



Diagramar Momento

1.- Reacciones

$\Sigma M_8 = 0$ ↷ ⊕ (abajo)

$4 - V_1(3) + H_1(3) = 0$

$\Sigma M_{⑤} = 0$ ↷ ⊕ (Izquierda)

$V_1(3) + H_1(6) + 4 - 2(6)(3) = 0$

$-3V_1 + 3H_1 = -4$

$3V_1 + 6H_1 = +32$

$+9H_1 = +28$

$H_1 = +3,111t$ //

$4 - V_1(3) + 3,111(3) = 0$

$V_1 = 4,444t$ //

$\Sigma F_V = 0$ ↑ ⊕

$4,444 - 2(6) - 4 + V_8 = 0$ $\quad V_8 = 11,556$ tm

$\Sigma F_H = 0$ → ⊕

$-3,111 + H_8 = 0$ $\quad \therefore H_8 = 3,111t$

$\Sigma M_{①} = 0$ ↷ ⊕

$4 + 4(7) - 11,556(5) + M_8 = 0$ $\quad \therefore M_8 = 25,78$ tm //

2.- Momentos

Barra 1-2

$M_1 = 0$

$M_2 = 4,444(3) = 13,332$ tm

Barra 2-8

$M_2 = -4,444(3) + 4 = -9,332$ tm

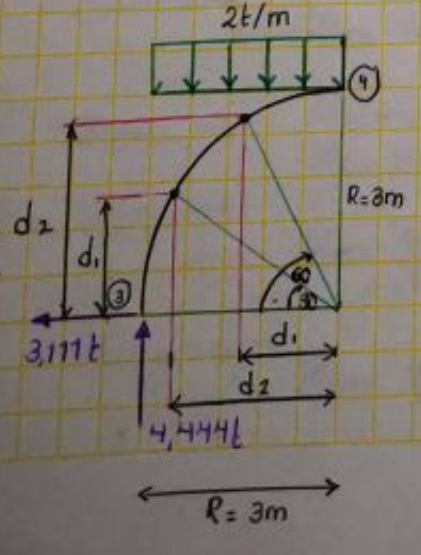
$M_2 = -9,332$ tm

$M_3 = 0$ tm

Barra 3-4

$d_1 = R\cos 60 = 3\cos 60 = 1,5m$

$d_2 = R\cos 30 = 3\cos 30 = 2,598m$



$M_{(\theta=0)} = 0$ //

$M_{(\theta=30)} = 4,444(3 - 2,598) + 3,111(1,5) - \frac{2(3-2,598)^2}{2} = 6,291$ tm //

$M_{(\theta=60)} = 4,444(3 - 1,5) + 3,111(2,598) - \frac{2(3-1,5)^2}{2} = 12,498$ tm //

$M_{(\theta=90)} = 4,444(3) + 3,111(3) - 2(3)(1,5) = 13,665$ tm //

Barra 4-5

$M_4 = M_{(8-90)}$ Por Continuidad de Momento

$M_4 = 13{,}665\ tm$ //

$M_5 = 0$ (articulación)

Barra 6-7

$M_6 = -4(2) = -8\ tm$

$M_7 = 0\ tm$

Barra 6-8

$M_6 = 3{,}111(3) - 25{,}78$

$M_6 = -16{,}447\ tm$

$M_8 = -25{,}78\ tm$

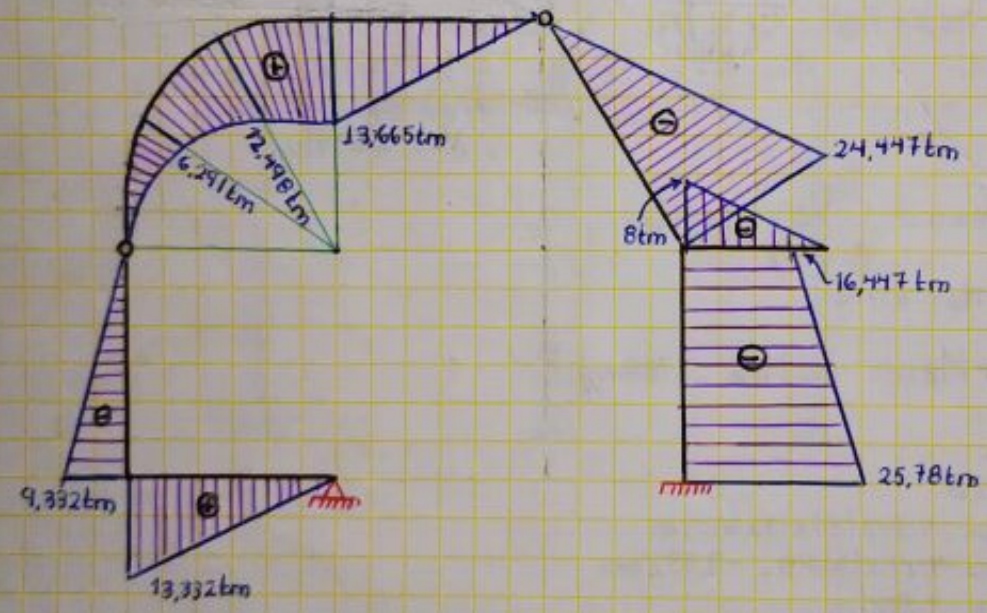
Barra 5-6

$M_5 = 0\ tm$

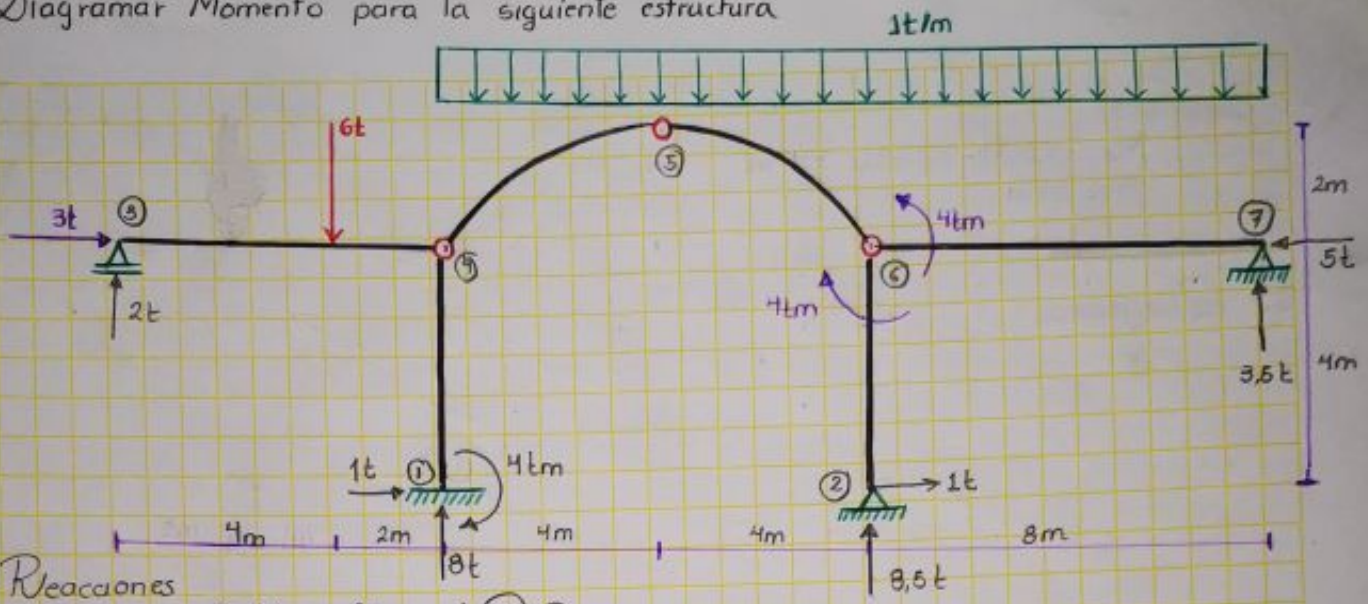
$M_6 = -4(2) - 25{,}78 + 3{,}111(3)$

$M_6 = -24{,}447\ tm$

3.- Diagramas



Diagramar Momento para la siguiente estructura



1- Reacciones

$\sum M_4 = 0$ (Izquierd) ↷ ⊕

$V_3(6) - 6(2) = 0$

$V_3 = 2t$ //

$\sum M_6 = 0$ ↷ ⊕ (derecha)

$-4 + 1(8)(4) - V_7(8) = 0$

$V_7 = 3{,}5t$

$\sum M_6 = 0$ ↷ ⊕ (abajo)

$4 - H_2(4) = 0 \quad \therefore H_2 = 1t$

$\sum M_4 = 0$ ↷ ⊕ (Derecha)

$1(16)(8) - 3{,}5(16) - 1(4) - V_2(8) = 0$

$V_2 = 8{,}5t$

$\sum F_V = 0$ ↑ ⊕

$2 - 6 + V_1 - 1(16) + 8{,}5 + 3{,}5 = 0$

$V_1 = 8t$

$\sum M_5 = 0$ ↷ ⊕ (derecha)

$1(12)(6) - 1(6) - 8{,}5(4) - 3{,}5(12) + H_7(2) = 0$

$H_7 = 5t$ //

$\sum F_H = 0$ → ⊕

$3 + 1 - 5 + H_1 = 0 \quad \therefore H_1 = 1t$ //

$\sum M_4 = 0$ ↷ ⊕ (abajo)

$-1(4) + M_1 = 0 \quad \therefore M_1 = 4tm$

2. Momentos

Barra 1-4

$M_1 = 4tm$

$M_4 = 0tm$

Barra 3-4

$M_p = \frac{Pab}{L} = \frac{6(4)(2)}{6} = 8tm$

Barra 2-6

$M_2 = 0tm$

$M_6 = 1(4) = 4tm$

Barra 4-5

$M_m = \frac{qL^2}{8} = \frac{1(4)^2}{8} = 2tm$

Barra 5-6

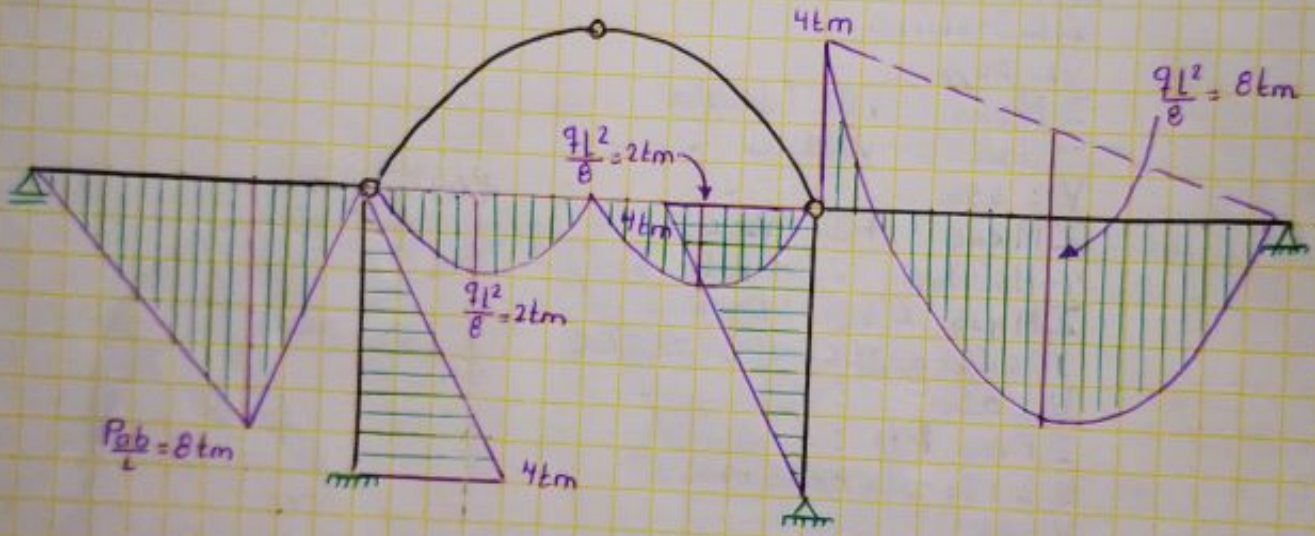
$M_m = \frac{qL^2}{8} = \frac{1(4)^2}{8} = 2tm$

Barra 6-7

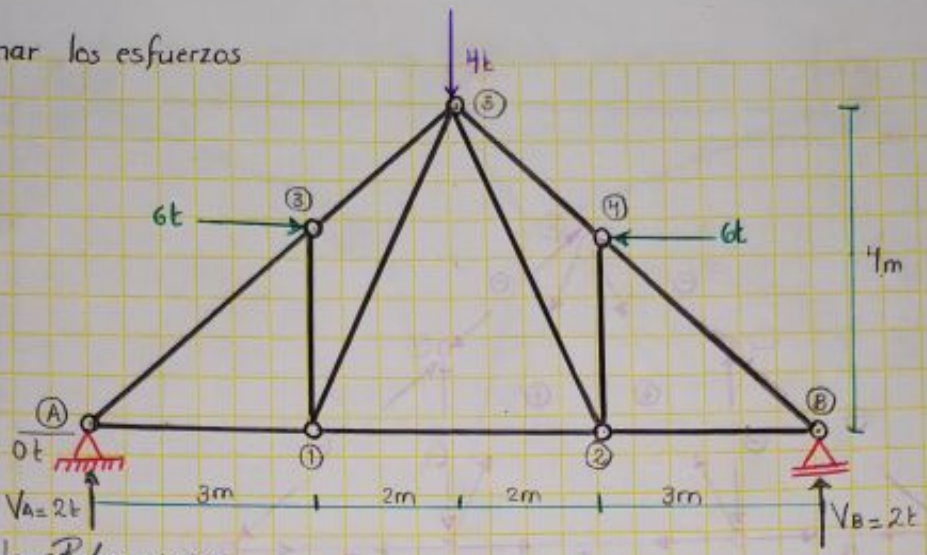
$M_6 = 3,5(8) - 1(8)(4) = -4tm$

$M_7 = 0tm$

3- Diagramas



Determinar los esfuerzos



1- Cálculo de Reacciones

$\Sigma M_A = 0 \quad \circlearrowright \oplus$

$4(5) - V_B(10) = 0 \quad \therefore V_B = 2t$

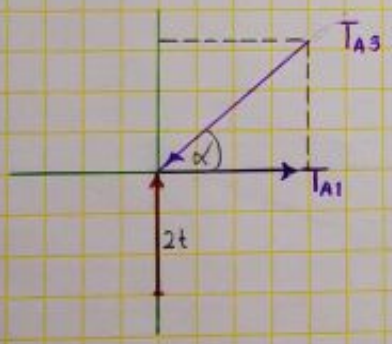
$\Sigma F_V = 0 \quad \uparrow \oplus$

$V_A - 4 + 2 = 0 \quad \therefore V_A = 2t$

2 Tensiones

Siendo La Estructura Simetrica y la carga Simetrica solo analizaremos Los nudos Ⓐ, ① y ③

* Nudo Ⓐ



$\alpha = \text{arctg}\left(\frac{4}{5}\right) = 38,66°$

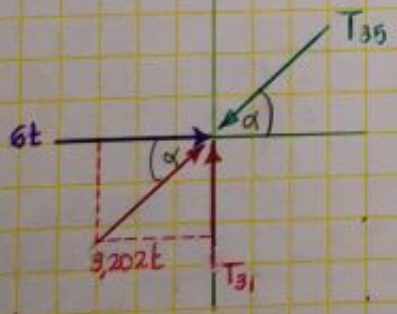
$\Sigma F_V = 0 \quad \uparrow \oplus$

$2 - T_{A3}\,\text{Sen}(38,66) = 0 \quad \therefore T_{A3} = 3,202\,t$ //

$\Sigma F_H = 0 \quad \rightarrow \oplus$

$T_{A1} - 3,202\,\text{Cos}(38,66) = 0 \quad \therefore T_{A1} = 2,5\,t$

* Nudo ③



$\Sigma F_V = 0 \quad \uparrow \oplus$

① $+T_{31} - T_{35}\,\text{Sen}(38,66) + 3,202 \cdot \text{Sen}(38,66) = 0$

$\Sigma F_H = 0 \quad \rightarrow \oplus$

$-T_{35}\,\text{Cos}(38,66) + 6 + 3,202\,\text{Cos}(38,66) = 0$

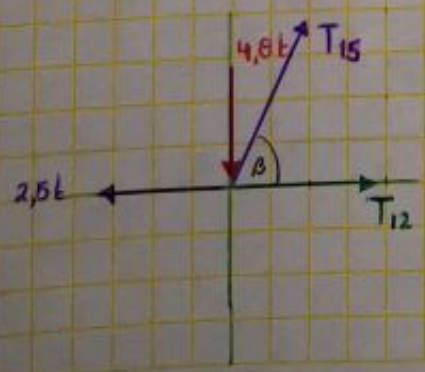
$T_{35} = 10,886\,t$ //

Reemplazamos en ①

$+T_{31} - 10,886\,\text{Sen}(38,66) + 3,202\,\text{Sen}(38,66) = 0$

$T_{31} = 4,8\,t$ //

* Nudo ①



$\Sigma F_V = 0 \quad \uparrow \oplus$ $\qquad \beta = \text{arctg}\left(\frac{4}{2}\right) = 63,435°$

$T_{15}\,\text{Sen}(63,435) - 4,8 = 0$

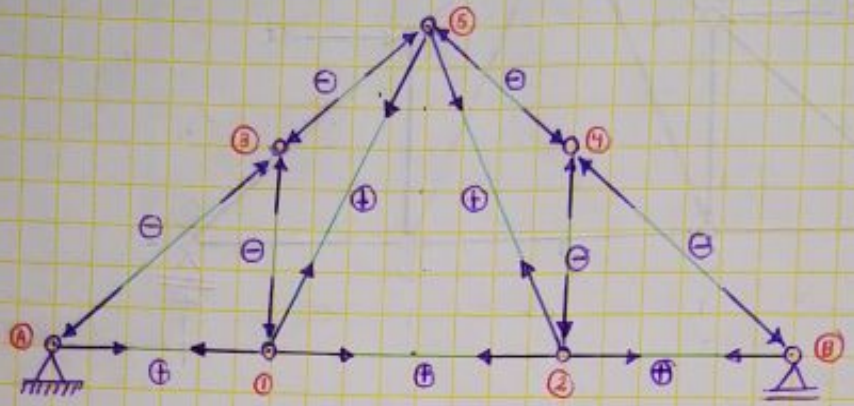
$T_{15} = 5,367\,t$ //

$\Sigma F_H = 0 \quad \rightarrow \oplus$

$-2,5 + T_{12} + 5,367\,\text{Cos}(63,435) = 0$

$T_{12} = -0,1\,t$

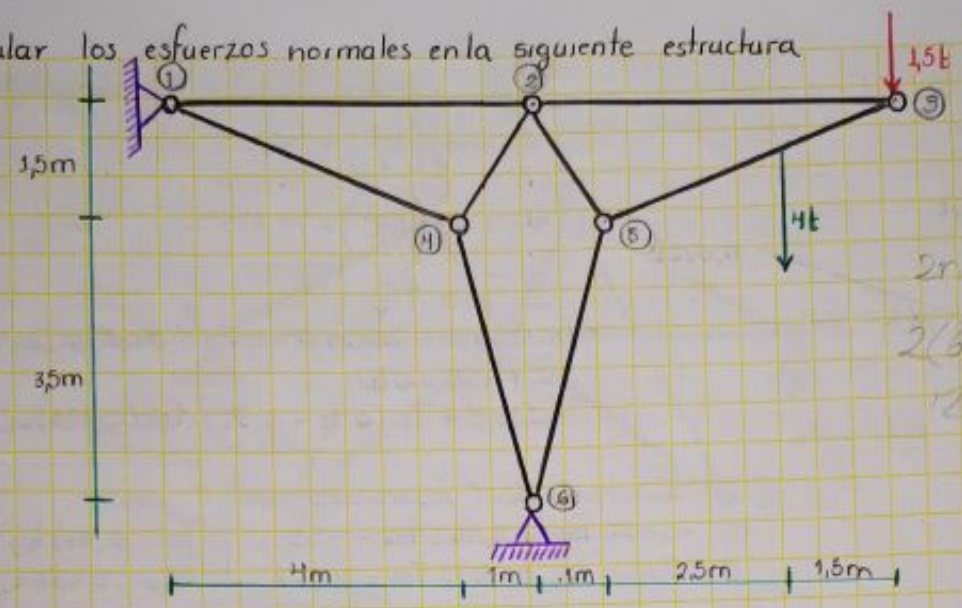
3- Diagrama de Tensiones



4.- Tabla Final

Barra	Tensión	Esfuerzos
A-1	2,5 t	Tracción
A-3	3,202 t	Compresión
1-2	0,1 t	Tracción
1-3	4,8 t	Compresión
1-5	5,367 t	Tracción
2-B	2,5 t	Tracción
2-4	4,8 t	Compresión
2-5	5,367 t	Tracción
B-4	3,202 t	Compresión
3-5	10,886 t	Compresión
4-5	10,886 t	Compresión

Calcular los esfuerzos normales en la siguiente estructura

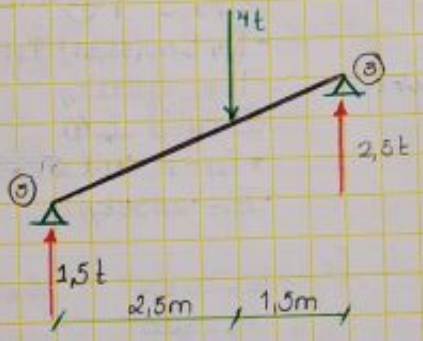


$2r - k = b$

$2(5) - 4 = 8$

$8 = 8$

1.- Idealización de cargas Barra 3-5



$\sum M_{⑤} = 0$ ↻ ⊕

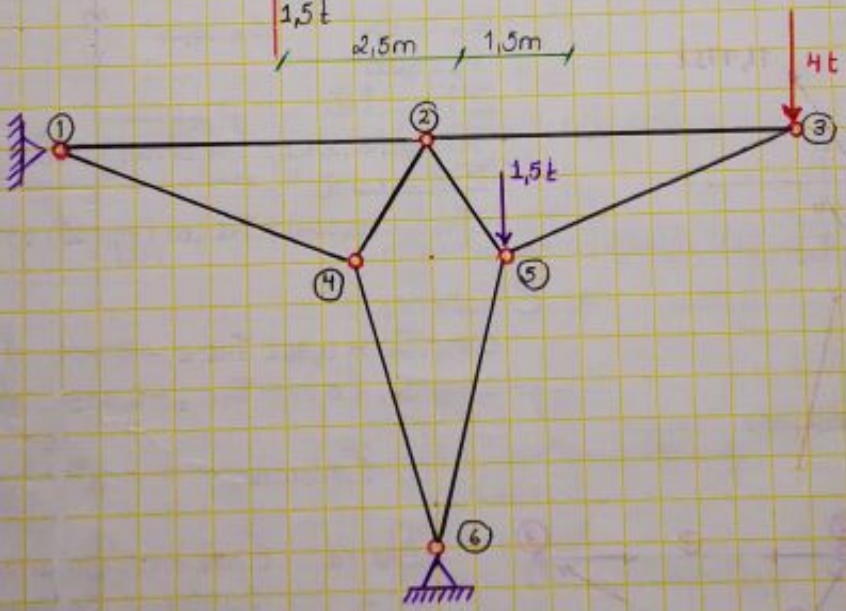
$4(2,5) - V_3(4) = 0$

$V_3 = 2,5t$

$\sum F_V = 0$ ↑ ⊕

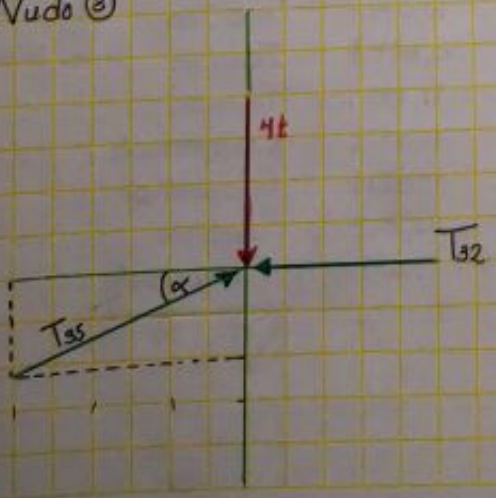
$V_5 - 4 + 2,5 = 0$

$V_5 = 1,5t$



2.- Tensiones

* Nudo ③



$\alpha = \text{arctg}\left(\frac{1,5}{4}\right) = 20,556°$

$\sum F_V = 0$ ↑ ⊕

$-4 + T_{35} \operatorname{Sen}(20,556) = 0$

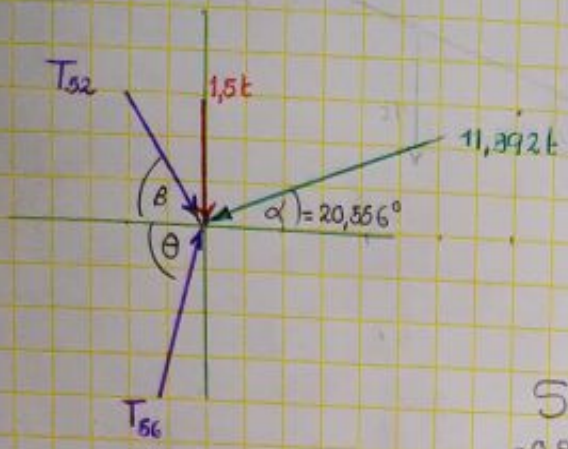
$T_{35} = 11,392t$ //

$\sum F_H = 0$ → ⊕

$-T_{32} + 11,392 \cdot \operatorname{Cos}(20,556) = 0$

$T_{32} = 10,667t$ //

Nudo ⑤



$\beta = arctg\left(\frac{1,5}{1}\right) = 56,31°$

$\theta = arctg\left(\frac{3,5}{1}\right) = 74,055°$

$\Sigma F_V = 0 \uparrow \oplus$

$-T_{52}\cdot Sen\,\beta + T_{56}\,Sen\,\theta - 1,5 - 11,392\,Sen(20,556) = 0$

$\Sigma F_H = 0 \rightarrow \oplus$

$T_{52}\,Cos\,\beta + T_{56}\,Cos\,\theta - 11,392\,Cos(20,556) = 0$

Sistema de Ecuaciones

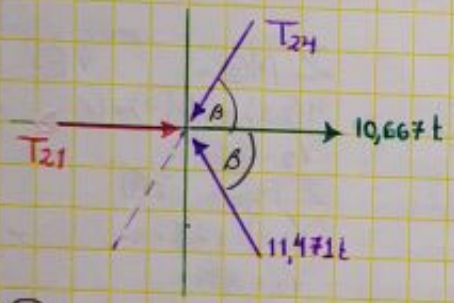
$-0,832\,T_{52} + 0,962\,T_{56} = +5,5$

$0,555\,T_{52} + 0,275\,T_{56} = 10,667$

$\Big\}\ T_{52} = 11,471\,t\ //$

$\Big\}\ T_{56} = 15,638\,t\ //$

Nudo ②



$\Sigma F_V = 0 \uparrow \oplus$

$-T_{24}\,Sen(56,31) + 11,471\cdot Sen(56,31) = 0$

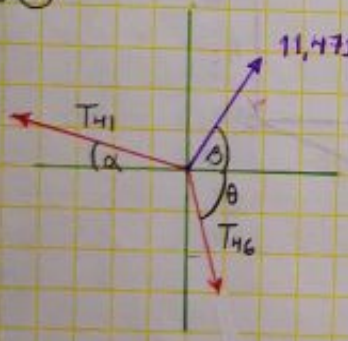
$T_{24} = 11,471\,t\ //$

$\Sigma F_H = 0 \rightarrow \oplus$

$+T_{21} - 11,471\cdot Cos(56,31) - 11,471\,Cos(56,31) + 10,667 = 0$

$T_{21} = 2,058\,t\ //$

Nudo ④



$\alpha = 20,556°$ $\quad \beta = 56,31°$

$\theta = 74,055°$

$\Sigma F_V = 0 \uparrow \oplus$

$T_{41}\,Sen(20,556) - T_{46}\,Sen(74,055) + 11,471\,Sen(56,31) = 0$

$\Sigma F_H = 0 \rightarrow \oplus$

$-T_{41}\,Cos(20,556) + T_{46}\,Cos(74,055) + 11,471\,Cos(56,31) = 0$

Ecuaciones

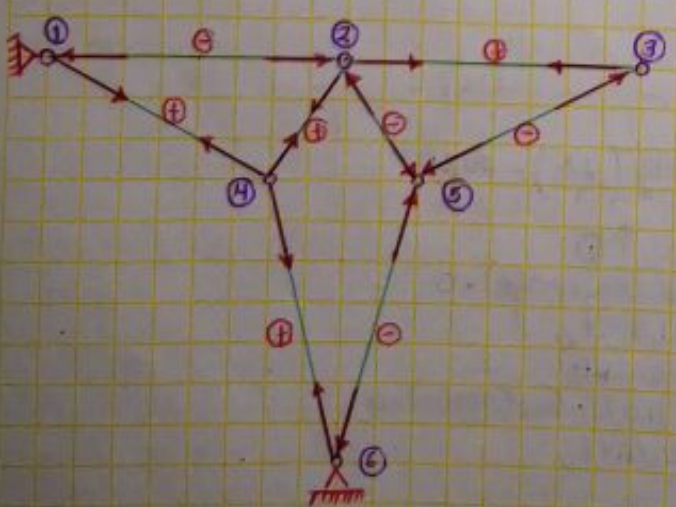
$0,351\,T_{41} - 0,962\,T_{46} = -9,544$

$-0,936\,T_{41} + 0,275\,T_{46} = -6,363$

$\Big\}\ T_{41} = 10,879\,t$

$\Big\}\ T_{46} = 13,89\,t$

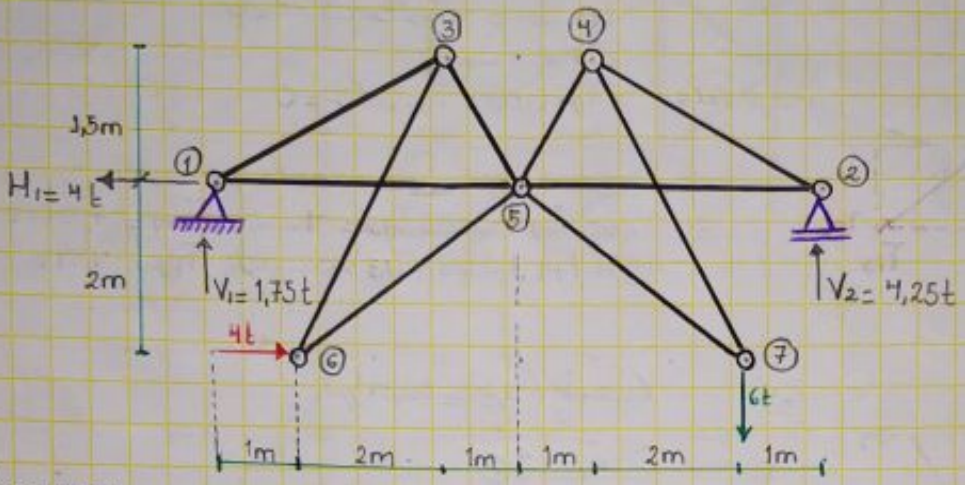
8 Diagrama de tensiones



4.- Tensiones

Barra	Tensión	Esfuerzo
1-2	2,058 t	Compresión
1-4	10,879 t	Tracción
2-3	10,667 t	Tracción
2-4	11,471 t	Tracción
2-5	11,471 t	Compresión
3-5	11,392 t	Compresión
4-6	13,89 t	Tracción
5-6	15,638 t	Compresión

Determinar los esfuerzos internos



1.- Reacciones

$\sum M_{①} = 0$ ↺ ⊕

$-4(2) + 6(7) - V_2(8) = 0 \quad \therefore V_2 = 4,25t$ //

$\sum F_V = 0$ ↑ ⊕

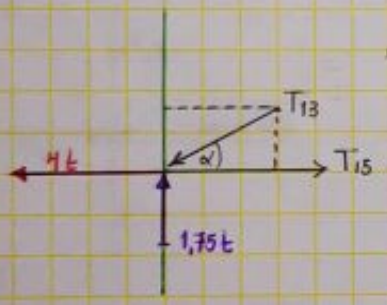
$V_1 + 4,25 - 6 = 0 \quad \therefore V_1 = 1,75t$ //

$\sum F_H = 0$ → ⊕

$H_1 = 4t$

2.- Tensiones por Nudos

Nudo ①



$\alpha = \text{arctg}\left(\frac{1,5}{3}\right) = 26,565°$

$\sum F_V = 0$ ↑ ⊕

$1,75 - T_{13} \cdot \text{Sen}(26,565) = 0$

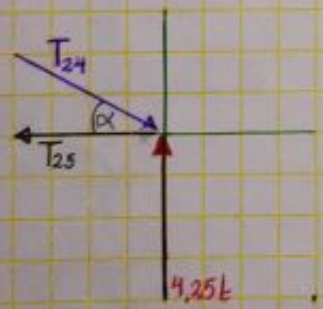
$T_{13} = 3,913t$ //

$\sum F_H = 0$ → ⊕

$-4 + T_{15} - 3,913 \cdot \text{Cos}(26,565) = 0$

$T_{15} = 7,499t$ //

Nudo ②



$\sum F_V = 0$ ↑ ⊕

$4,25 - T_{24} \cdot \text{Sen}(26,565) = 0$

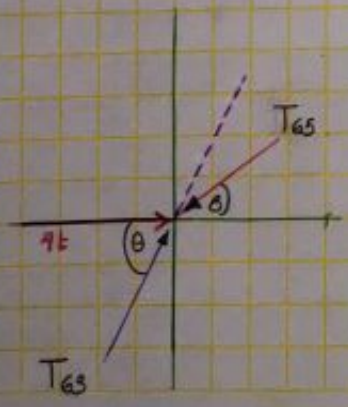
$T_{24} = 9,503t$ //

$\sum F_H = 0$ → ⊕

$-T_{25} + (9,503)\text{Cos}(26,565) = 0$

$T_{25} = 8,5t$ //

Nudo ⑥



$\beta = \text{arctg}\left(\frac{2}{3}\right) = 33,69°$

$\theta = \text{arctg}\left(\frac{3,5}{2}\right) = 60,255°$

$\sum F_V = 0$ ↑ ⊕

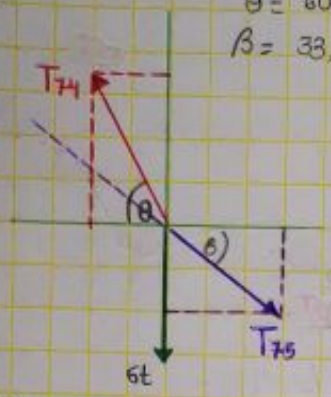
$T_{63}\,\text{Sen}\theta - T_{65}\,\text{Sen}\beta = 0$

$\sum F_H = 0$ → ⊕

$4 + T_{63}\,\text{Cos}\theta - T_{65}\text{Cos}\beta = 0$

$\begin{cases} 0,868\,T_{63} - 0,555\,T_{65} = 0 \\ 0,496\,T_{63} - 0,832\,T_{65} = -4 \end{cases}$ $\quad T_{63} = 4,968t \quad T_{65} = 7,769t$

Nudo ⑦



$\Sigma F_V = 0 \uparrow \oplus$

$T_{74} \operatorname{Sen}\theta - T_{75} \operatorname{Sen}\beta - 4 = 0$

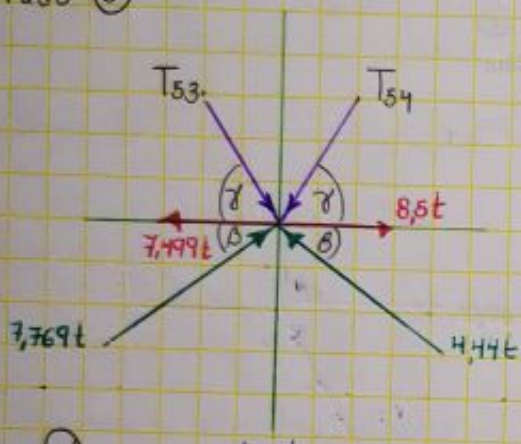
$\Sigma F_H = 0 \rightarrow \oplus$

$-T_{74} \operatorname{Cos}\theta + T_{75} \operatorname{Cos}\beta = 0$

Sistema de Ecuaciones

$$\left.\begin{array}{l} 0{,}868\, T_{74} - 0{,}555\, T_{75} = 4 \\ -0{,}496\, T_{74} + 0{,}832\, T_{75} = 0 \end{array}\right\} \begin{array}{l} T_{74} = 7{,}447\,t \\ T_{75} = 4{,}44\,t \end{array}$$

Nudo ⑤



$\gamma = \operatorname{arctg}\left(\frac{1{,}5}{1}\right) = 56{,}31°$ //

$\Sigma F_V = 0 \uparrow \oplus$

$7{,}769 \operatorname{Sen}\beta + 4{,}44 \operatorname{Sen}\beta - T_{53} \operatorname{Sen}\gamma - T_{54} \operatorname{Sen}\gamma = 0$

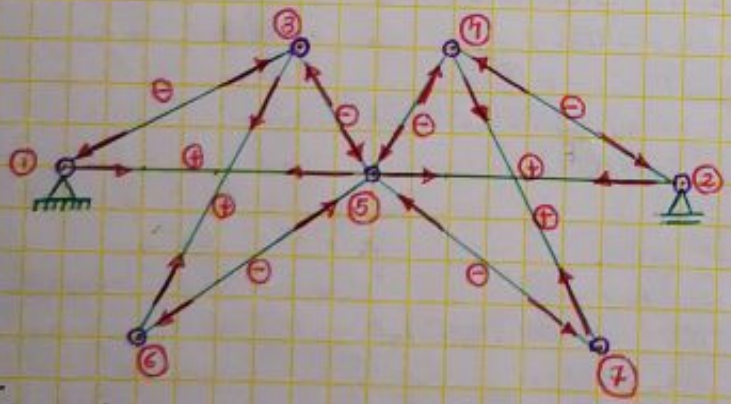
$\Sigma F_H = 0 \rightarrow \oplus$

$7{,}769 \operatorname{Cos}\beta - 4{,}44 \operatorname{Cos}\beta + 8{,}5 - 7{,}499 + T_{53} \operatorname{Cos}\gamma - T_{54} \operatorname{Cos}\gamma = 0$

Sistema de Ecuaciones

$$\left.\begin{array}{l} 0{,}832\, T_{53} + 0{,}832\, T_{54} = 6{,}772 \\ 0{,}555\, T_{53} - 0{,}555\, T_{54} = -3{,}771 \end{array}\right\} \begin{array}{l} T_{53} = 0{,}672\,t \\ T_{54} = 7{,}467\,t \end{array}$$

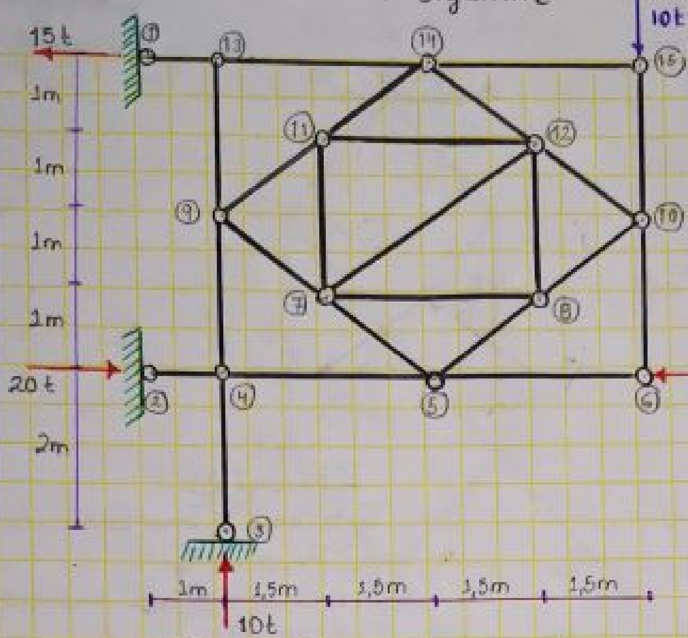
3 - Diagrama de tensiones



4 - Tensiones

Barra	Tensión	Esfuerzo
1-3	3,913 t	Compresión
1-5	7,499 t	Tracción
2-4	9,503 t	Compresión
2-5	8,5 t	Tracción
3-5	0,672	Compresión
3-6	4,968	Tracción
4-5	7,467 t	Compresión
4-7	7,447 t	Tracción
5-6	7,769 t	Compresión
5-7	4,440 t	Compresión

Cálcular las Normales del reticulado siguiente



1. Reacciones

$\Sigma M_{(4)}=0$ ↻ ⊕

$10(6)-H_1(4)=0$ ∴ $H_1=15t$

$\Sigma F_H=0$ → ⊕

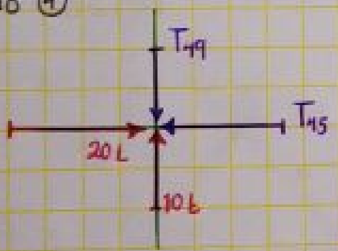
$-15+H_2-5=0$ ∴ $H_2=20t$

$\Sigma F_V=0$ ↑ ⊕

$V_3=10t$

2. Tensiones

* Nudo ④



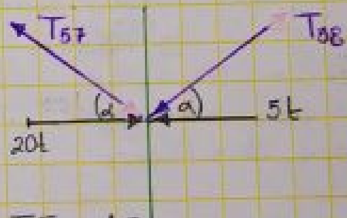
$\Sigma F_V=0$ ↑ ⊕

$T_{49}=10t$

$\Sigma F_H=0$ → ⊕

$T_{45}=20t$

* Nudo 5



$\alpha=arctg\left(\frac{2}{3}\right)$

$\alpha=33,69°$

$\Sigma F_V=0$ ↑ ⊕

$+T_{57}Sen(33,69)-T_{58}\cdot Sen(33,69)=0$

$T_{57}-T_{58}=0$

$\Sigma F_H=0$ → ⊕

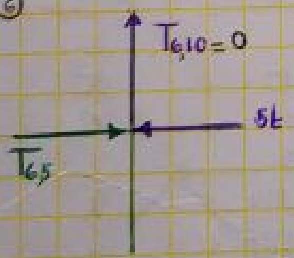
$-T_{57}\cdot Cos(33,69)-T_{58}Cos(33,69)+20-5=0$

$T_{57}+T_{58}=18,028$

Sistema de Ecuaciones

$$\left.\begin{array}{l}T_{57}-T_{58}=0\\T_{57}+T_{58}=18,028\end{array}\right\}\begin{array}{l}T_{57}=9,014t\\T_{58}=9,014t\end{array}$$

* Nudo ⑥



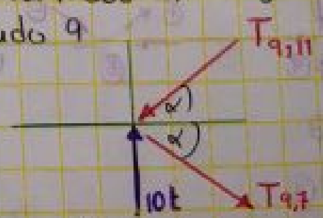
$\Sigma F_V=0$ ↑ ⊕

$T_{6,10}=0$

$\Sigma F_H=0$ → ⊕

$T_{6,5}=5t$

* Nudo 9



$\Sigma F_V=0$ ↑ ⊕

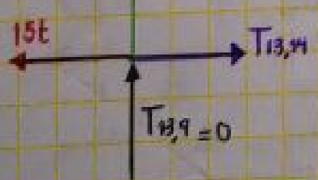
$-T_{9,11}\,Sen\,\alpha-T_{9,7}\,Sen\,\alpha+10=0$

$\Sigma F_H=0$ → ⊕

$-T_{9,11}\,Cos\,\alpha+T_{97}\,Cos\,\alpha=0$

$$\left\{\begin{array}{l}T_{9,7}+T_{9,11}=18.028\\T_{9,7}-T_{9,11}=0\end{array}\right.\Rightarrow\begin{array}{l}T_{9,7}=9,014t\\T_{9,11}=9,014t\end{array}$$

* Nudo 13



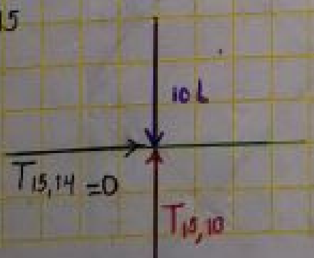
$\Sigma F_V=0$ ↑ ⊕

$T_{13,9}=0$

$\Sigma F_H=0$ → ⊕

$T_{13,14}=15t$

* Nudo 15



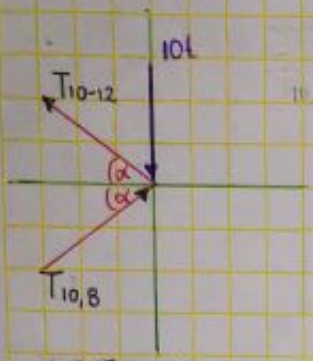
$\Sigma F_V=0$ ↑ ⊕

$T_{15,10}=10t$

$\Sigma F_H=0$ → ⊕

$T_{15,14}=0t$

✱ Nudo 10 $\alpha = 33{,}69°$



$\Sigma F_V = 0 \uparrow \oplus$

$T_{10,8} \operatorname{Sen}\alpha + T_{10,12} \cdot \operatorname{Sen}\alpha - 10 = 0$

$\Sigma F_H = 0 \rightarrow \oplus$

$T_{10,8} \cos\alpha - T_{10,12} \cos\alpha = 0$

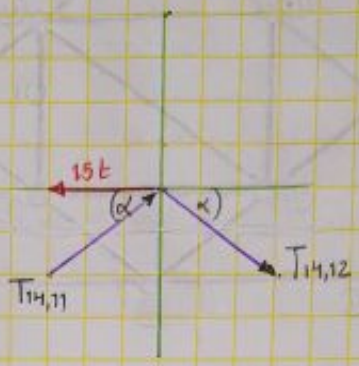
Sistema de Ecuaciones

$T_{10,8} + T_{10,12} = 18{,}028$

$T_{10,8} - T_{10,12} = 0$

$\Big\} \begin{matrix} T_{10,8} = 9{,}014\,t \\ T_{10,12} = 9{,}014\,t \end{matrix}$

✱ Nudo 14



$\Sigma F_V = 0 \uparrow \oplus$

$T_{14,11} \operatorname{Sen}\alpha - T_{14,12} \operatorname{Sen}\alpha = 0$

$\Sigma F_H = 0 \rightarrow \oplus$

$T_{14,11} \cos\alpha + T_{14,12} \cos\alpha - 15 = 0$

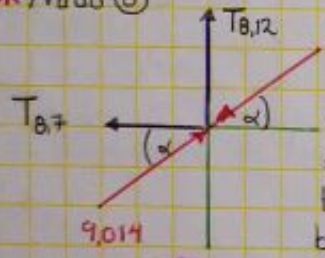
Ecuaciones

$T_{14,11} - T_{14,12} = 0$

$T_{14,11} + T_{14,12} = 18{,}028$

$\Big\} \begin{matrix} T_{14,11} = 9{,}014\,t \,// \\ T_{14,12} = 9{,}014\,t \,// \end{matrix}$

✱ Nudo ⑧



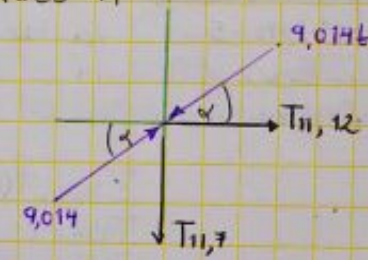
Como las fuerzas se equilibran las tensiones en las barras son nulas

$T_{8,7} = 0$; $T_{8,12} = 0$

✱ Nudo ⑦

Idem a las anteriores ∴ $T_{7,12} = 0$

✱ Nudo 11

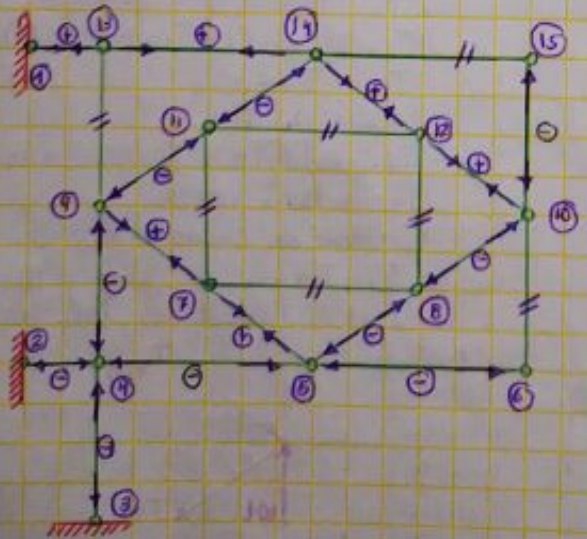


Idem al anterior

$T_{11,12} = 0$

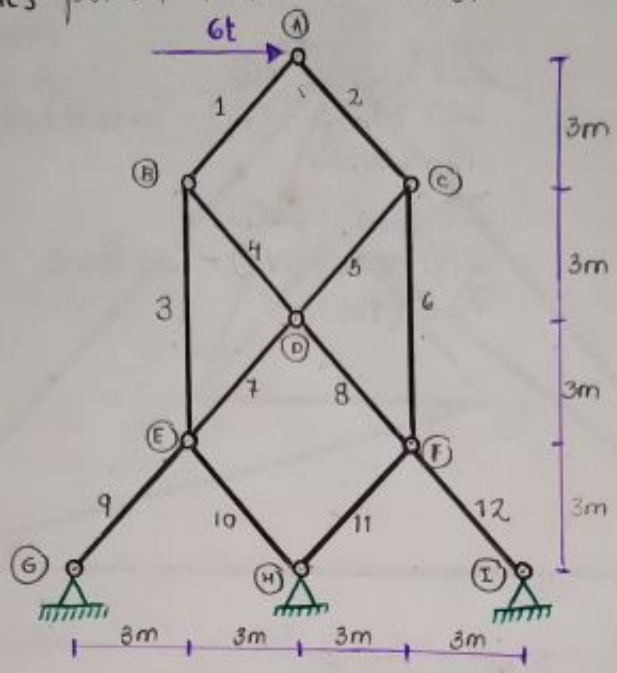
$T_{11,7} = 0$

4. Diagrama de tensiones

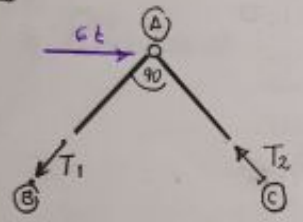


Barra	Tension	Esfuerzo
4-5	20	C
4-9	10	C
6-6	5	C
5-7	9,014	T
5-8	9,014	C
6-10	0	Nulo
7-8	0	Nulo
7-9	9,014	T
7-11	0	Nulo
8-10	9,014	C
8-12	0	Nulo
9-11	9,014	C
9-13	0	Nulo
10-12	9,014	T
10-15	10	C
11-12	0	Nulo
11-14	9,014	C
12 14	9,014	T
11-14	9,014	C
14-15	0	Nulo

Calcular las Normales por el Metodo de Ritter



Tensiones



$\sum M_B = 0 \curvearrowright +$

$6(3) - T_2 \cdot 3\sqrt{2} = 0$

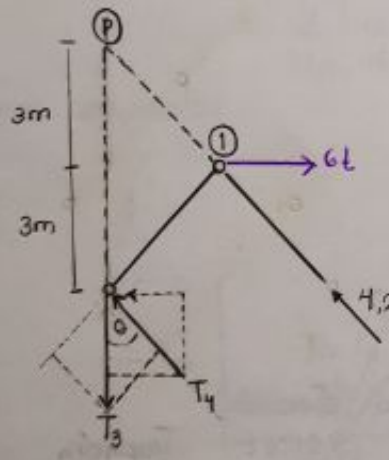
$T_2 = 4{,}2426\ t$ //

$\sum M_C = 0 \curvearrowright +$

$6(3) - T_1(3\sqrt{2}) = 0$

$T_1 = 4{,}2426\ t$ //

$\theta = \text{arctg}\left(\frac{3}{3}\right) = 45°$



$\sum M_P = 0 \curvearrowright +$

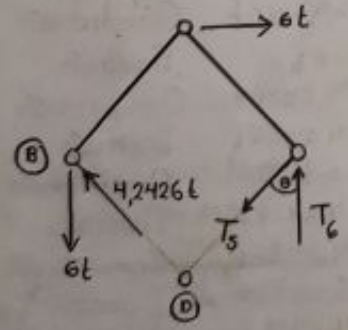
$-6(3) + T_4 \operatorname{Sen}45\,(6) = 0$

$T_4 = 4{,}2426\ t$ //

$\sum M_1 = 0 \curvearrowright +$

$4{,}2426\,(3\sqrt{2}) - T_3 \operatorname{Cos}45°\,(3\sqrt{2}) = 0$

$T_3 = 6\ t$ //



$\sum M_D = 0 \curvearrowright +$

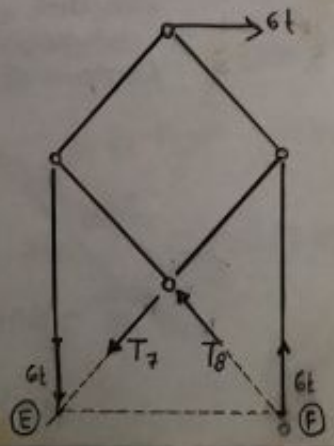
$6(6) - 6(3) - T_6(3) = 0$

$T_6 = 6\ t$ //

$\sum M_B = 0 \curvearrowright +$

$6(3) - 6(6) + T_5 \operatorname{Cos}45 \cdot 6 = 0$

$T_5 = 4{,}2426\ t$



$\sum M_F = 0 \curvearrowright +$

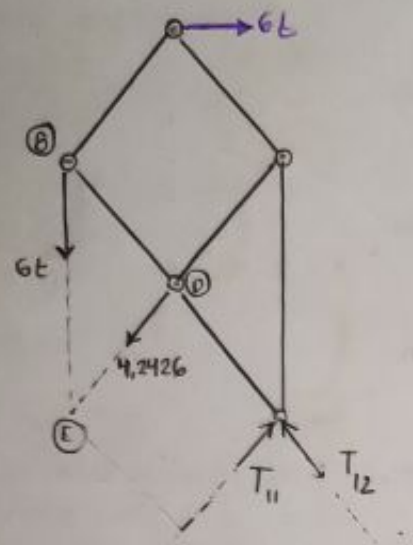
$-6(6) + 6(9) - T_7(3\sqrt{2}) = 0$

$T_7 = 4{,}2426\ t$

$\sum M_E = 0 \curvearrowright +$

$6(9) - 6(6) - T_8(3\sqrt{2}) = 0$

$T_8 = 4{,}2426\ t$



$\sum M_{B=0}$ ↷ ⊕

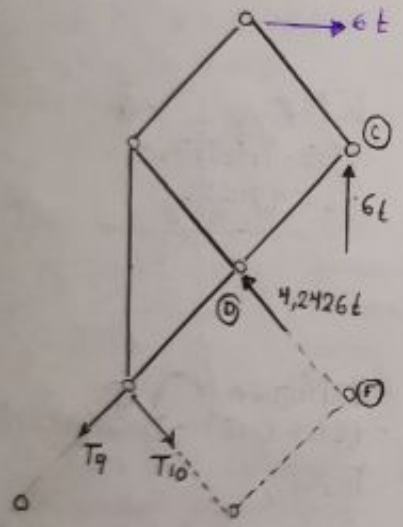
$6(3) + 4{,}2426\,(3\sqrt{2}) - T_{11}\,(6\sqrt{2}) = 0$

$T_{11} = 4{,}2426\ t$

$\sum M_{E=0}$ ↷ ⊕

$6(9) - 4{,}2426\,(3\sqrt{2}) + T_{12}\,3\sqrt{2} = 0$

$T_{12} = 8{,}4853\ t$



$\sum M_{C=0}$ ↷ ⊕

$6(3) + 4{,}2426(3\sqrt{2}) - T_{10}(6\sqrt{2}) = 0$

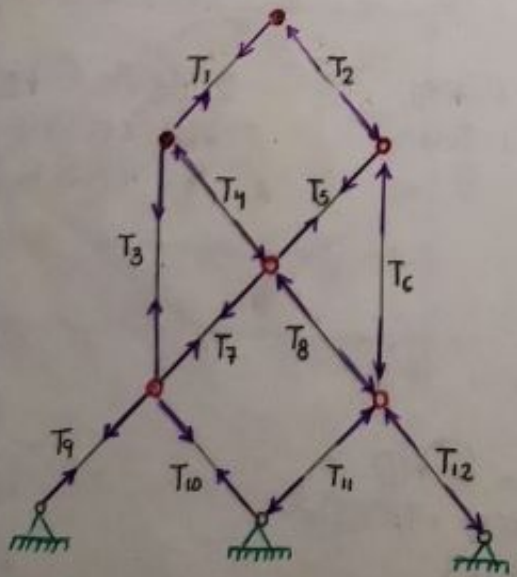
$T_{10} = 4{,}2426\ t$

$\sum M_{F=0}$ ↷ ⊕

$6(9) - 4{,}2426\,(3\sqrt{2}) - T_9(3\sqrt{2}) = 0$

$T_9 = 8{,}4853\ t$

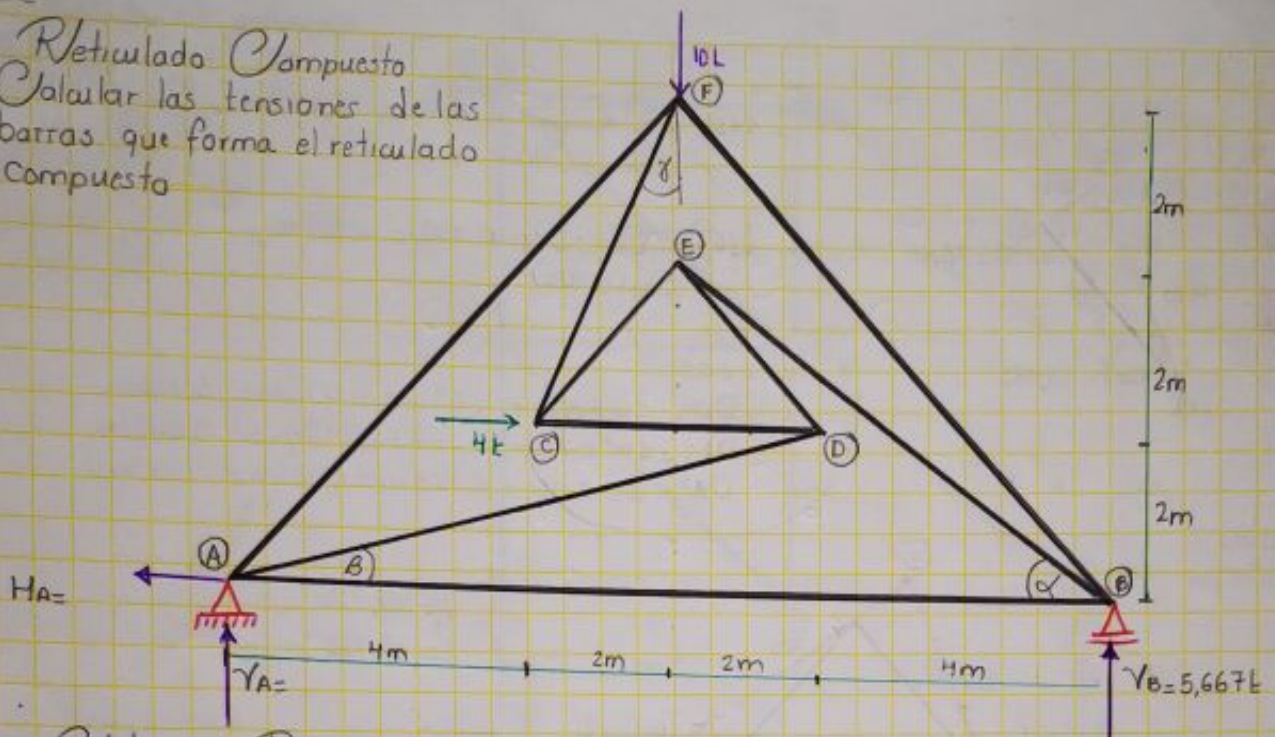
2- Diagrama de Tensiones



Barra	Tensión	
1	4,2426 t	Tracción
2	4,2426 t	Compresión
3	6 t	Tracción
4	4,2426 t	Compresión
5	4,2426 t	Tracción
6	6 t	Compresión
7	4,2426 t	Tracción
8	4,2426 t	Compresión
9	8,4853 t	Tracción
10	4,2426 t	Tracción
11	4,2426 t	Compresión
12	8,4853 t	Compresión

Reticulado Compuesto

Calcular las tensiones de las barras que forma el reticulado compuesto



1.- Cálculo de Reacciones

$\sum M_{A=0}$ ↻ ⊕

$4(2)+10(6)-V_B(12)=0 \quad \therefore V_B = 5,667t$ //

$\sum F_{V=0}$ ↑ ⊕

$V_A - 10 + 5,667 = 0 \quad \therefore V_A = 4,333t$ //

$\sum F_{H=0}$ → ⊕

$H_A = 4t$ //

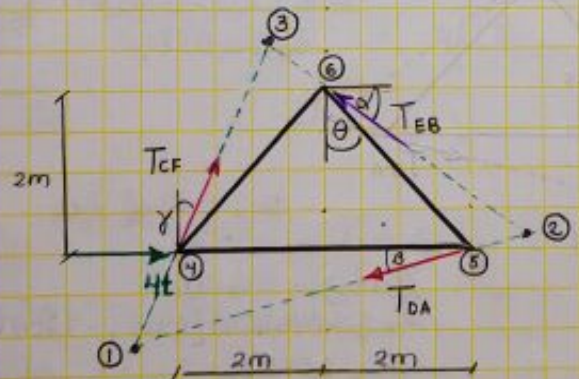
2.- Tensiones compuestas

$\alpha = \text{arctg}\left(\frac{4}{6}\right) = 33,69°$

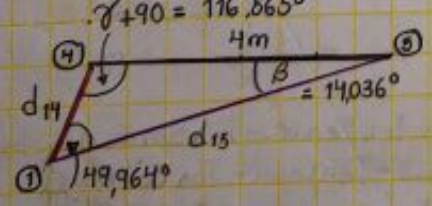
$\beta = \text{arctg}\left(\frac{2}{8}\right) = 14,036°$

$\gamma = \text{arctg}\left(\frac{2}{4}\right) = 26,565°$

$\theta = \text{arctg}\left(\frac{2}{2}\right) = 45°$

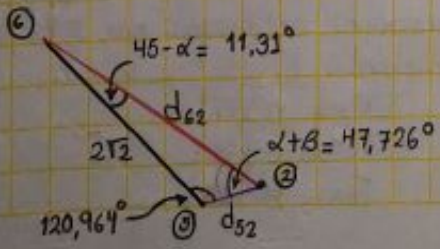


a) Cálculo de Longitudes



$d_{14} = \frac{4m \cdot \text{Sen}(14,036)}{\text{Sen}(49,964)} = 1,267\ m$

$d_{15} = \frac{4 \cdot \text{Sen}(116,565)}{\text{Sen}(49,964)} = 4,673m$

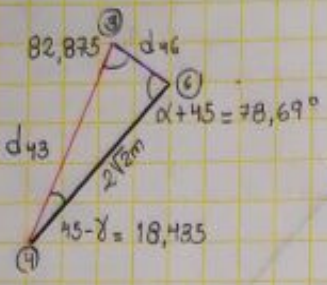


$d_{52} = \frac{2\sqrt{2}}{\text{Sen}(47,726)} \cdot \text{Sen}(11,31) = 0,75m$

$d_{62} = \frac{2\sqrt{2}}{\text{Sen}(47,726)} \cdot \text{Sen}(120,964) = 3,278m$

$$d_{43} = \frac{2\sqrt{2} \cdot \text{Sen}(78,69)}{\text{Sen}(82,875)} = 2,795\,m \;//$$

$$d_{36} = \frac{2\sqrt{2} \cdot \text{Sen}(18,435)}{\text{Sen}(82,875)} = 0,901\,m \;//$$



* Distancias

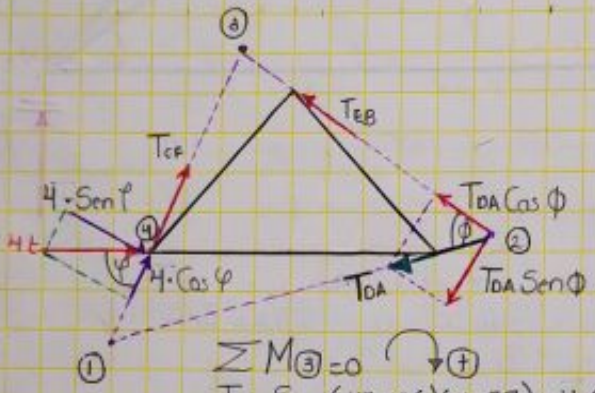
$D_{12} = d_{15} + d_{25} = 4,673m + 0,75m = 5,423\,m$

$D_{13} = d_{14} + d_{43} = 1,267 + 2,795m = 4,062m$

$D_{23} = d_{26} + d_{63} = 3,278\,m + 0,901m = 4,179\,m$

3 Tensiones

* Tensión DA



$\Phi = 47,726°$

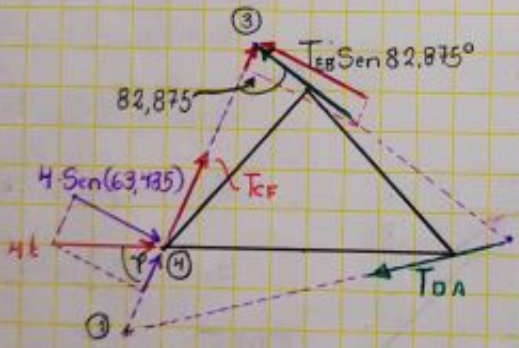
$\wp = 63,435°$

$$\sum M_{③} = 0 \quad \circlearrowright (+)$$

$$T_{DA}\,\text{Sen}(47,726)(4,179) - 4 \cdot \text{Sen}(63,435)(2,795) = 0$$

$$T_{DA} = 3,234t \;//$$

* Tensión EB

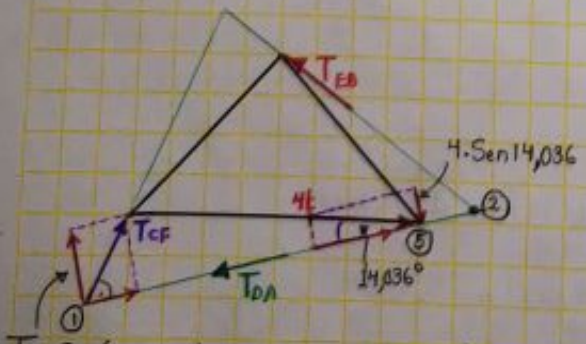


$$\sum M_{①} = 0 \quad \circlearrowright (+)$$

$$4 \cdot \text{Sen}(63,435) \cdot (1,267) - T_{EB} \cdot \text{Sen}\,82,875\,(4,062) = 0$$

$$T_{EB} = 1,125\,t \;//$$

* Tensión CF



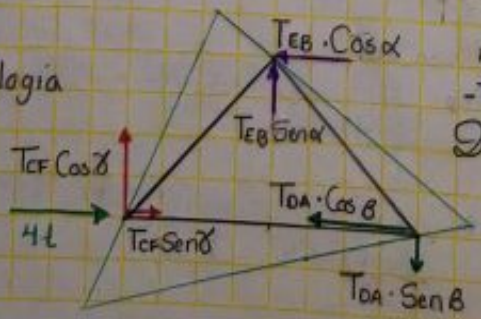
$$\sum M_{②} = 0 \quad \circlearrowright (+)$$

$$T_{CF}\,\text{Sen}(49,964)[5,423] - 4\,\text{Sen}(14,036)(0,75) = 0$$

$$T_{CF} = 0,175\,t \;//$$

$T_{CF}\,\text{Sen}(49,964)$

4.- 2da Metodología



$$\left.\begin{aligned} &T_{CF}\cos\gamma + T_{EB}\,\text{Sen}\,\alpha - T_{DA}\,\text{Sen}\,\beta = 0 \\ &4 + T_{CF}\,\text{Sen}\,\gamma - T_{EB}\cos\alpha - T_{DA}\cos\beta = 0 \\ &-T_{EB}\,\text{Sen}\,\alpha\,(2) - T_{EB}\cos\alpha\,(2) + T_{DA}\,\text{Sen}\,\beta\,(4) = 0 \end{aligned}\right\} 3\text{ ec}$$

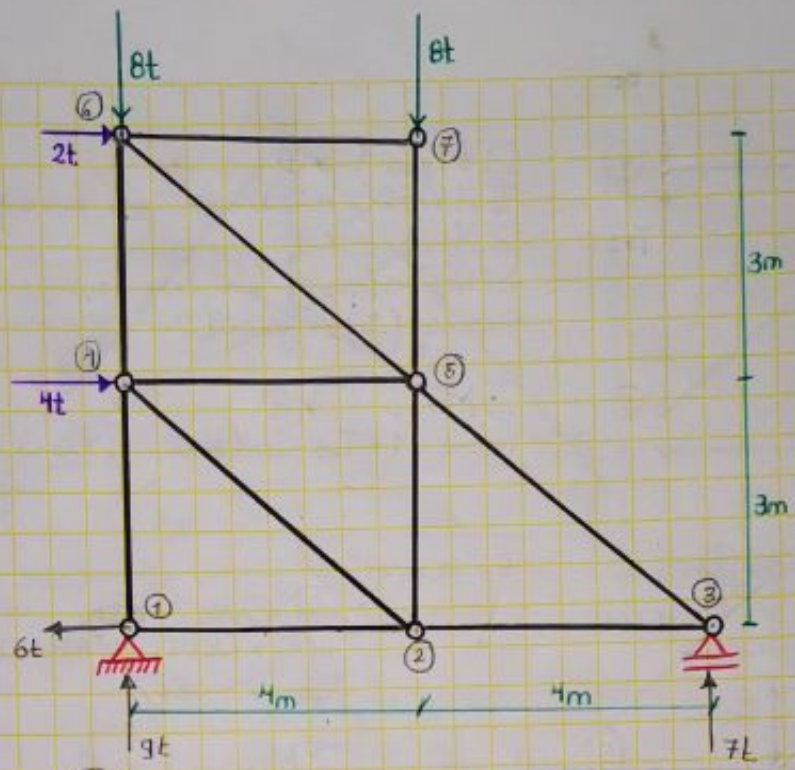
De las 3 ecuaciones obtenidas por ΣF_V, ΣF_H y ΣM obtenemos:

$T_{CF} = 0,177\,t$

$T_{EB} = 1,131\,t$

$T_{DA} = 3,235\,t$

Calcular los esfuerzos



1- Reacciones

$\sum M_{①}=0 \circlearrowright \oplus$

$4(3)+2(6)+8(4)-V_3(8)=0 \quad \therefore V_3=7t$

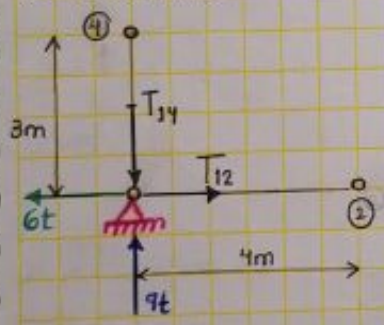
$\sum F_V=0 \uparrow \oplus$

$V_1-8-8+7=0 \quad \therefore V_1=9t$

$\sum F_H=0 \rightarrow \oplus$

$H_1=6t$

2- Tensiones

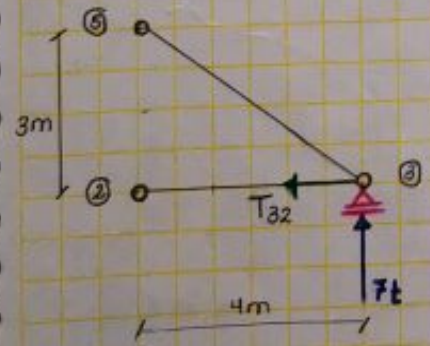


$\sum M_{④}=0 \circlearrowright \oplus$

$6\cdot 3-T_{12}\cdot 3=0 \quad \therefore T_{12}=6t$ //

$\sum M_{②}=0 \circlearrowright \oplus$

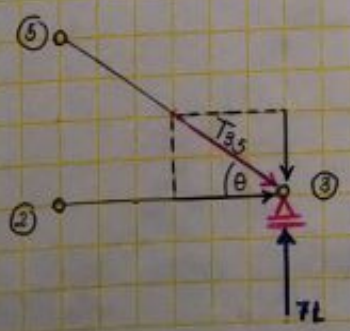
$9(4)-T_{14}(9)=0 \quad \therefore T_{14}=9t$ //



$\sum M_{③}=0 \circlearrowright \oplus$

$T_{32}(3)-7(4)=0$

$T_{32}=9.333t$ //

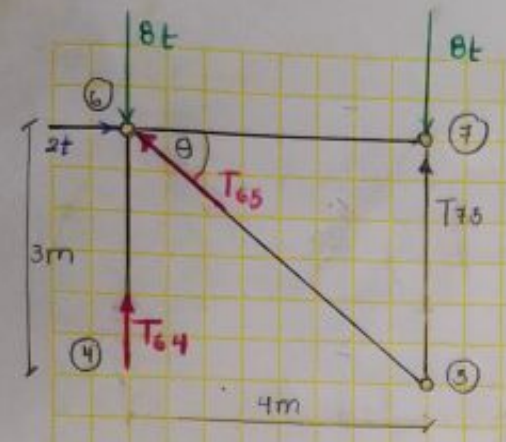


$\theta=\text{arctg}\left(\frac{3}{4}\right)=36,8699°$

$\sum M_{②}=0 \circlearrowright \oplus$

$T_{35}\,\text{Sen}(36,8699)(4)-7(4)=0$

$T_{35}=11,667t$ //



$\sum M_{5=0}$ ↷ ⊕

$T_{64}(4) - 8(4) + 2(3) = 0 \quad \therefore T_{64} = 6{,}5t$ //

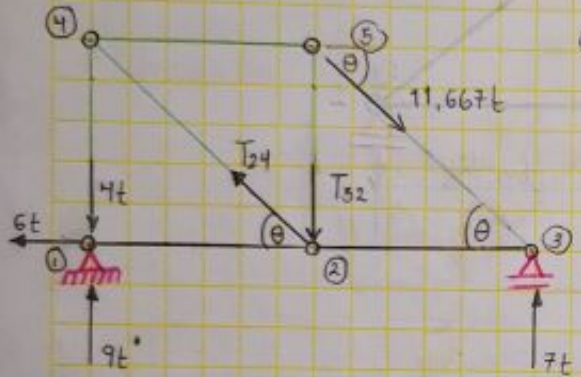
$\sum M_{6=0}$ ↷ ⊕

$8(4) - T_{75}(4) = 0 \quad \therefore T_{75} = 8t$ //

$\theta = \arctg\left(\frac{3}{4}\right) = 36{,}869°$

$\sum M_{7=0}$ ↷ ⊕

$(6{,}5 - 8)\cdot 4 + T_{65}\cdot \operatorname{Sen}(36{,}869)\cdot[4] = 0 \quad \therefore T_{65} = 2{,}5t$ //



$\theta = 36{,}8699°$

$\sum M_{④=0}$ ↷ ⊕

$6(3) + T_{25}(4) - 7(8) + 11{,}667\cdot \operatorname{Sen}(36{,}8699)(4) = 0$

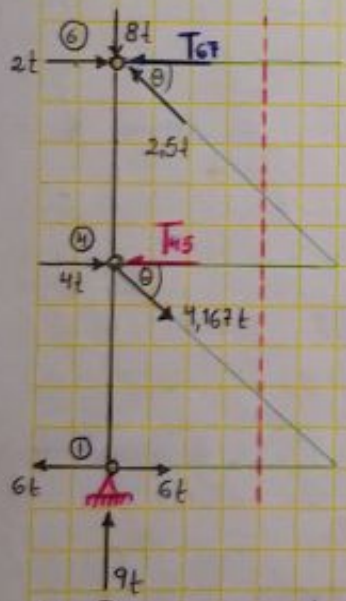
$T_{25} = 2{,}4998t$ //

$\sum M_{①=0}$ ↷ ⊕

$2{,}4998\cdot(4) - 7(8) + 11{,}667\ \operatorname{Sen}(36{,}8699)(8) - T_{24}*$

$* \operatorname{Sen}(36{,}8699)(4) = 0$

$T_{24} = 4{,}167t$ //



$\sum M_{②=0}$ ↷ ⊕

$T_{45}(3) - 4(3) - 4{,}167(\operatorname{Cos}(36{,}8699))\cdot 3 = 0$

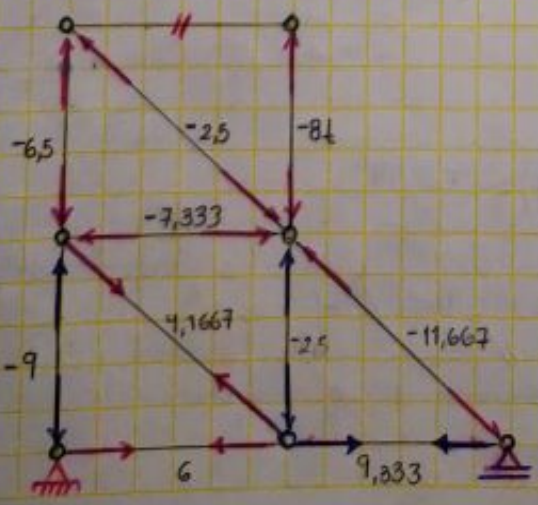
$T_{45} = 7{,}333t$ //

$\sum M_{④=0}$ ↷ ⊕

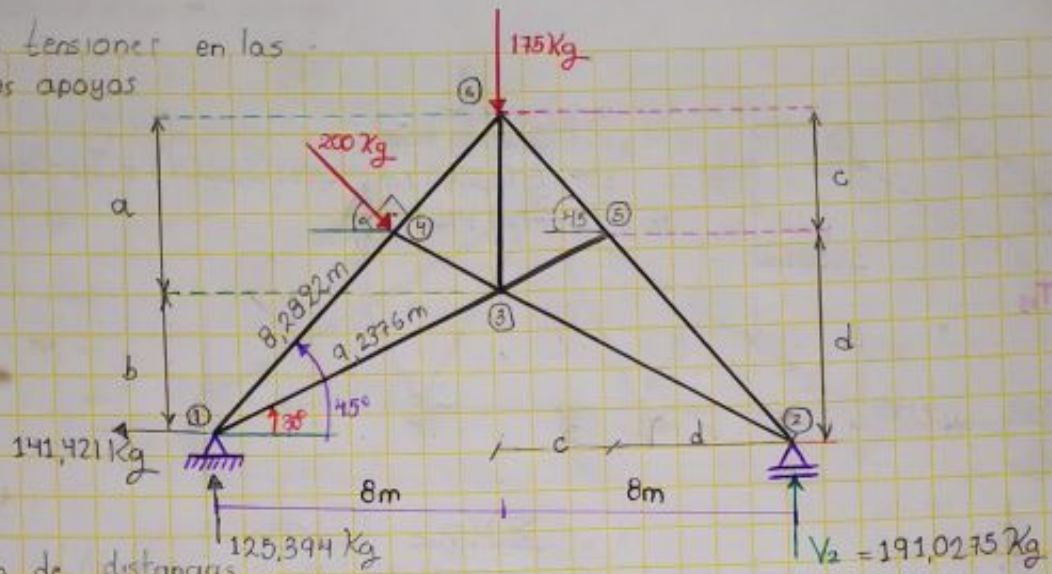
$2(3) - 2{,}5\operatorname{Cos}(36{,}8699)(3) - T_{67}(3) = 0$

$T_{67} = 0$ //

3 - Diagrama de Tensiones



Hallar las tensiones en las barras de los apoyos



1. Cálculo de distancias

$Tag\,30 = \frac{b}{8}$ ∴ $b = 8\,Tg\,30$

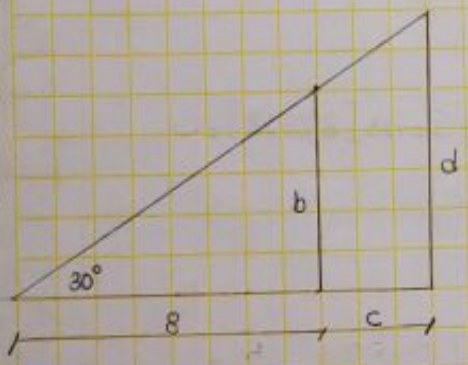
$b = 4,6188\ m$ //

$Tag\,45 = \frac{a + 4,6168}{8}$ ∴ $a = 8\,Tg\,45 - 4,6168$

$a = 3,3812\ m$ //

$c + d = a + b$

① $c + d = 8\ m \Rightarrow c = 8 - d$



$\frac{d}{8+c} = \frac{8\,Tg\,30}{8}$

$8d = 8\,Tg\,30\,(8 + c)$

$8d = 64\,Tg\,30 + 8c\,Tag\,30$

$8d = 64\,Tg\,30 + 8(8-d)Tag\,30$

$d = 5,8564$

$c = 8 - 5,8564$

$c = 2,1436$ //

2. Reacciones

$\sum M_{①} = 0$ ↻ (+)

$200(8,2822) + 175(8) - V_2(16) = 0$

$V_2 = 191,0275\ Kg$

$\sum F_V = 0$ ↑ (+)

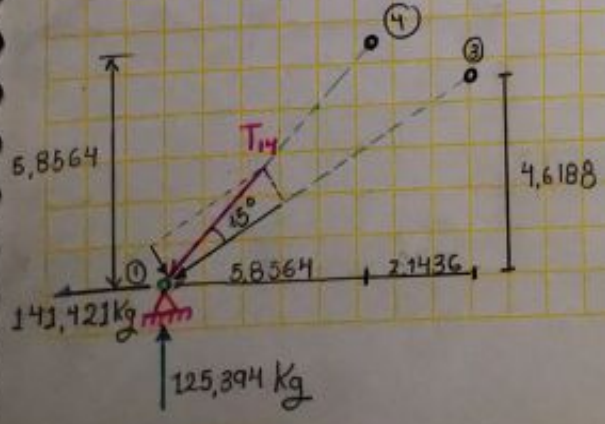
$V_1 - 200\,Sen\,45 - 175 + 191,0275 = 0$

$V_1 = 125,3938\ Kg$

$\sum F_H = 0$ → (+)

$H_1 = 200\,Cos\,45 = 141,421\ Kg$

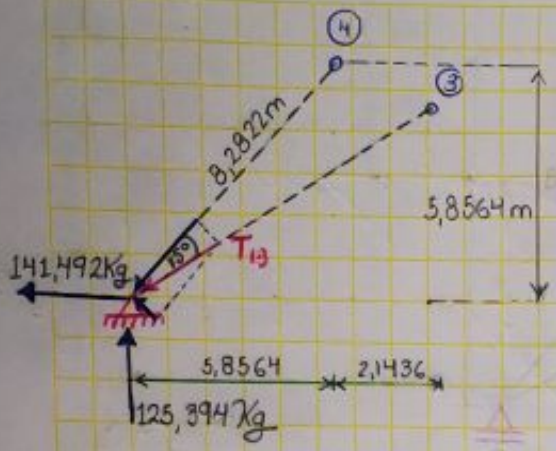
3. Tensiones



$\sum M_{③} = 0$ ↻ (+)

$125,394(8) + 141,421(4,6188) - T_{14} \cdot Sen\,15 \cdot (9,2376) = 0$

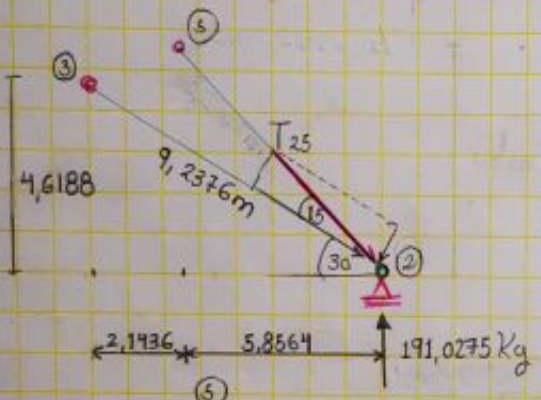
$T_{14} = 692,781\ Kg$



$\sum M_{④}=0 \circlearrowright +$

$125,894(5,8564) + 141,492(5,8564) + T_{13} \cdot Sen\,15\,(8,2822)=0$

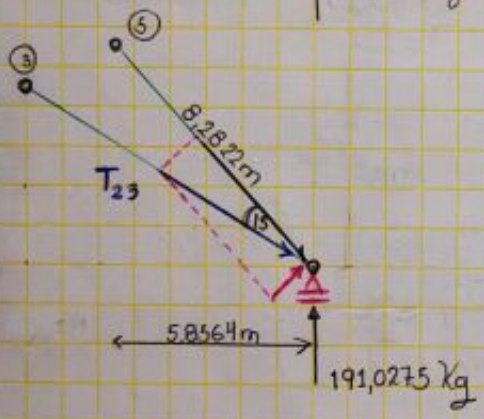
$T_{13} = -729,146\ Kg$



$\sum M_{③}=0 \circlearrowright +$

$-191,0275\,(8) + T_{25}\,Sen\,15\,(9,2376)=0$

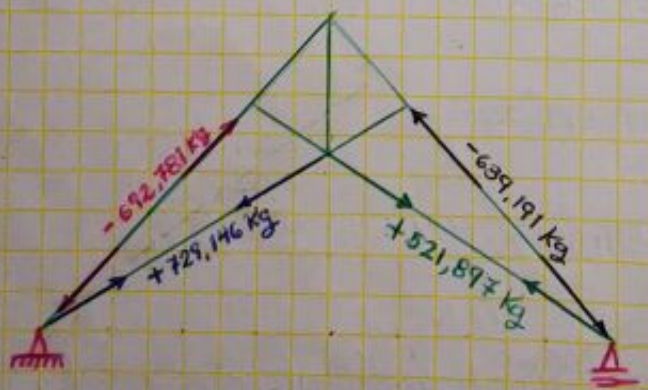
$T_{25} = 639,191\ Kg$



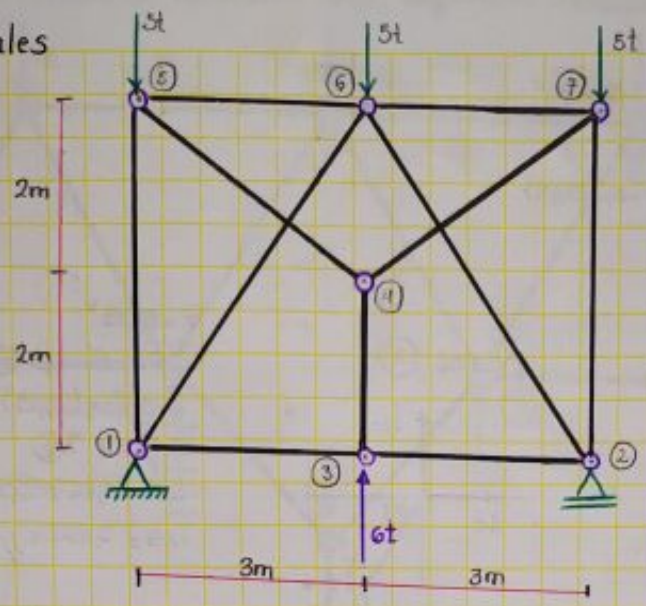
$\sum M_{⑤}=0 \circlearrowright +$

$-191,0275\,(5,8564) - T_{23}\,Sen\,15\,[8,2822]=0$

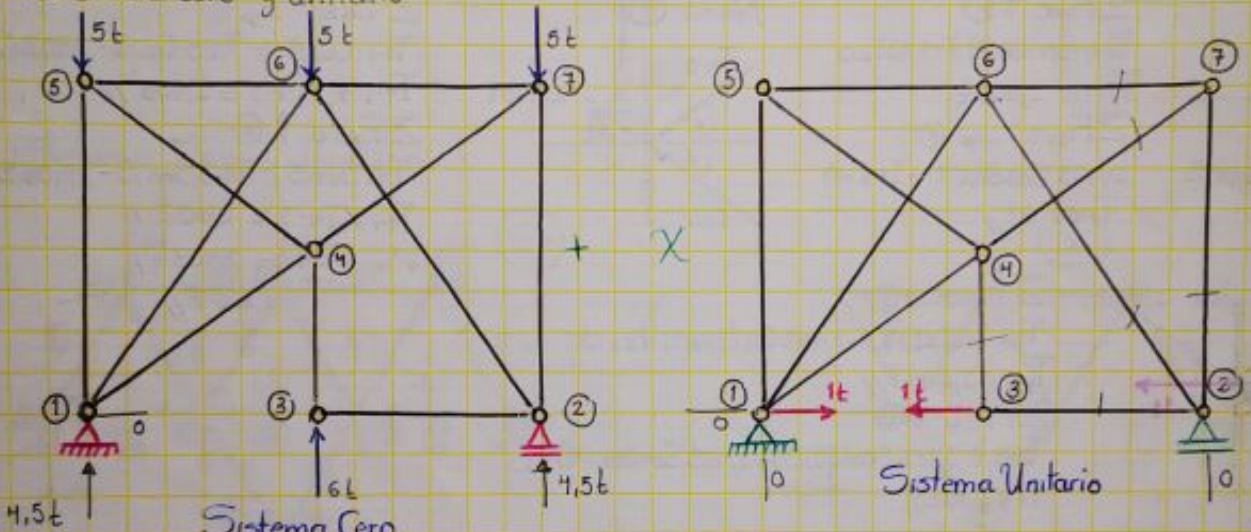
$T_{23} = -521,897\ Kg$



Calcular las Normales

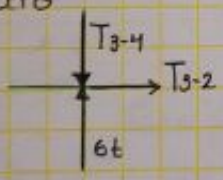


1- Sistema cero y unitario



2- Sistema Cero

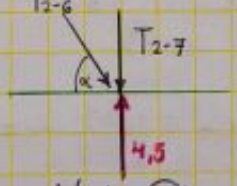
Nudo ③



$T_{3-4} = 6t$

$T_{3-2} = 0t$

Nudo ②



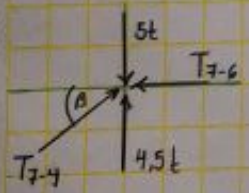
$\sum F_H = 0 \rightarrow \oplus$

$T_{2-6} \cos\alpha = 0 \quad \therefore T_{2-6} = 0$

$\sum F_V = 0 \uparrow \oplus$

$T_{2-7} = 4,5t$

Nudo ⑦



$\beta = \text{arctg}(2/3) = 33,69°$

$\sum F_V = 0 \uparrow \oplus$

$T_{7-4} \cdot \text{Sen}(33,69) + 4,5 - 5 = 0$

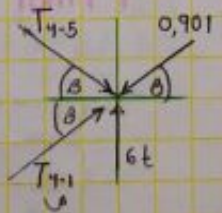
$T_{7-4} = 0,901\ t$ //

$\sum F_H = 0 \rightarrow \oplus$

$+ T_{7-4} \cos(33,69) - T_{7-6}$

$T_{7-6} = 0,7497\ t$ //

Nudo ④



$\sum F_V = 0 \uparrow \oplus$

$T_{4-1} \text{Sen}\beta + T_{4-5} \text{Sen}\beta + 6 - 0,901 \cdot \text{Sen}\beta = 0$

$T_{4-1} - T_{4-5} = -9,916$

$\sum F_H = 0 \rightarrow \oplus$

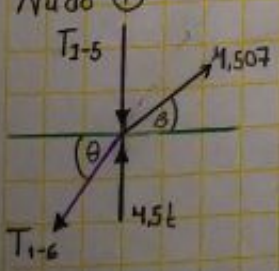
$T_{41} \cos\beta + T_{4-5} \cos\beta - 0,901 \cdot \cos\beta = 0$

$T_{41} + T_{4-5} = 0,901$

$T_{41} = -4,507$ //

$T_{4-5} = 5,408$ //

Nudo ①



$\theta = \text{arctg}(4/3) = 53,13°$

$\sum F_H = 0 \rightarrow \oplus$

$-T_{1-6} \cos(53,13) + 4,507 \cdot \cos(33,69) = 0$

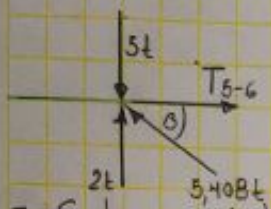
$T_{1-6} = 6,25\ t$ //

$\sum F_V = 0 \uparrow \oplus$

$-6,25 \cdot \text{Sen}(53,13) + 4,5 + 4,507 \cdot \text{Sen}(33,69) - T_{1-5} = 0$

$T_{1-5} = 2t$ //

Nudo ⑤



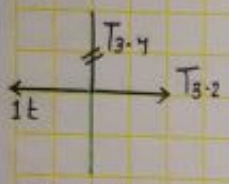
$\Sigma F_H = 0 \rightarrow +$

$T_{5-6} = 5{,}408 \cos(33{,}69)$

$T_{5-6} = 4{,}5\,t$

3 - Sistema Unitario

Nudo ③



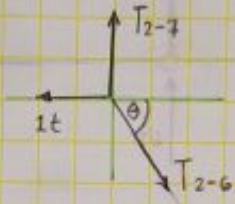
$\Sigma F_V = 0 \uparrow +$

$T_{3-4} = 0\,t$

$\Sigma F_H = 0 \rightarrow +$

$T_{3-2} = 1\,t$ //

Nudo ②



$\theta = 53{,}13°$

$\Sigma F_H = 0 \rightarrow +$

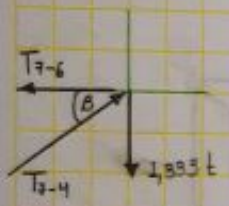
$T_{2-6} \cdot \cos(53{,}13) - 1 = 0 \quad \therefore T_{2-6} = 1{,}667\,t$

$\Sigma F_V = 0 \uparrow +$

$T_{2-7} - 1{,}667 \cdot \operatorname{Sen}(53{,}13) = 0$

$T_{2-7} = 1{,}333\,t$ //

Nudo ⑦



$\beta = 33{,}69°$

$\Sigma F_V = 0 \uparrow +$

$T_{7-4} \operatorname{Sen}(33{,}69) - 1{,}333 = 0$

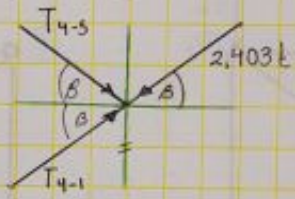
$T_{7-4} = 2{,}403\,t$

$\Sigma F_H = 0 \rightarrow +$

$2{,}403 \cos(33{,}69) - T_{7-6} = 0$

$T_{7-6} = 2\,t$ //

Nudo ④



$\Sigma F_H = 0 \rightarrow +$

$T_{4-1} \cdot \cos\beta + T_{4-5} \cos\beta - 2{,}403 \cos\beta =$

$T_{4-1} + T_{4-5} = 2{,}403$ //

$\Sigma F_V = 0 \uparrow +$

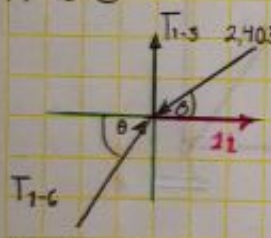
$T_{41} \operatorname{Sen}\beta - T_{4-5} \operatorname{Sen}\beta - 2{,}403 \operatorname{Sen}$

$T_{4-1} - T_{4-5} = 2{,}403$ //

$\therefore \quad T_{4-1} = 2{,}403\,t$ //

$T_{4-5} = 0\,t$ //

Nudo ①



$\Sigma F_H = 0 \rightarrow +$

$T_{1-6} \cos(53{,}13) - 2{,}403 \cos(33{,}69) + 1 = 0$

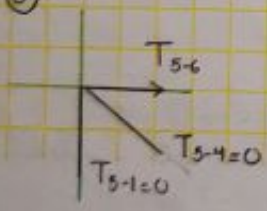
$T_{1-6} = 1{,}666\,t$ //

$\Sigma F_V = 0 \uparrow +$

$T_{1-5} + 1{,}666 \operatorname{Sen}(53{,}13) - 2{,}403 \operatorname{Sen}(33{,}69) = 0$

$T_{1-5} = 0\,t$ //

Nudo ⑤



$T_{5-6} = 0\,t$ //

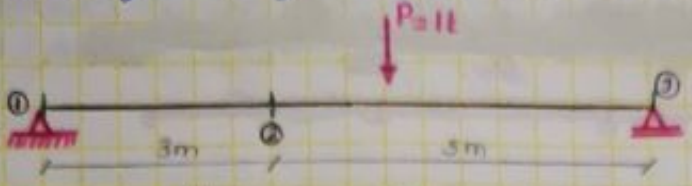
4 - Cálculo de χ

5 - Tensiones Finales

$$\chi = \frac{-T^{\circ}_{1-4}}{T^{\circ}_{1-4}} = \frac{-4{,}507}{-2{,}403} = 1{,}8756\ //$$

Barra	T "o"	χ	T "U"	T. Final
1-3	0	1,8756	1	1,8756 t
1-5	-2	"	0	-2 t
1-6	-6,25	"	1,667	-3,1234 t
2-3	0	"	1	1,8756 t
2-6	0	"	-1,667	-3,126 t
2-7	-4,5	"	1,333	-2 t
3-4	-6	"	0	-6 t
4-5	-5,408	"	0	-5,408 t
4-7	-0,901	"	-2,403	-5,408 t
5-6	4,5	"	0	4,5 t
6-7	0,7497	"	2	4,5 t

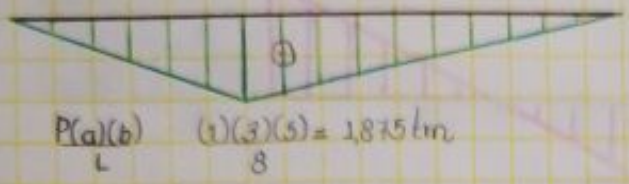
117.- Para la siguiente viga calcular las Líneas de influencias sgtes:



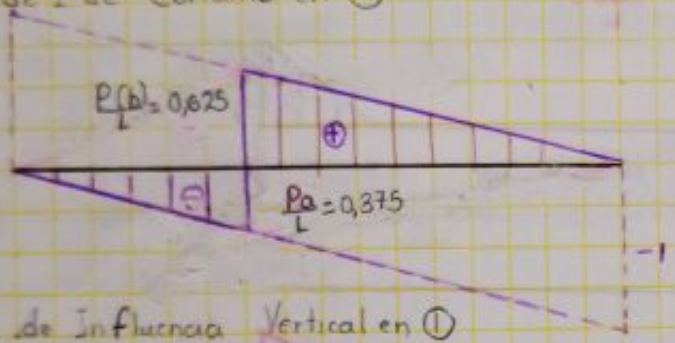
a) M_{2-2}
b) Q_{2-2}
c) V_1
d) V_3

} Utilizar el método Gráfico - Numérico

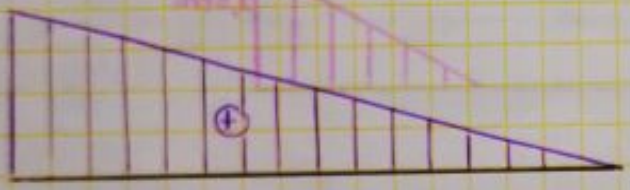
a) Línea de I. de Momentos en ②



b) Línea de I de Cortante en ②

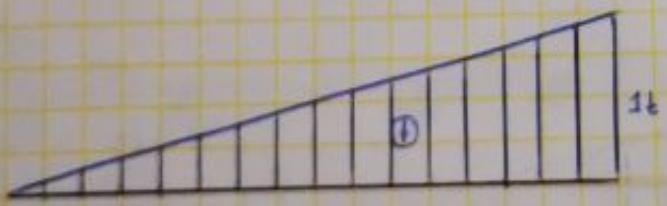


c) Línea de Influencia Vertical en ①



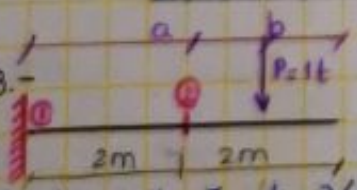
Si P=1t esta en ① ⇒ V_1 = 1t
Si P=1t " " ③ ⇒ V_1 = 0t

d) Línea de Influencia Vertical en ③



Si P=1t esta ① ⇒ V_3 = 0
Si P=1t " ③ ⇒ V_3 = 1t

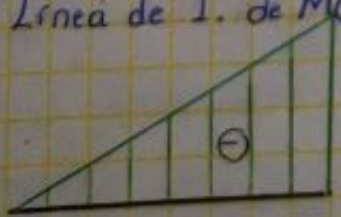
118.- Para la viga mostrada calcular las L.I sgtes:



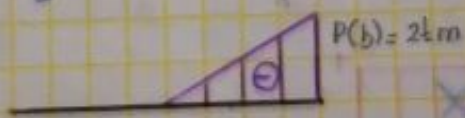
a) V_1
b) M_1
c) M_{2-2}
d) Q_{2-2}

a) Línea de I. de V en ①

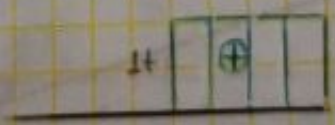
b) Línea de I. de M ①



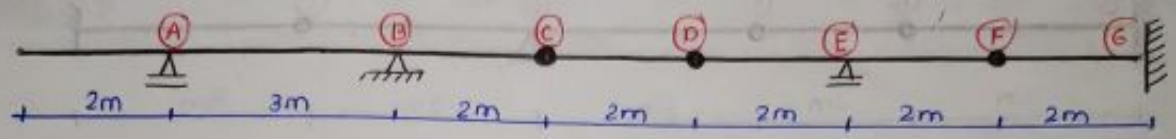
d) Línea de I. de M_{2-2}



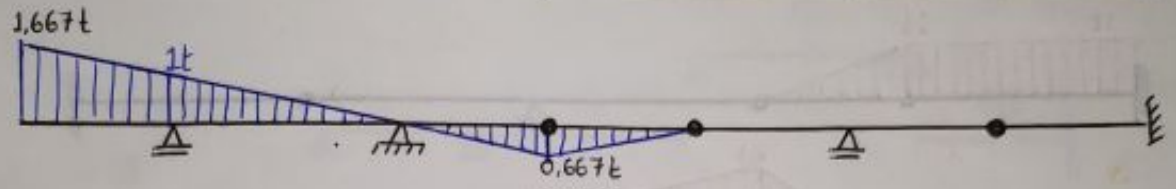
c) Línea de I. de Q_{2-2}



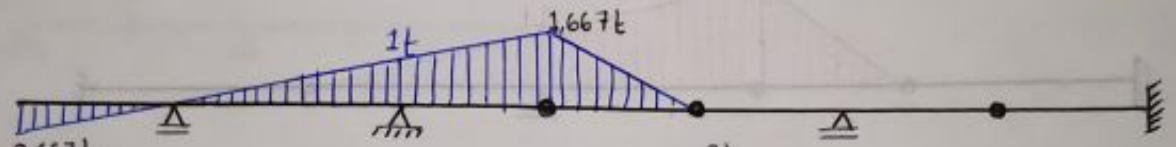
Hallar los siguientes diagramas de Linea de Influencia



V_A

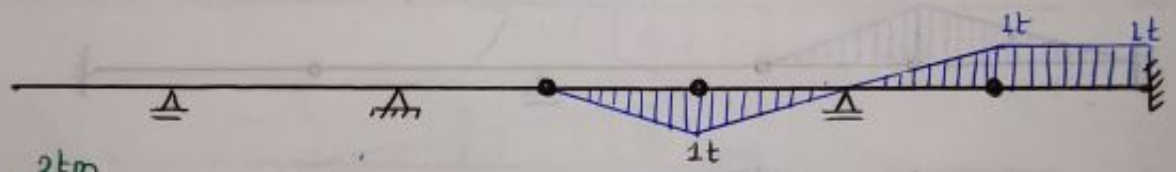


V_B



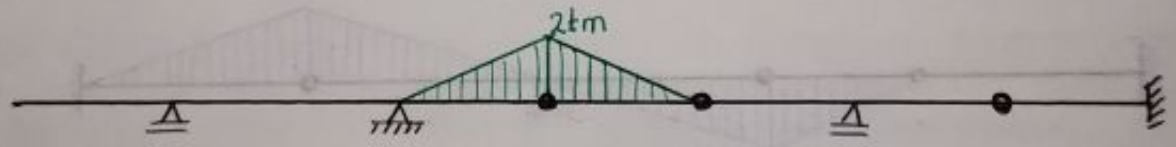
V_E

V_G

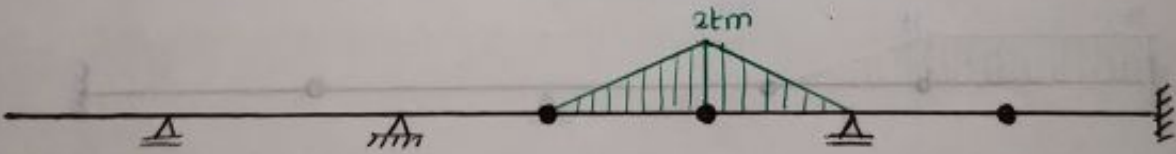


M_A

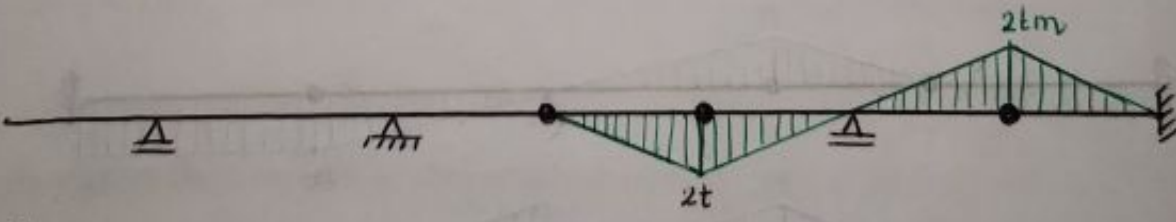
M_B



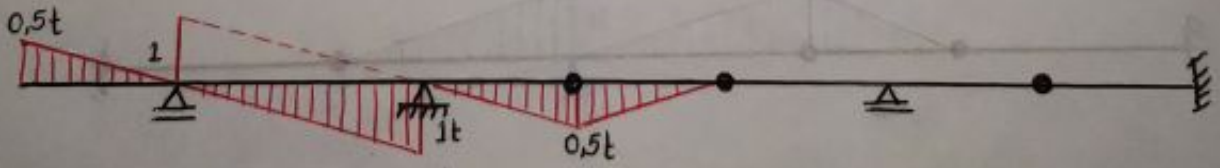
M_E



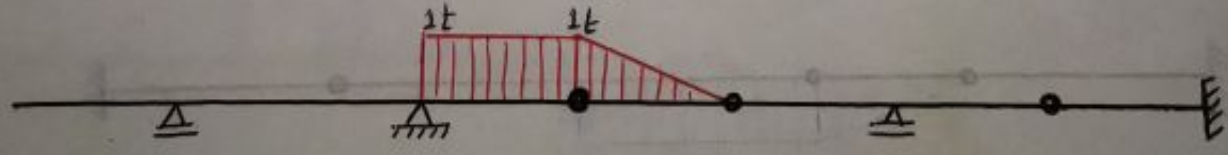
M_G



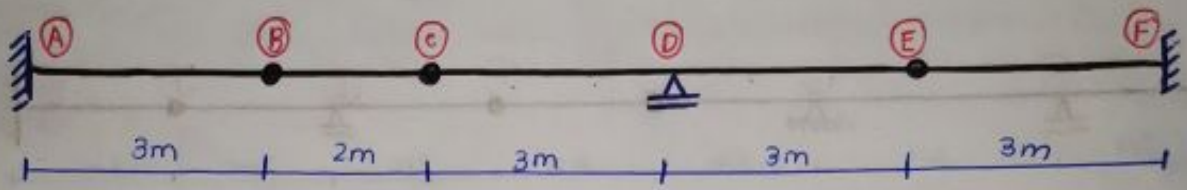
Q_B^{IZQ}



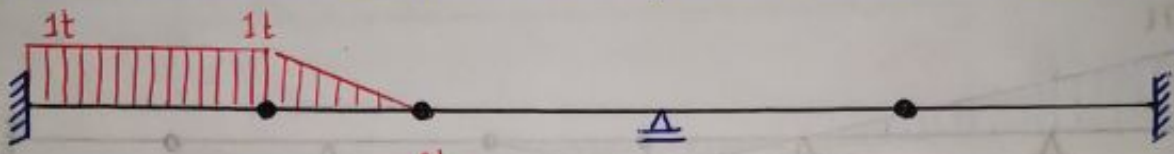
$Q_B^{DER.}$



Hallar las siguientes Líneas de Influencia



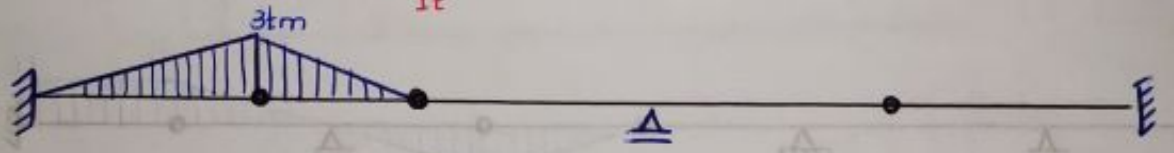
V_A



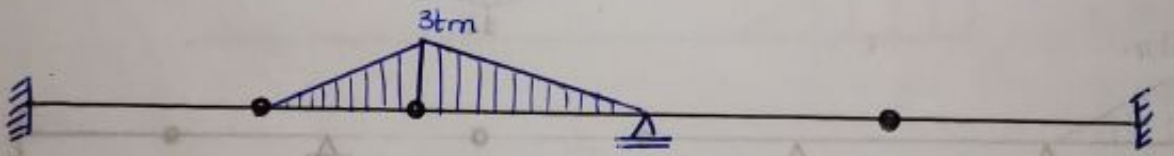
V_D

V_F

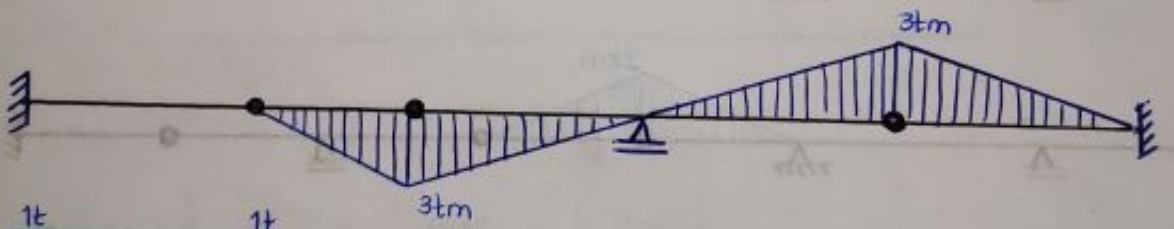
M_A



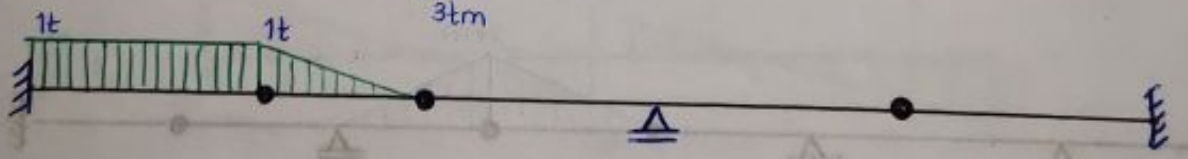
M_D



M_F

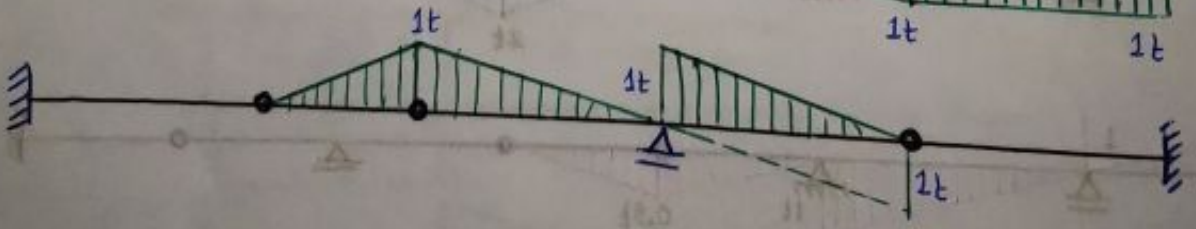


Q_A

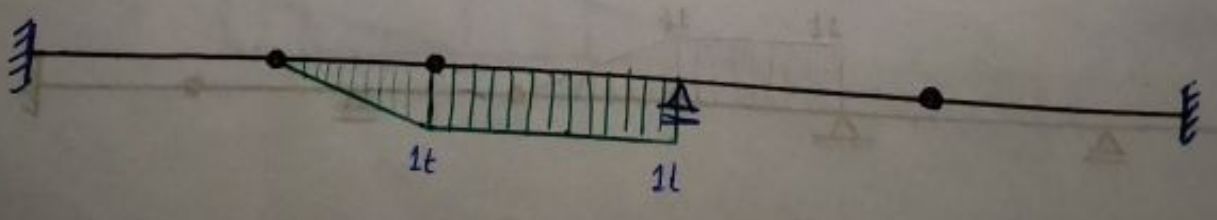


Q_F

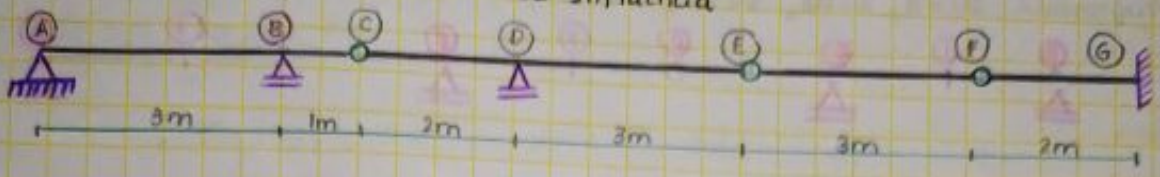
Q_D^{DER}

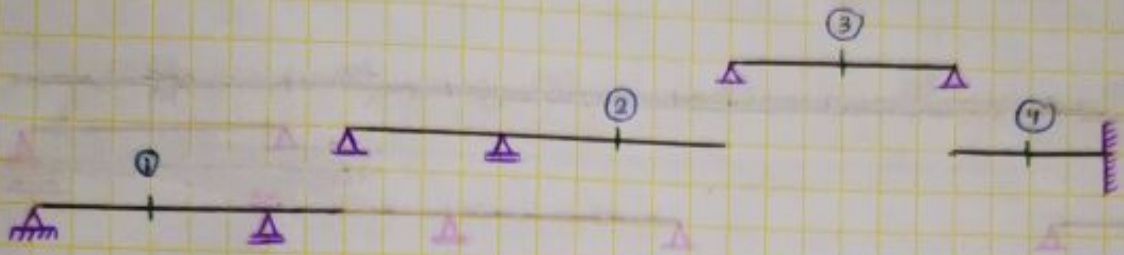


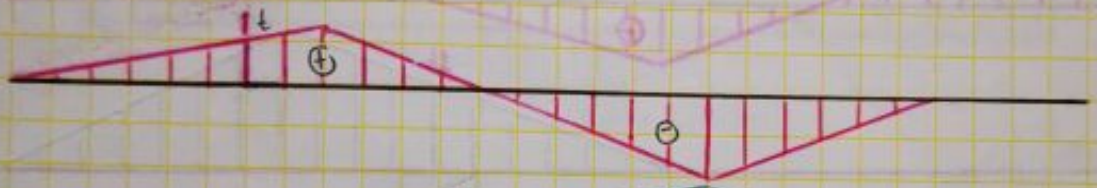
Q_D^{IZQ}

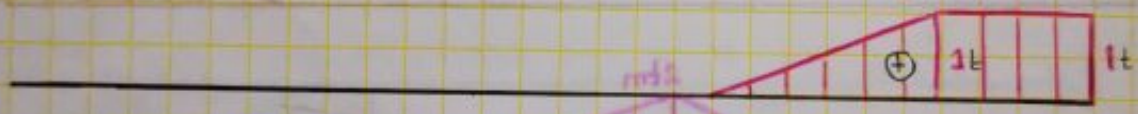


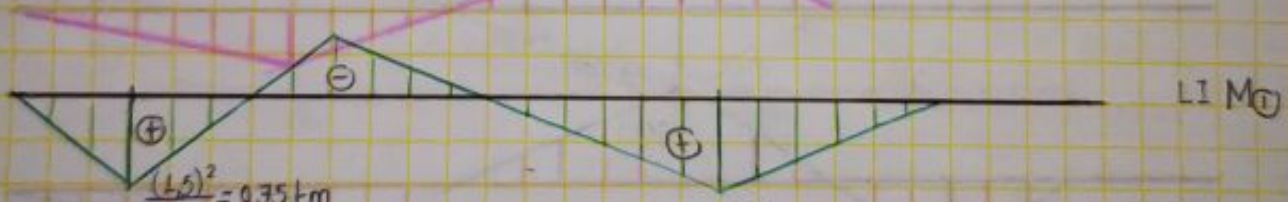
120.- Diagramar las siguientes Líneas de Influencia

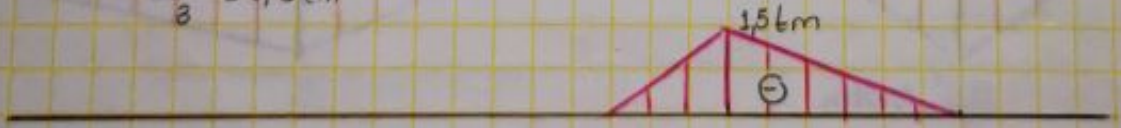


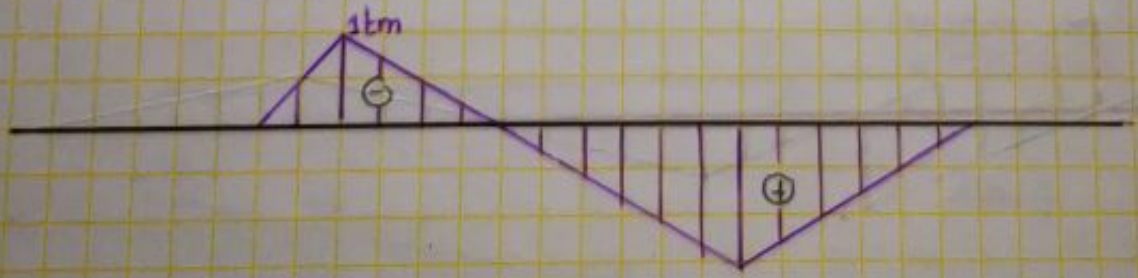


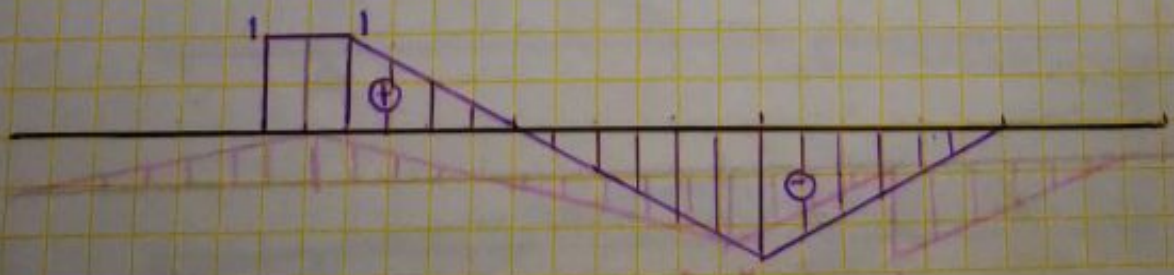










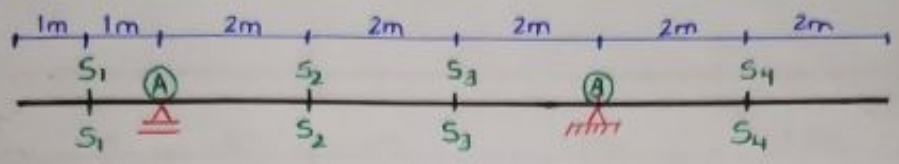


Por el Método Gráfico- Numérico diagramar las Lineas de Influencias siguientes:

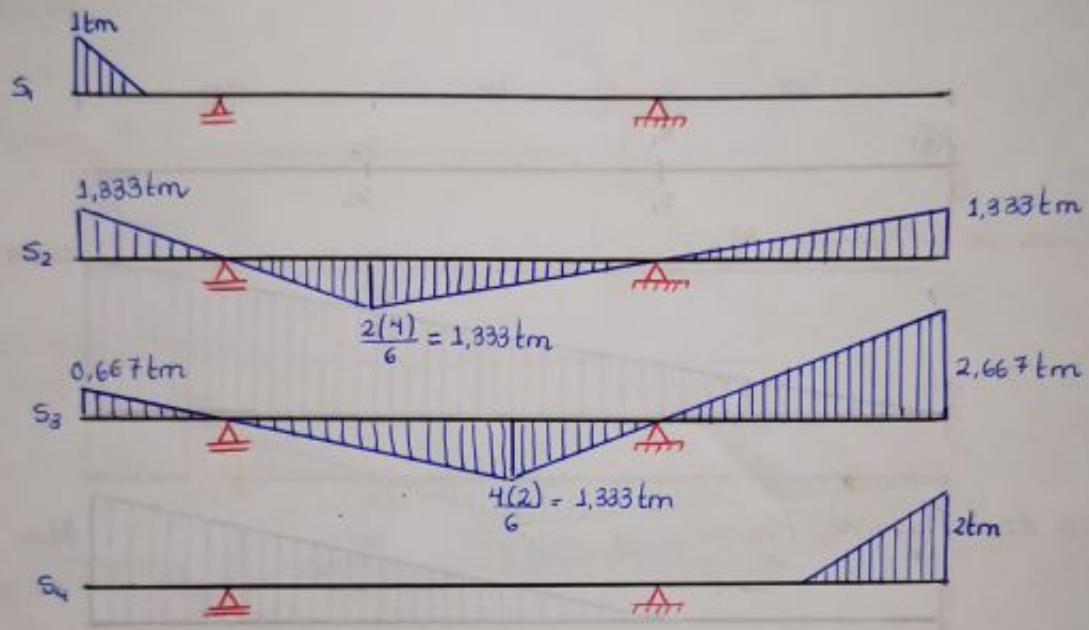
a) Momento - S_1, S_2, S_3, S_4

b) Cortante - S_1, S_2, S_3, S_4

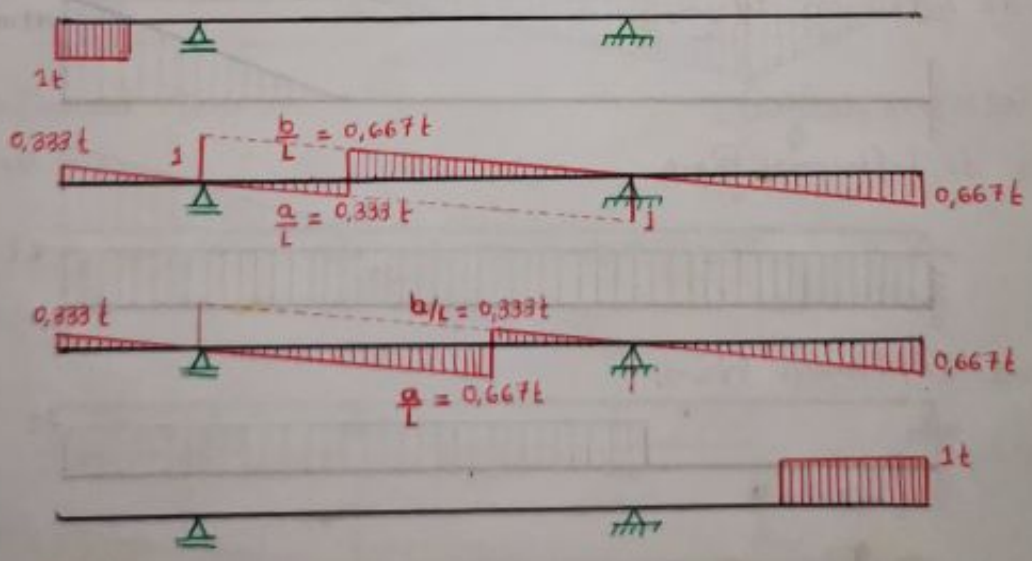
c) Reacción - A, B



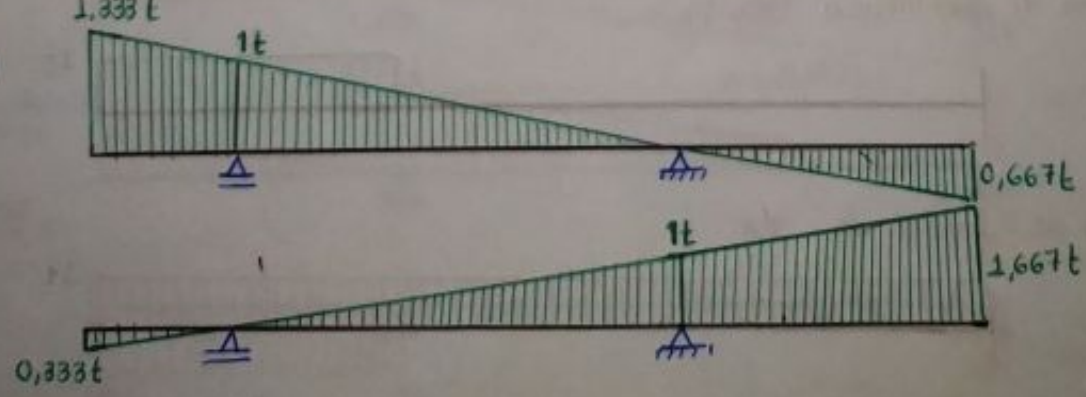
a) Momentos



b) Cortantes

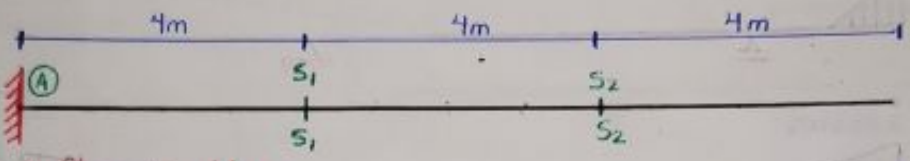


c) Reacción



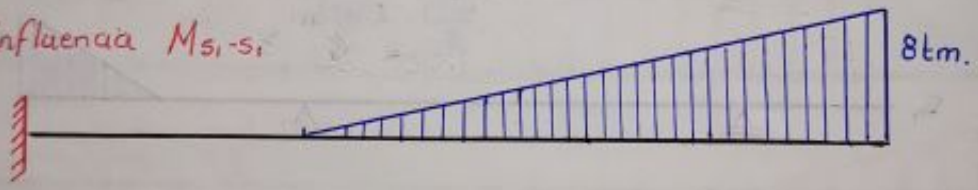
Por el método gráfico numérico diagramar las siguientes Lineas de influencia

a) M_{A-A}
b) $M_{S_1-S_1}$
c) $M_{S_2-S_2}$
d) Q_{A-A}
e) $Q_{S_1-S_1}$
f) $Q_{S_2-S_2}$
g) V_A

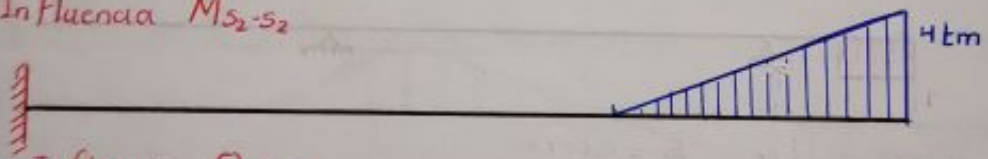


a) Linea de Influencia M_{A-A}

b) Linea de Influencia $M_{S_1-S_1}$

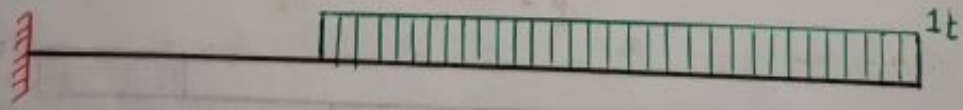


c) Linea de Influencia $M_{S_2-S_2}$

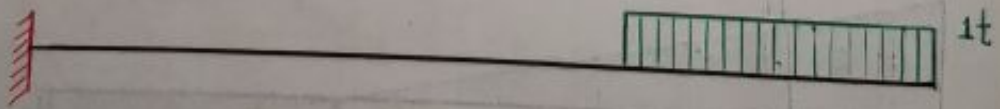


e) Linea de Influencia Q_{A-A}

f) Linea de Influencia $Q_{S_1-S_1}$



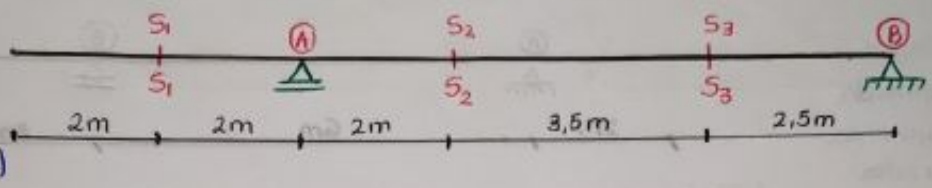
g) Linea de Influencia $Q_{S_2-S_2}$



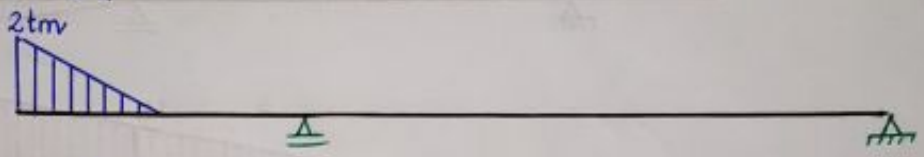
g) Linea de Influencia V_A

Utilizando el metodo Gráfico-Numérico obtener las siguientes Líneas de Influencia:

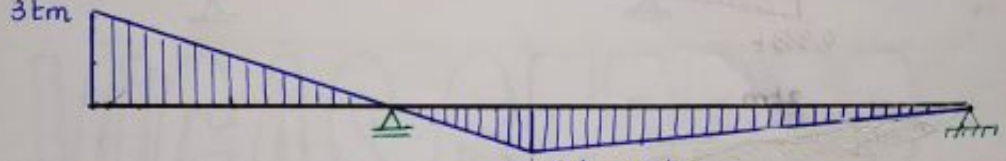
a) $M_{S_1-S_1}$
b) $M_{S_2-S_2}$
c) $M_{S_3-S_3}$
d) M_{A-A}
e) $Q_{S_1-S_1}$
f) $Q_{S_3-S_3}$
g) Q_{A-A} (derecha)
h) Q_{A-A} (izquierda)



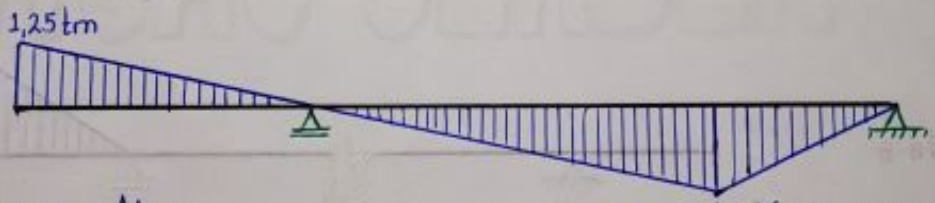
a) Línea de Influencia $M_{S_1-S_1}$



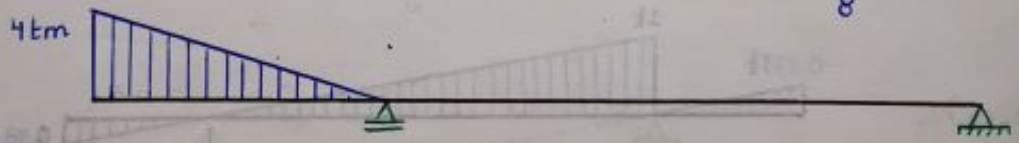
b) Linea de Influencia $M_{S_2-S_2}$



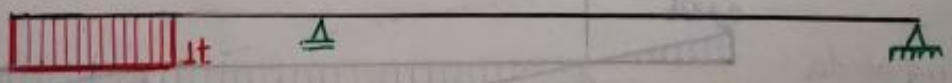
c) Linea de Influencia $M_{S_3-S_3}$



d) Linea de Influencia M_{A-A}



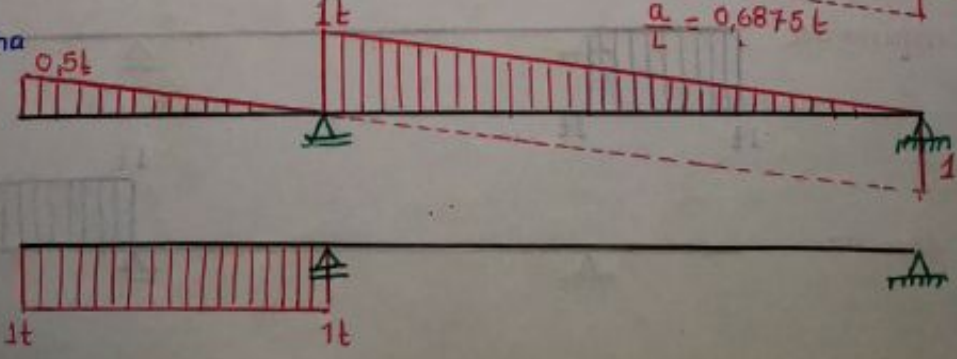
e) linea de I. $Q_{S_1-S_1}$



f) Linea de I. $Q_{S_3-S_3}$

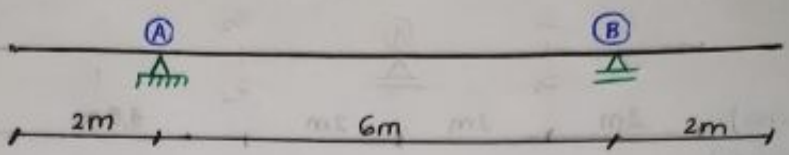
g) linea de I $Q_{AA}^{derecha}$

h) Linea de I. $Q_{A-A}^{IZQ.}$

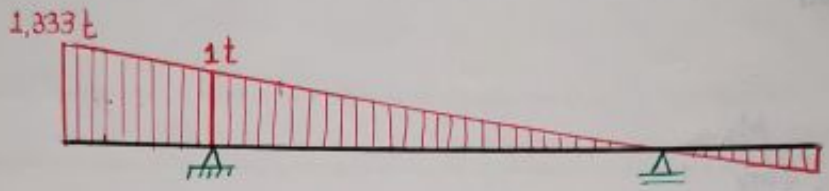


Obtener los diagramas de Linea de influencia siguientes:

a) V_A
b) V_B
c) M_{A-A}
d) M_{B-B}
e) Q_{A-A} derecha
f) Q_{BB} izquierda
g) Q_{A-A} izquierda
h) Q_{B-B} derecha

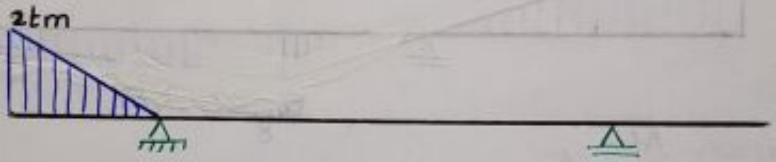


a) L.I. V_A



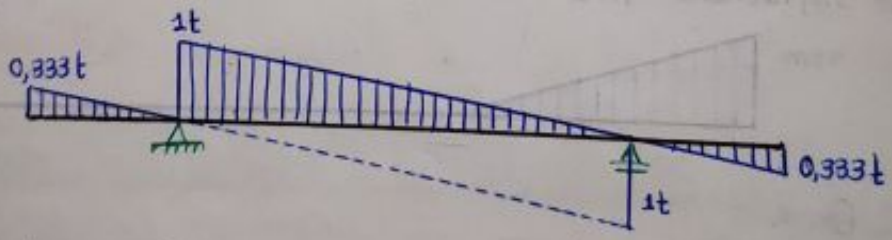
b) L.I. V_B

c) L.I. M_{A-A}

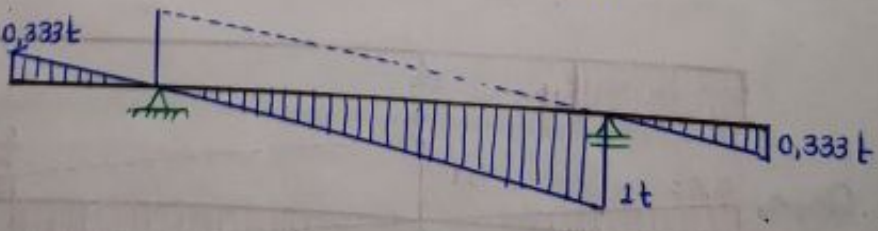


d) L.I. M_{B-B}

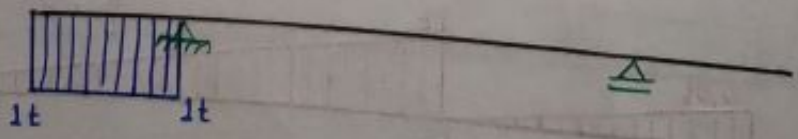
e) Q_{A-A} derecha



f) Q_{B-B} izquierda



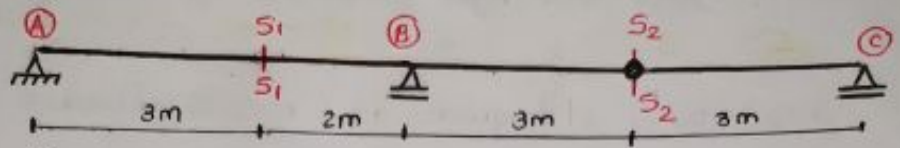
g) Q_{A-A} izquierda



h) Q_{B-B} derecha

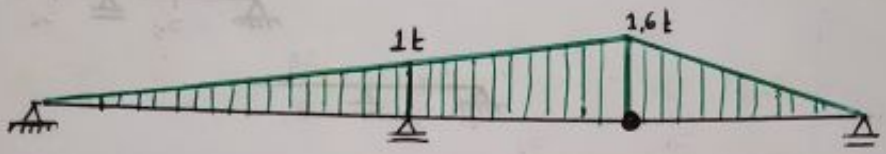
Diagramar las siguientes Líneas de Influencia

a) V_A
b) V_B
c) V_C
d) $M_{S_1-S_1}$
e) $M_{S_2-S_2}$
f) M_B
g) Q_B^{IZQ}
h) Q_B^{DER}

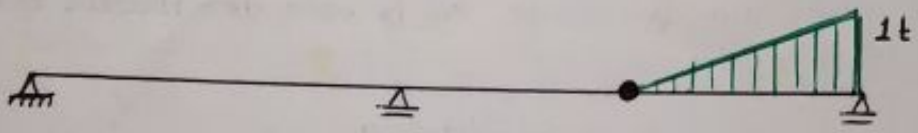


a) L.I. V_A

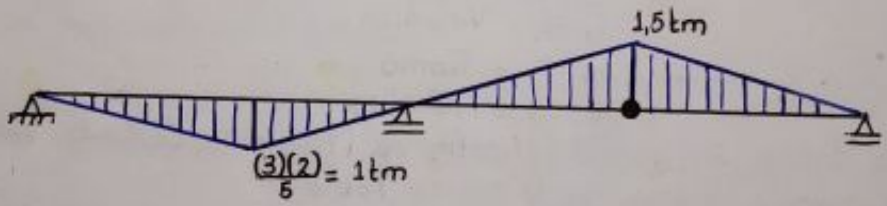
b) L.I. V_B



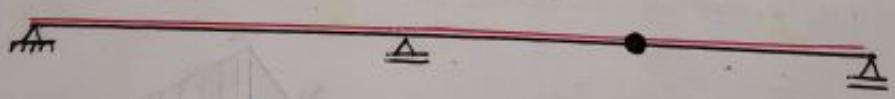
c) L.I. V_C



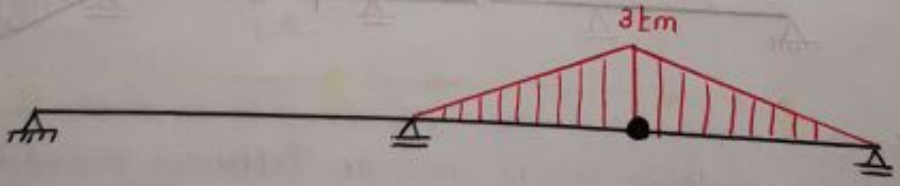
d) L.I. $M_{S_1-S_1}$



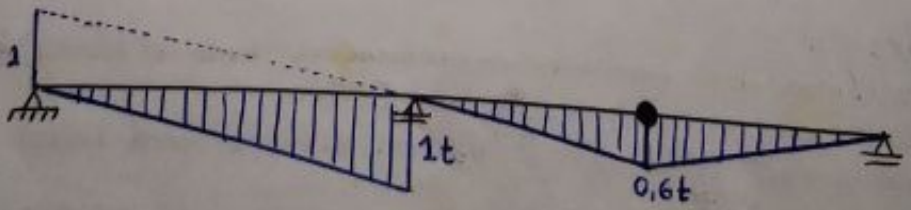
e) L.I. $M_{S_2-S_2}$



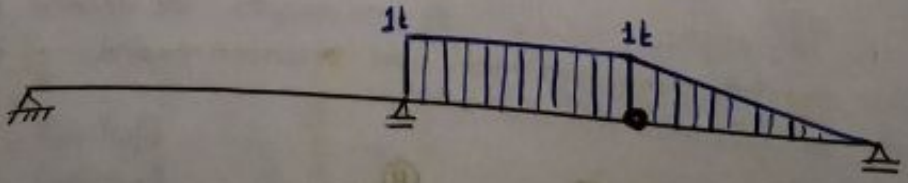
f) L.I. M_B



g) L.I. Q_B^{IZQ}



h) L.I. Q_B^{DER}



Hallar las siguientes Líneas de Influencia

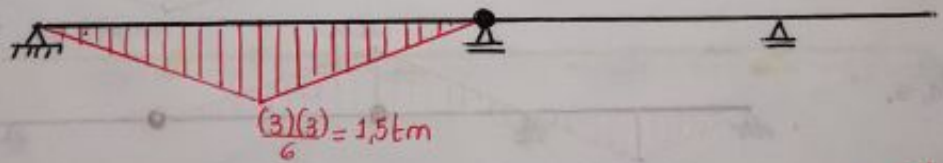
a) V_A
b) V_B
c) V_C
d) $M_{S_1-S_1}$
e) $M_{S_2-S_2}$
f) $M_{S_3-S_3}$
g) $Q_{S_2-S_2}$
h) $Q_{S_1-S_1}$

a) L.I. V_A

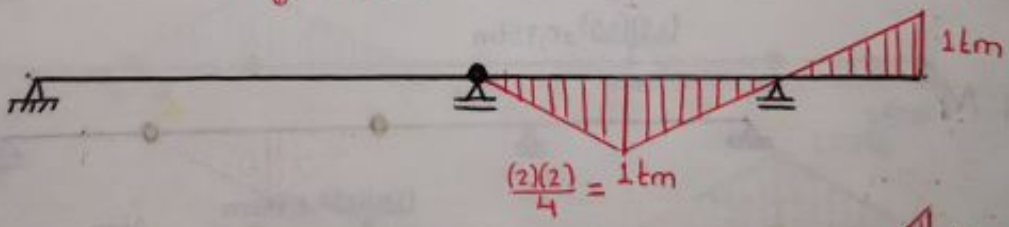
b) L.I. V_B

c) L.I. V_C

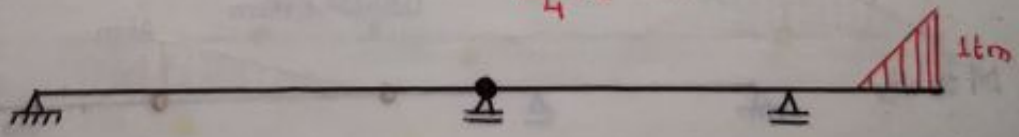
d) L.I. $M_{S_1-S_1}$



e) L.I. $M_{S_2-S_2}$

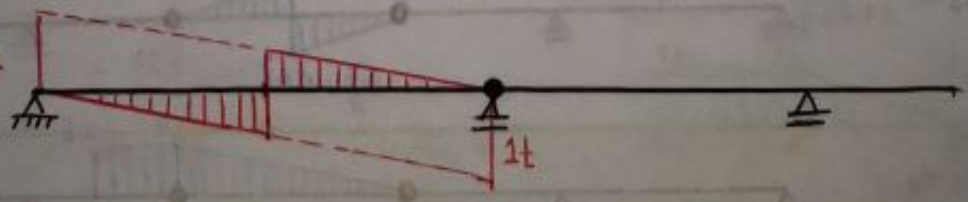


f) L.I. $M_{S_3-S_3}$

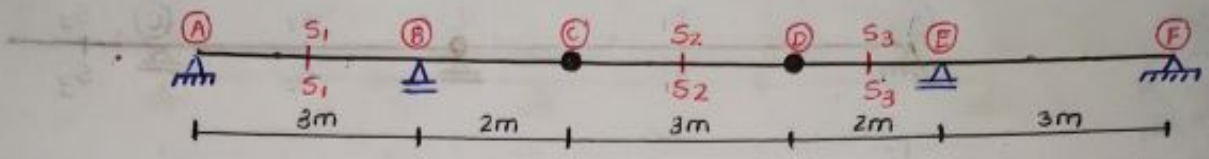


g) L.I. $Q_{S_2-S_2}$

h) L.I. $Q_{S_1-S_1}$

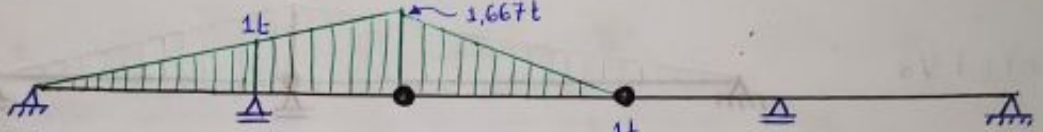


Hallar las siguientes Líneas de Influencia



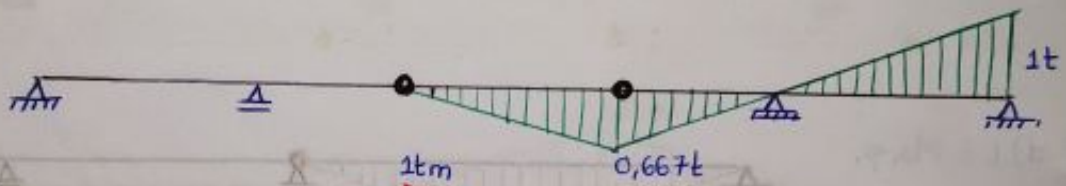
a) L.I. V_A

b) L.I. V_B

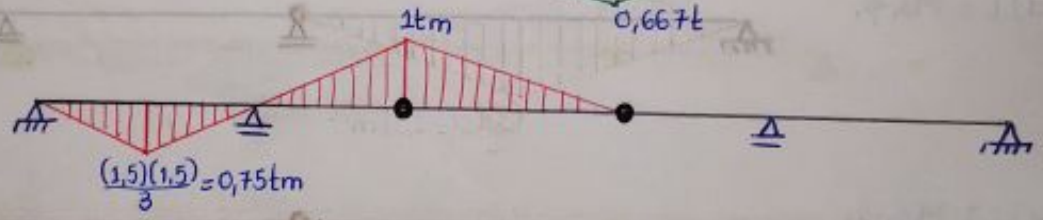


c) L.I. V_E

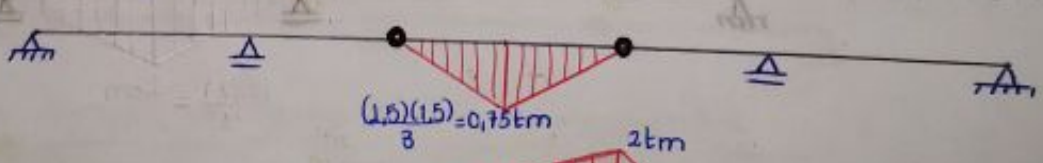
d) L.I. V_F



e) L.I. $M_{S_1-S_1}$

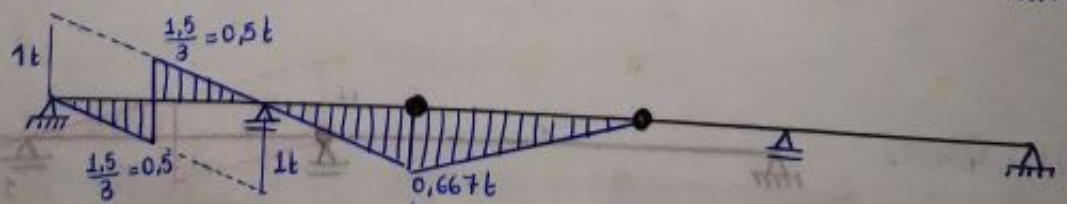


f) L.I. $M_{S_2-S_2}$

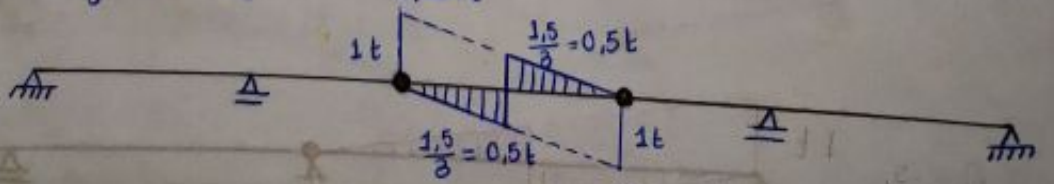


g) L.I. $M_{S_3-S_3}$

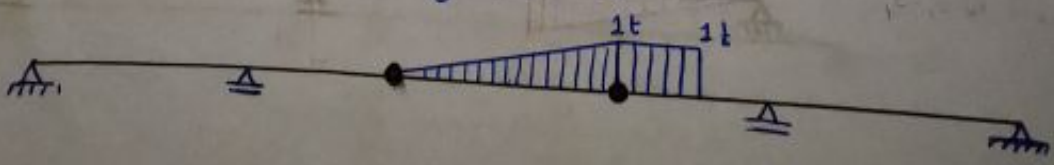
h) L.I. $Q_{S_1S_1}$



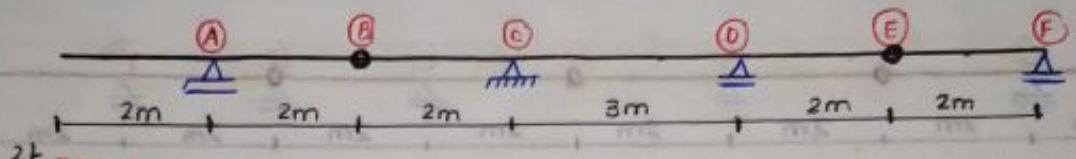
i) L.I. $Q_{S_2-S_2}$



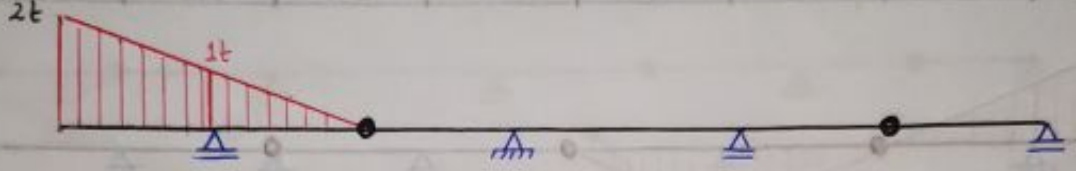
j) L.I. $Q_{S_3-S_3}$



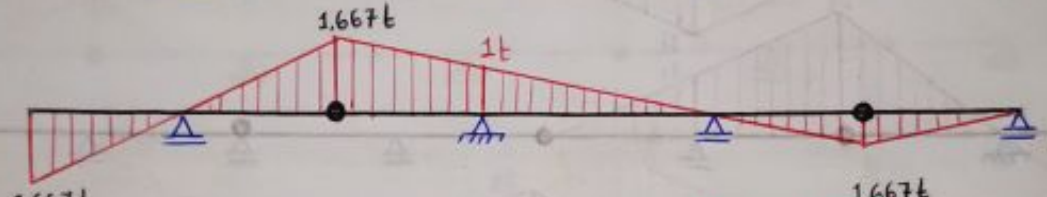
Hallar las siguientes Líneas de Influencia



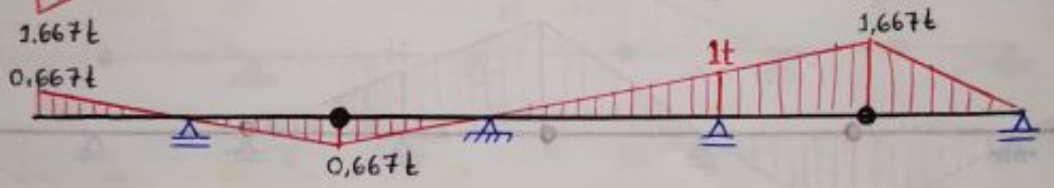
a) L.I. V_A



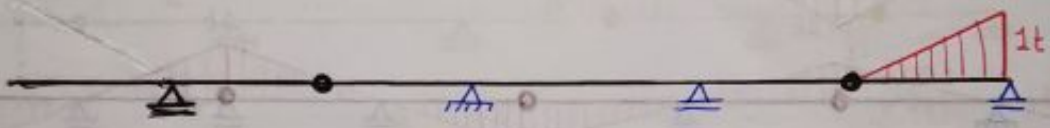
b) L.I. V_C



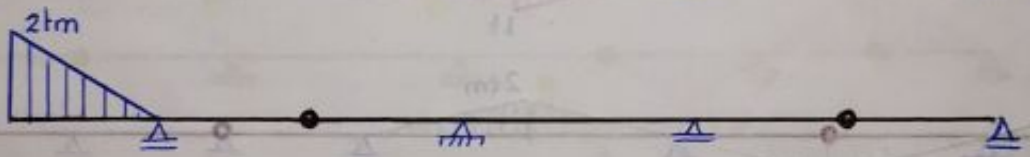
c) L.I. V_D



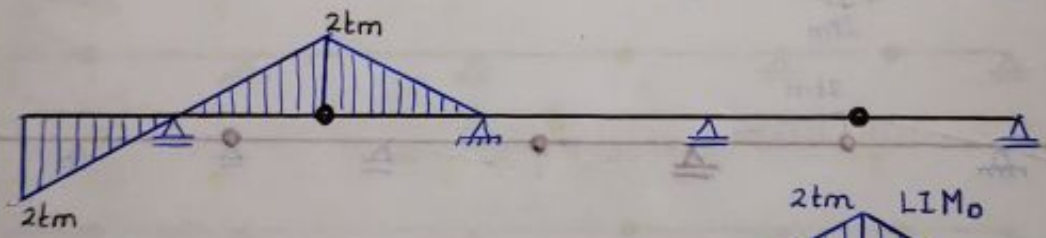
d) L.I. V_F



e) L.I. M_A

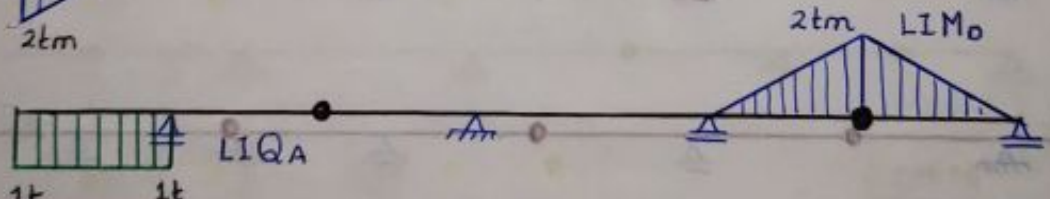


f) L.I. M_C

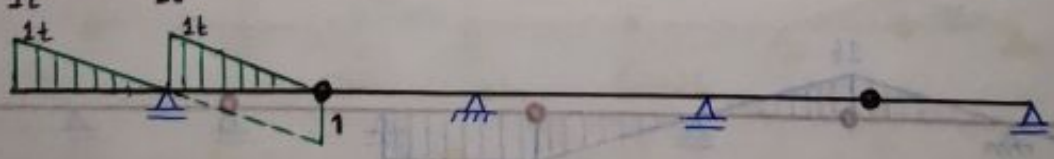


g) L.I M_D

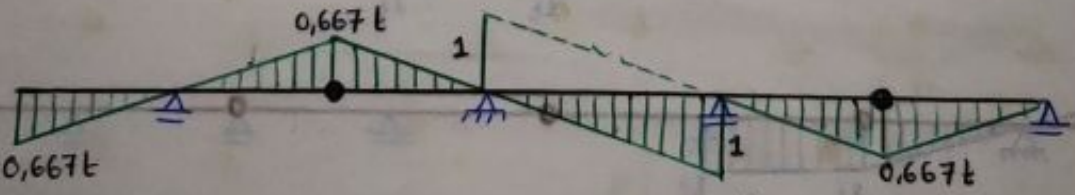
L.I. $Q_{A(IZQ)}$



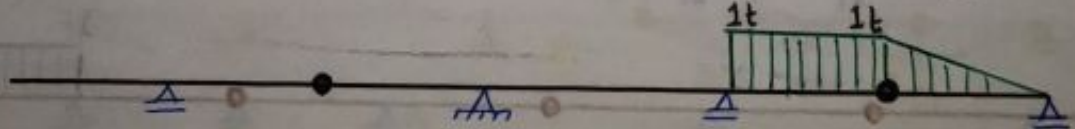
h) L.I $Q_{A\,(DER)}$



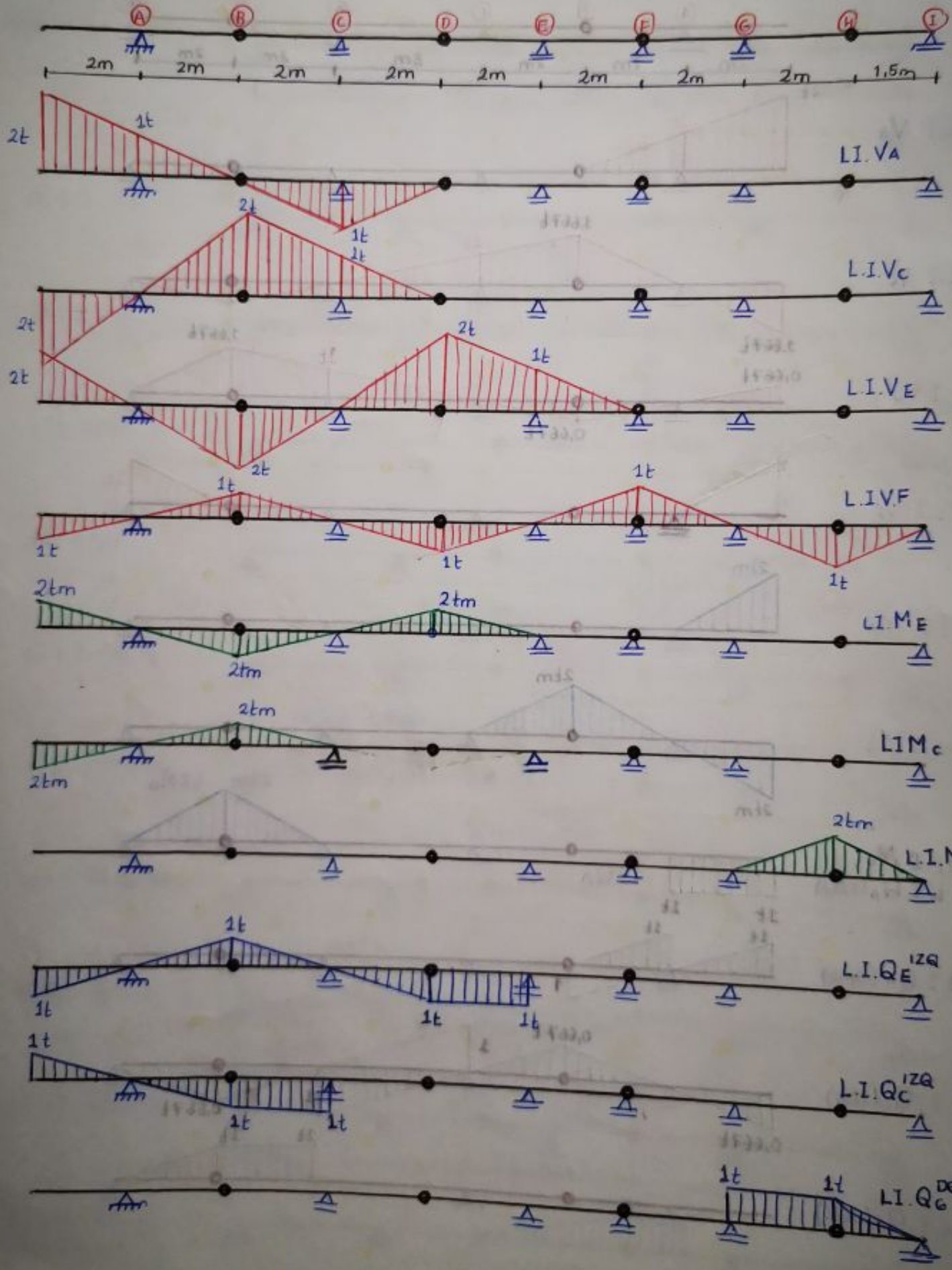
i) L.I. $Q_{D(IZQ)}$



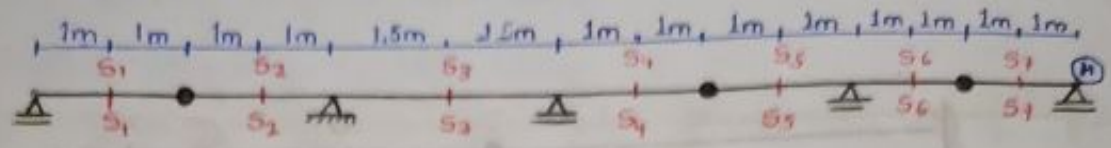
j) L.I. $Q_{D(DER)}$



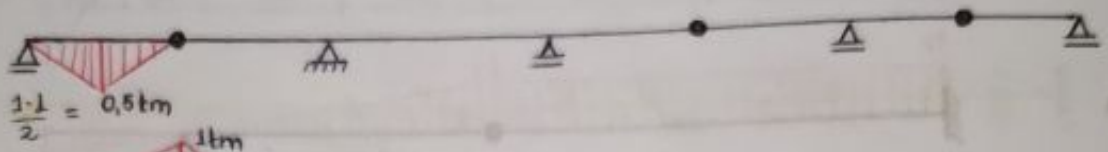
Hallar las siguientes Líneas de Influencia:



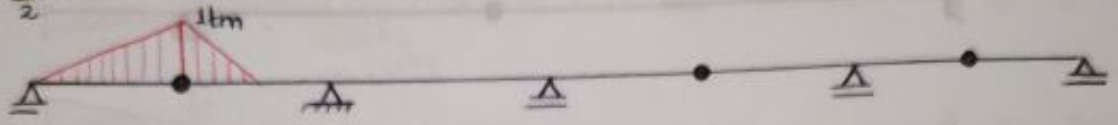
Calcular las siguientes Líneas de Influencia:



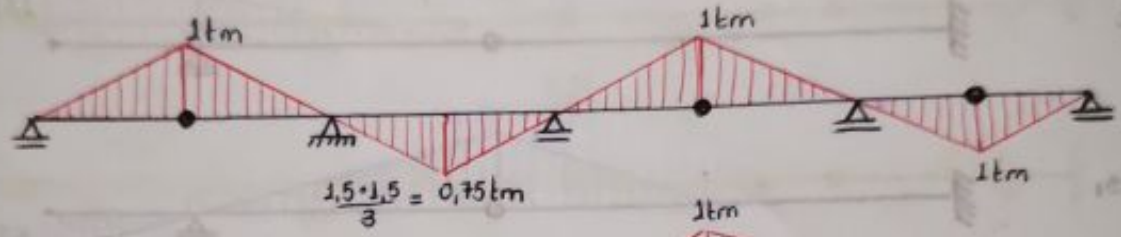
LI M_{S1}



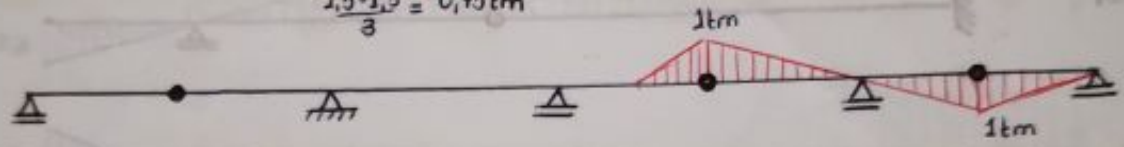
LI M_{S2}



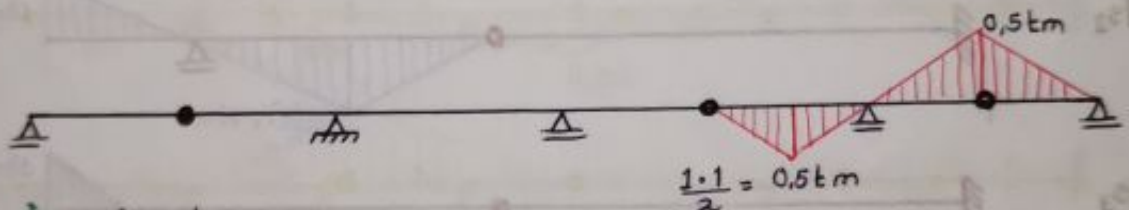
LI M_{S3}



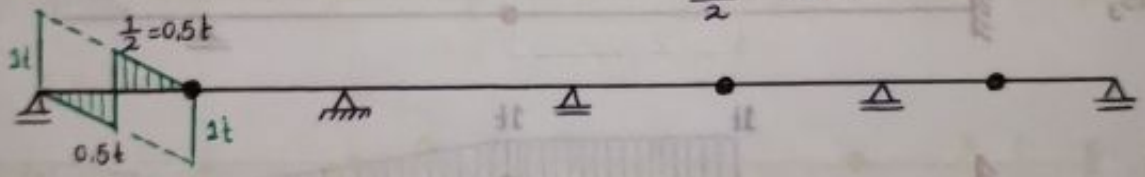
LI M_{S4}



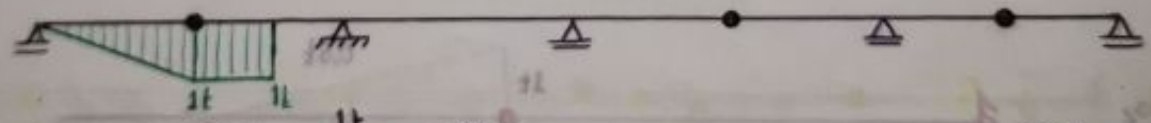
LI M_{S5}



LI Q_{S1}

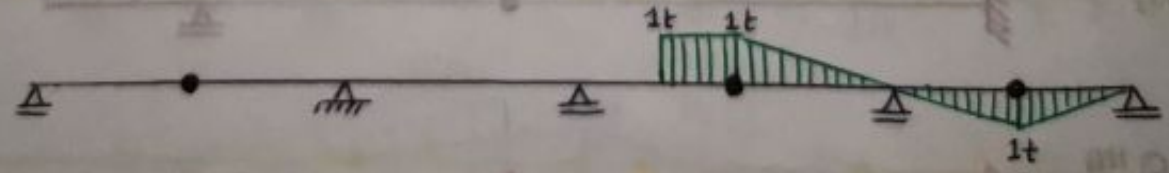


LI Q_{S2}

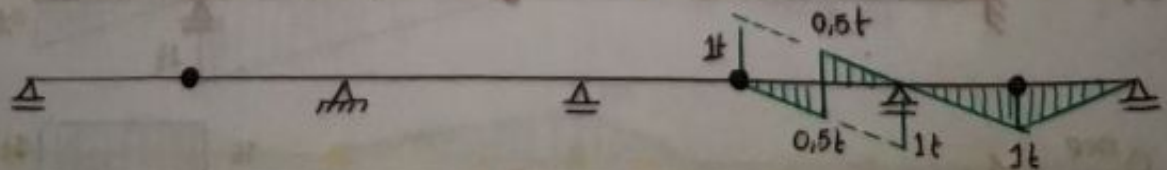


LI Q_{S3}

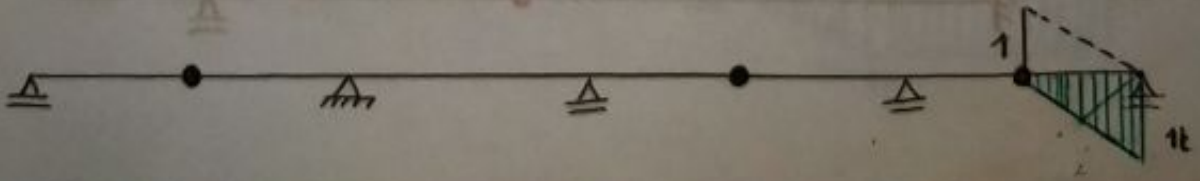
LI Q_{S4}



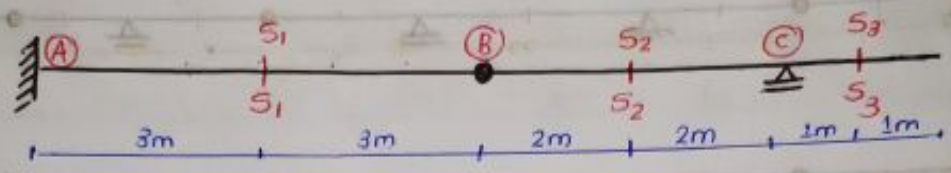
LI Q_{S5}



LI Q_H



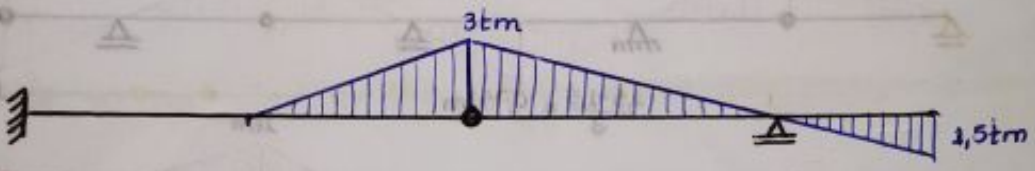
Hallar las siguientes Lineas de Influencia



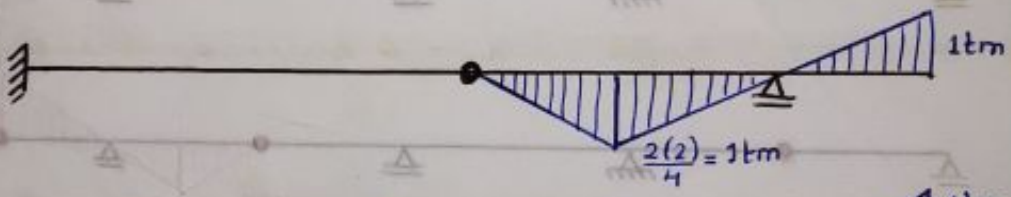
a) L.I. V_A

b) L.I. V_c

c) L.I. M_{S_1}

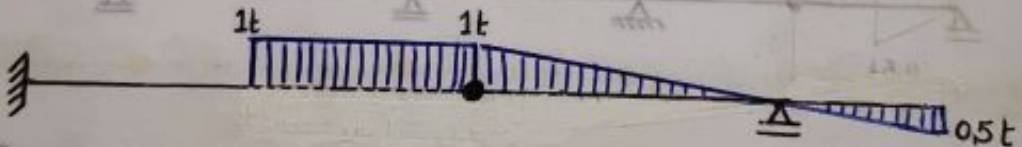


d) L.I. M_{S_2}

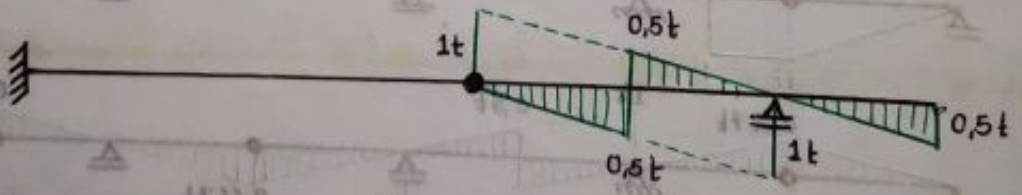


e) L.I M_{S_3}

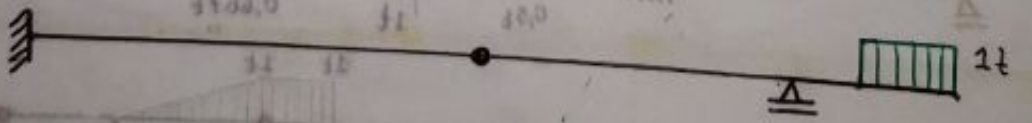
f) L.I O



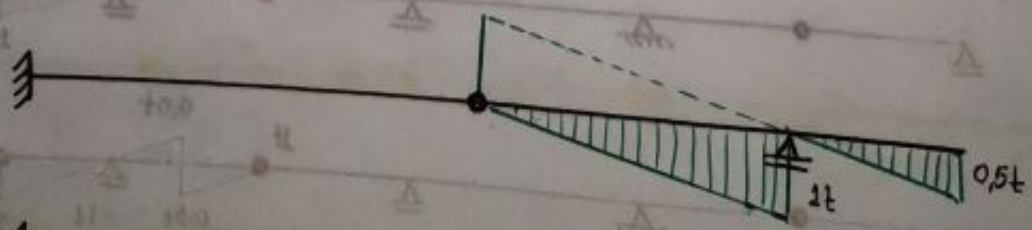
g) L.I. Q_{S_2}



h) L.I. Q_{S_3}

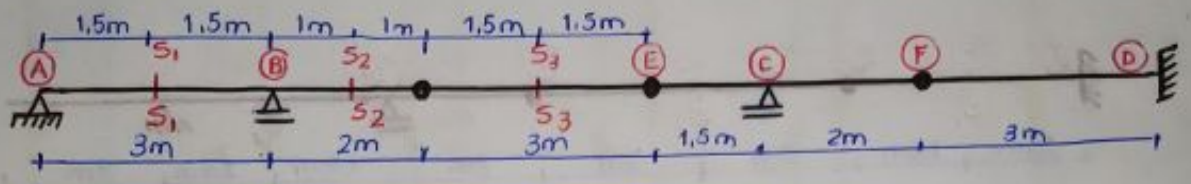


i) L.I. Q_c^{IZQ}



j) L.I. Q_c^{DER}

Hallar las siguientes Lineas de Influencia

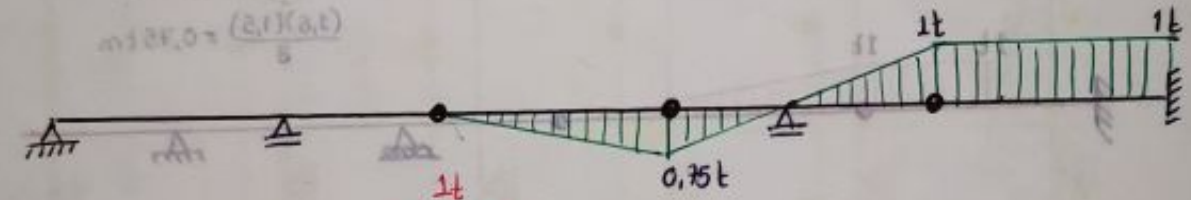


V_A

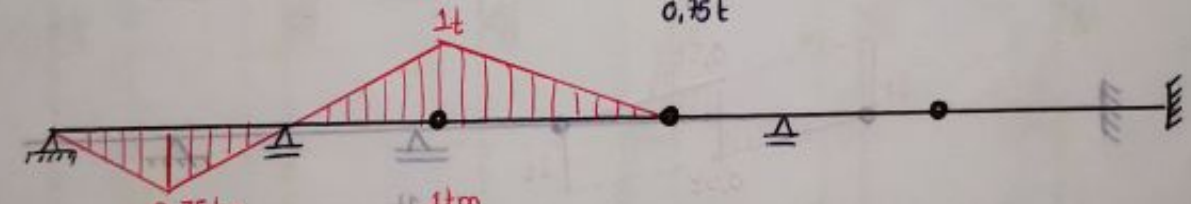
V_B

V_C

V_D

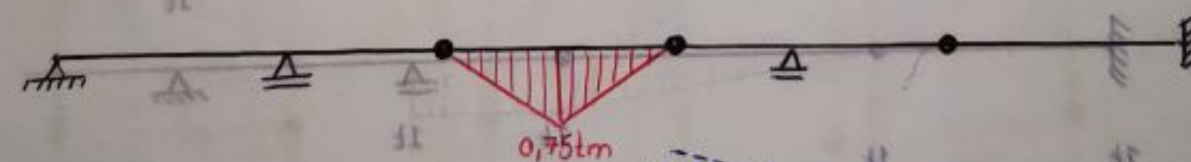


M_{S_1}

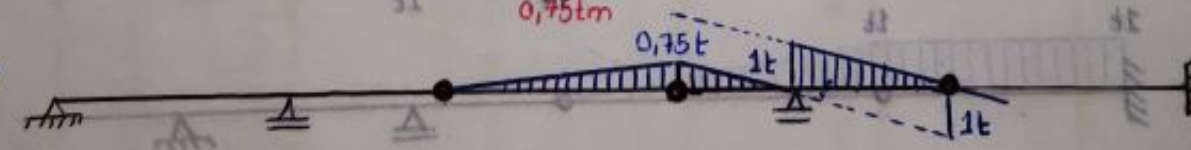


M_{S_2}

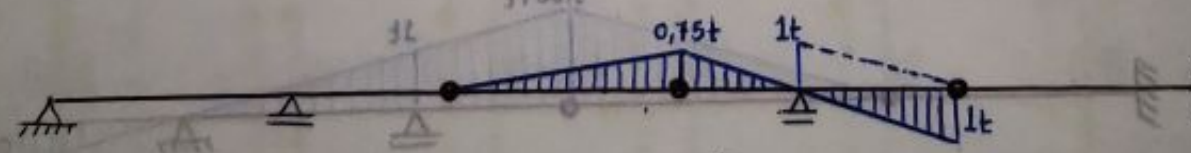
M_{S_3}



Q_C^{DER}

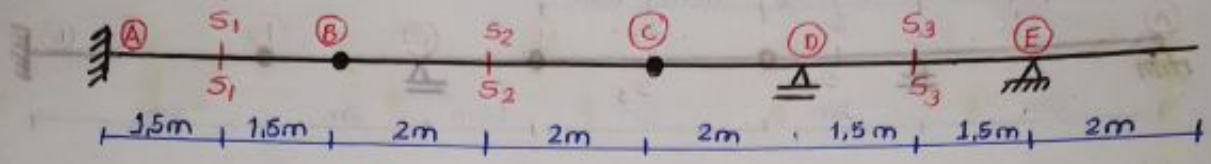


Q_F



Q_D

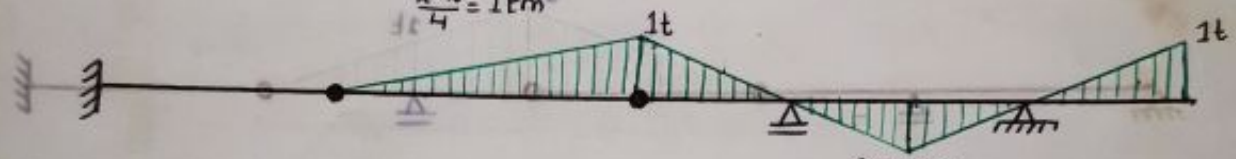
Diagramar las siguientes Lineas de Influencia



M_{S_1}

M_{S_2}

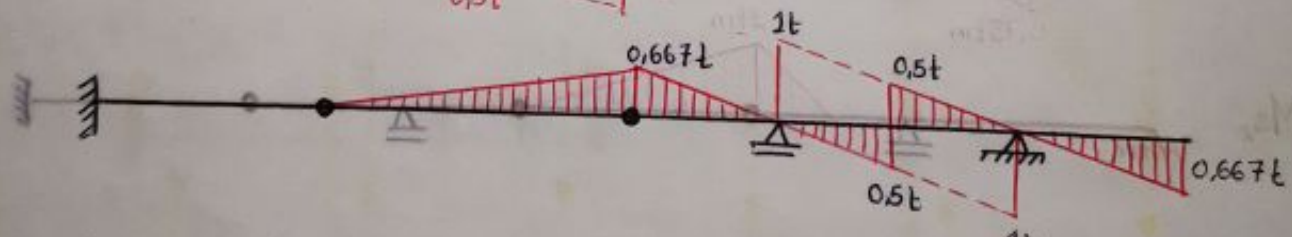
M_{S_3}



Q_{S_1}

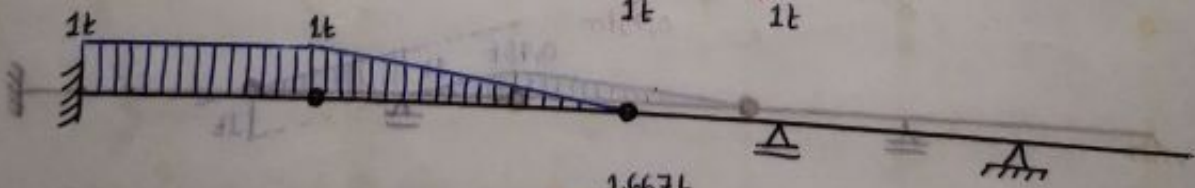
Q_{S_2}

Q_{S_3}

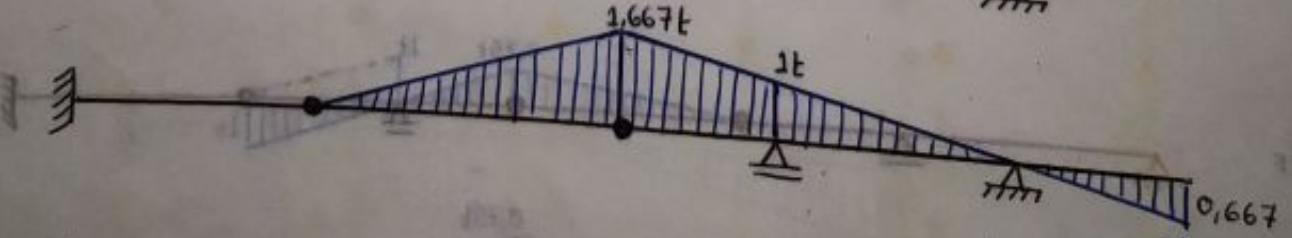


Q_D^{IZQ}

V_A



V_D



V_E